MOTION CONTROL

BY

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CONTENTS

I. Introduction

The use of microprocessors (MP) in motion control systems is expanding rapidly. Most control systems that are being designed now use MP to perform all or part of the control functions of the system. The main advantage of the MP over the design with discrete components is that the MP design results in fewer components. This translates into lower cost, reduced circuit size and improved reliability.

Another advantage of the MP is the flexibility it offers. The MP enables the user to perform complex control algorithms, such as nonlinear control functions, feedforward and adaptive control. All of which, result in higher performance.

The third advantage, regarding motion control systems, is that the MP has no component variations. The gain and the bandwidth of the filter, generated by the MP, do not drift. As a result, the designer does not need to allow additional design margins for component drift. This results in a less conservative design and higher performance.

Most motion control requirements can be performed by position control systems. As a result, position control systems are the most common motion control system type and they are described in this book extensively.

A second type of motion control systems, velocity control systems, can also be implemented by a MP. Such systems are described in Chapter 6.

II. Elements of Position Control Systems

The functional elements of the basic position control system are shown in Figure 2-1.

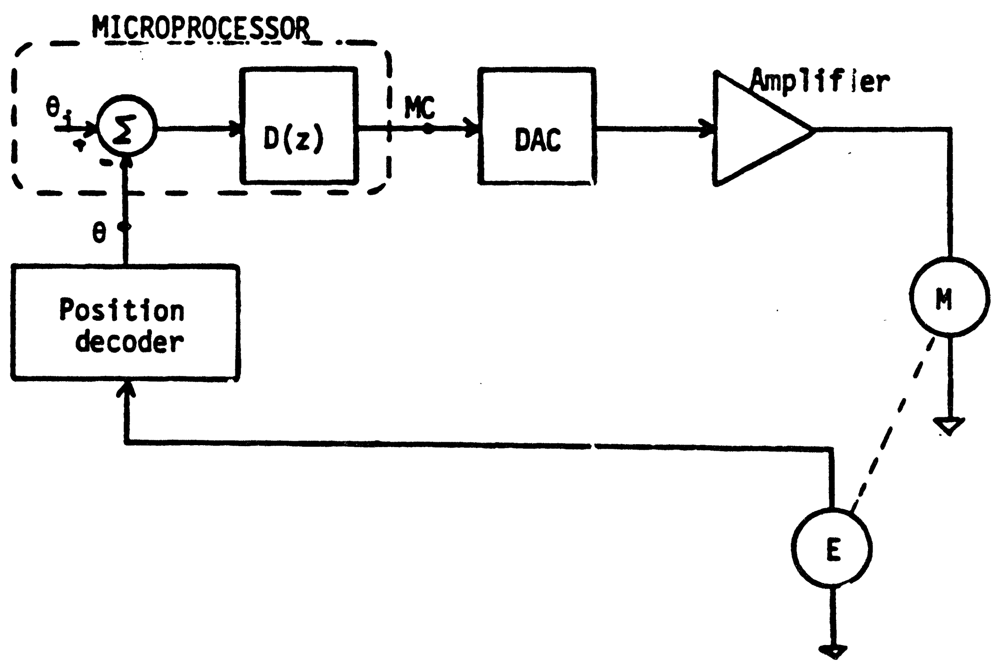

Fig. 2.1 Position control system with no velocity feedback

The motor is a dc motor and it is driven by an amplifier. The motor position is sensed by an encoder E. The encoder signals are applied to a special circuit, the position decoder, which construct the position feedback signal, θ . The microprocessor determines the deserved position, θ_i , and the position error, $\Delta\theta$, which is the difference between the desired position and the position feedback. The position error is then filtered by the microprocessor and the filter output is applied to the amplifier via a digital-to-analog converter (DAC).

The digital filter, which is generated by the MP is sufficient to create a stable motion control system.

In some applications where the requirements call for a very smooth velocity at low speeds, it may be necessary to add an analog velocity feedback. Such a feedback signal can be generated by a tachometer, as shown in Fig. 2-2, or it can be derived from the encoder signal, as will be described later.

