G-15 TECHNICAL BULLETIN • EQUIPMENT SERIES

ACCESSORY DA-1
DIGITAL DIFFERENTIAL ANALYZER
FOR THE
BENDIX G-15 COMPUTER

DIGITAL DIFFERENTIAL ANALYZER DA-1

Purpose of Accessory

The Digital Differential Analyzer, Model DA-1, is designed to operate in conjunction with the Bendix G-15 General Purpose Digital Computer. It permits the simple programming of the numerical solution of problems which can be expressed in the form of ordinary differential equations. These include the solutions of linear and non-linear ordinary differential equations, linear and non-linear simultaneous equations, solutions for roots of transcendental equations, and the simulation of real systems.

In order to solve differential equations on a general purpose computer without a differential analyzer attachment, it is necessary to transform the equation into suitable arithmetic form by use of techniques of numerical analysis and then to formulate coded instructions for the computer. The programming time required is extensive and the programming knowledge required is specialized and often sophisticated. By use of Accessory DA-1, the differential equations, in a slightly altered form, may be placed in the computer and their digital solutions realized. Since the solution is digital, much greater accuracy may be obtained than is possible from earlier-designed differential analyzers, which are of an analog nature.

Background

The basic principles of a differential analyzer for the solution of differential equations are not new - they were conceived by Lord Kelvin 100 years ago, described by Sir William Thomson 80 years ago, and put into practical mechanical form by Dr. Vannevar Bush 30 years ago. But the means of execution embodied in the DA-1, the use of digital techniques and the association with a general purpose computer, permit an accuracy, versatility, and facility which greatly extend the range of usefulness of this type of machine.

The basic computing unit in the differential analyzer is the "Integrator". An integrator may be thought of as a physical unit (electronic digital registers in the DA-1; mechanical shaft assemblies in the early Bush machine) which has two input lines and one output line. The units may be interconnected so that the output lines of some integrators become the input lines of other integrators.

The inputs are differentials of dependent and independent variables arising from the differential equation being solved; they may be called dy and dx respectively. The output of the integrator is a differential, dz, so that

$$dz = y\,dx$$

The value y, which is the accumulated value of the dy inputs, is held in a register within the integrator. Therefore, if the output line of the integrator is connected to the dependent variable input line of a second integrator, we may obtain in the second integrator

$$z = \int dz = \int y\,dx$$

Since differential equations usually involve constants, the output of an integrator may be multiplied by any arbitrarily chosen constant value before being used. We may, then, obtain with two integrators

$$dz = K\,y\,dx \qquad \text{(Equation 1)}$$

$$z = K \int y\,dx \qquad \text{(Equation 2)}$$

By programming interconnection of integrators of this nature and by selection of initial values for the "y" term in each, any ordinary differential equation may be solved.

Method of Solution

A frequently-used approach is to isolate the highest-order derivative in terms of the other variables. The existence of this derivative in an integrator is assumed and by repeated integration is used to provide all the lower-order derivatives. The lower-order derivatives are then combined according to the conditions of the equation to find the highest-order derivative. The resulting closed loop may then be put in the computer in coded form and solved. An illustration of the technique is given in the example below.

Example

The equation to be solved is:

$$\frac{d^2v}{du^2} + .297\,uv^2\,\frac{dv}{du} - u^2v + \sin u = 0$$

Isolating the highest-order derivative:

$$\frac{d^2v}{du^2} = -.297\ uv^2\ \frac{dv}{du} + u^2v - \sin u$$

By integrating once with respect to u:

$$\int \frac{d^2v}{du^2}\,du = -.297 \int uv^2\ \frac{dv}{du}\,du + \int u^2v\,du - \int \sin u\,du$$

or

$$\frac{dv}{du} = -.297 \int uv^2\,dv + \int u^2v\,du + \cos u$$

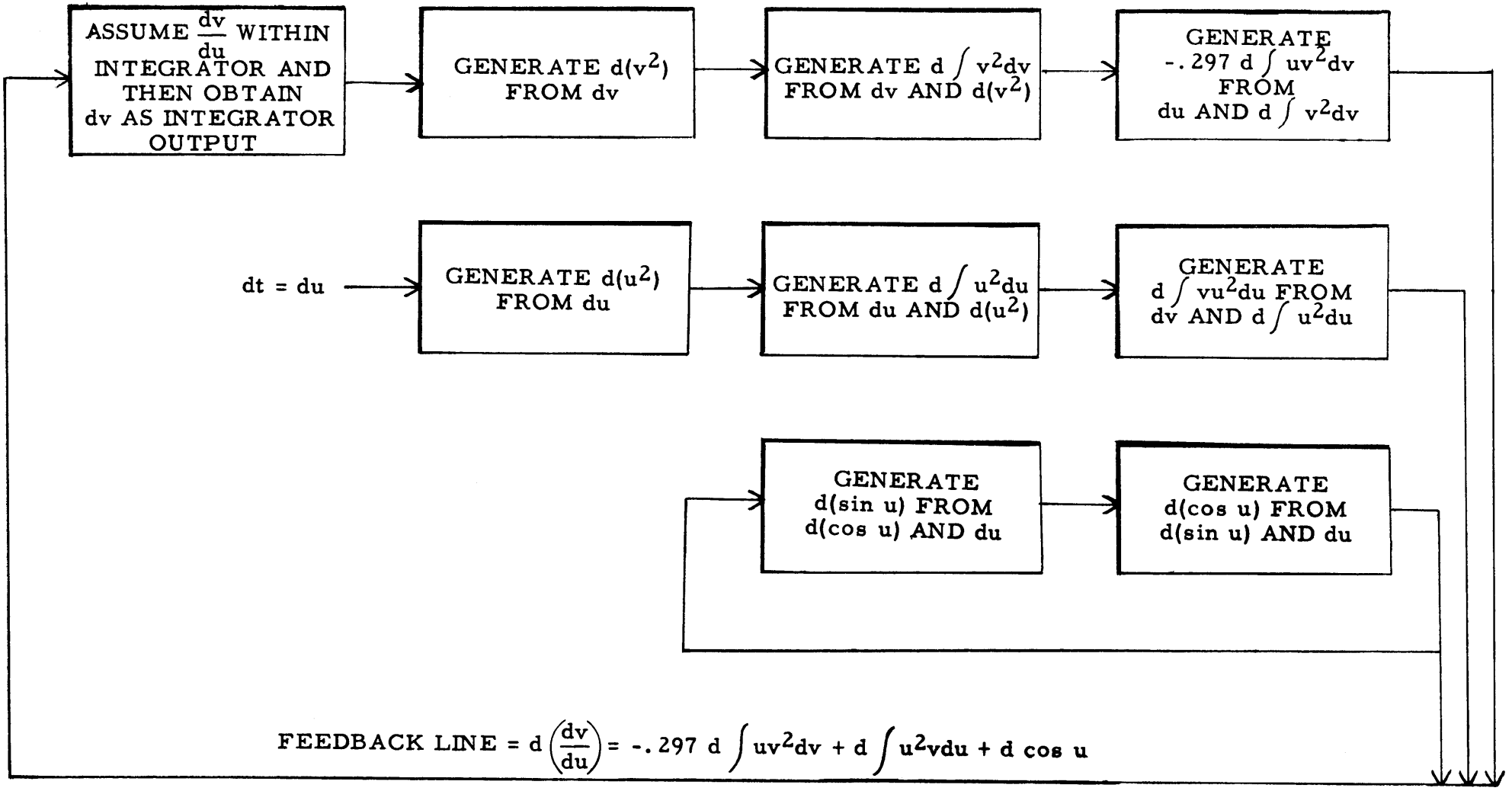
This equation may be put in differential form:

$$d\frac{dv}{du} = -.297\ d\int uv^2\,dv + d\int u^2v\,du + d\cos u \quad \text{(Eq. 3)}$$

Since the input and outputs of integrators are differentials, Equation 3 now describes a relationship between integrators. A machine independent variable called dt is available in the DA-1. This variable provides du. $\frac{dv}{du}$ is assumed to exist in the y register of one integrator. By development from $\frac{dv}{du}$ and du, the right-hand terms in the equation may be obtained as the outputs of integrators. These outputs may then be combined in the manner of Equation 3 to form $d\frac{dv}{du}$. By making $d\frac{dv}{du}$ the dependent differential input into the integrator in which $\frac{dv}{du}$ was assumed to exist, the feed-back loop is closed and the layout of the solution is completed.

The procedure is summarized in block form on the following page. Each block represents one integrator and each functions in the manner of Equations 1 and 2. The solution of this problem is described in detail in the DA-1 Programming Manual.

The diagram of the solution is then put in coded form and inserted into the computer. By use of a standard



METHOD OF SOLUTION FOR EQUATION $\frac{d^2v}{du^2} + .297\, uv^2\frac{dv}{du} - u^2v + \sin u = 0$

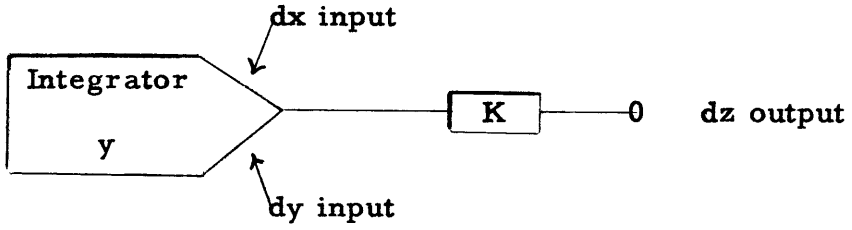
EACH BLOCK REPRESENTS ONE INTEGRATOR

program preparation routine, the initial values of the "y" terms in the integrators and the values of the constant multipliers may be expressed as decimal numbers. The interconnection paths of the integrators, scale factors, and operating instructions are specified numerically in the program.

Since Accessory DA-1 provides 108 integrators and 108 constant multipliers (a much greater number than heretofore available in digital differential analyzers), it may be used to obtain numerical solutions of complex problems involving high-order differential equations or sets of simultaneous equations.

A Complete Example

An example of the complete solution of a problem is presented in another technical bulletin entitled: "Evaluation of the Fresnel Integrals by the G-15 Computer with Accessory DA-1." The bulletin shows the flow diagram of integrator interconnection, the coding sheet for the computer obtained from the flow diagram, and the computed results. In reading this bulletin, note that an integrator and its constant multiplier are represented in a standard form:



$$dz = K\,y\,dx$$

Note also that each numerical value is written as a floating decimal point number in which the exponent, in excess-fifty form, precedes the mantissa.

Characteristics

Distinctive features of the DA-1 are listed below.

There are 108 integrators and 108 constant multipliers provided.

The primary functions of an integrator are those described in Equations 1 and 2; but, an integrator may also be used as a servo, as a decision element, or for the addition of variables.

The outputs of all integrators are available to be used as inputs by every integrator. Interconnection is by numerical coding and not by patchboard.

During a single step of computation, an integrator receives dy and dx inputs and accumulates both dy and ydx products. Whenever the sum of ydx products exceeds a pre-assigned quantity, the integrator has a dz output. Accuracy and speed of computation have been increased by making the dz output ternary rather than binary; dz may assume a value of +one increment, 0, or -one increment.

An integrator may be coded to receive a dx input from one other integrator; it may be coded to receive dy inputs from any number of other integrators. The value of the dy input during one step of computation must range between -7 and +7 increments.

The set of 108 integrators is processed 34 times per second. The speed of computation is independent of the number of integrators used in the program. The speed of solution of a problem is inversely proportional to the degree of precision required.

The general computing facilities of the G-15 may be used in conjunction with the DA-1 for the programming of highly involved problems.

Empirical functions may be used.

A standard program preparation routine provides the following input-output characteristics:

Initial values of integrands and values of constant multipliers are expressed in decimal form as positive or negative floating decimal point numbers. Precision of computation remains optional. Up to 7 decimal digits of precision are available. Exponent range is from 10^{-4} to 10^{+9}. Computed results are typed out as floating point decimal numbers.

Physical and Electrical Specifications

The DA-1 is controlled from the G-15 General Purpose Computer. The DA-1 has no controls of its own and may not be used independently.

Input can be programmed from normal G-15 equipment. This includes typewriter, punched paper tape, punched cards and magnetic tape. Empirical function input may be from any G-15 input device. Output may be tabulated by normal G-15 equipment or plotted graphically.

The graph-plotter plots the relationships between any two variables in 0.01 inch increments on a standard, sprocketed roll of paper, 1 foot by 100 feet, at a rate of 34 increments per second. It is an optional accessory to the DA-1.

The DA-1 is 22 inches deep by 24 inches wide and is 60 inches high. It weighs 300 lbs. Power input is 1 kva, 110-120 volts, 60 cycles, single phase. Cooling is by internal forced air.

Left to right are pictured the G-15 Computer, the DA-1 Accessory, the input-output typewriter supplied with the computer, and the PA-3 x-y Plotter used for graphical output from the DA-1.

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