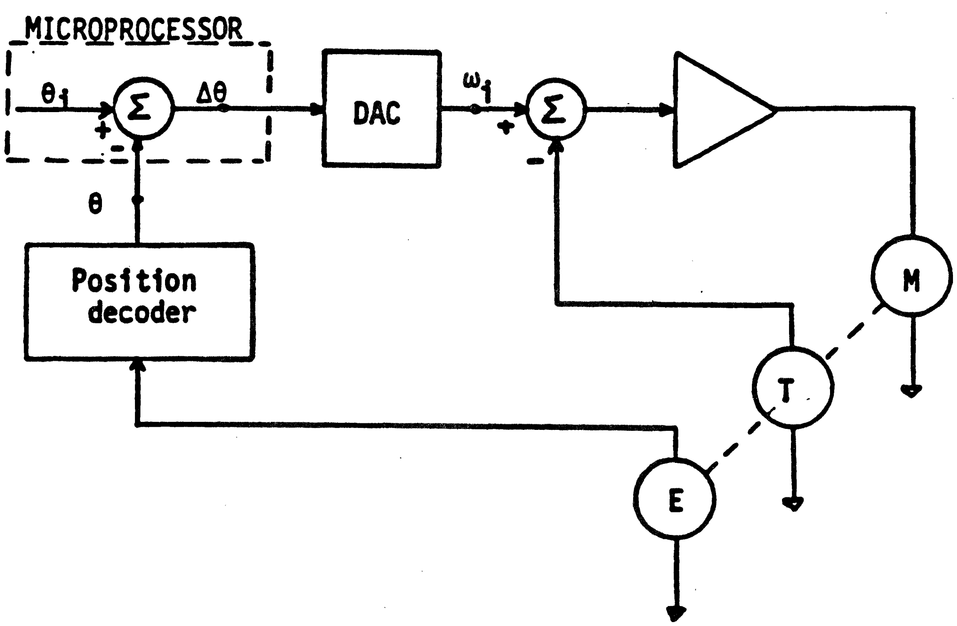

Fig 2-2 Position control loop with velocity feedback

The system elements can be listed according to their functions:

controller

position feedback

DAC

motor

amplifier

velocity feedback

A detailed discussion of the system elements follows.

Controller

The controller is the "brain" of the motion control system. Its function is performed by the MP. The controller performs the following functions:

- Receive commands and reports status
- Generates the function of the desired position $\theta_i(t)$
- Reads the position feedback, θ, and determines the position error.
- Performs the digital filtering
- Outputs the motor command.

The functional elements of the controller are illustrated by Fig. 2.3.

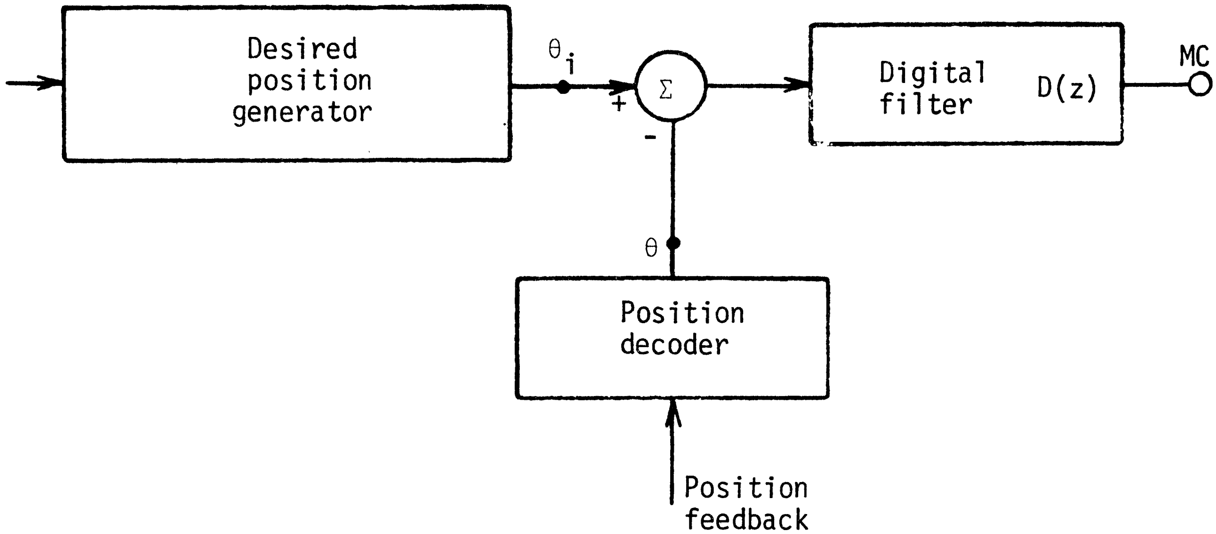

Fig. 2.3 Functional elements of the controller

The controller is often commanded to move to a final position, θ_f. It is the task of the controller to generate a smooth trajectory θ_i (t) that leads from the current position to the final position θ_f .

For example, let the required rotation be $\theta_f = 10$ rad. In addition, the acceleration and the slew rate are specified as 10,000 rad/s^2 and 200 rad/s respectively. The resulting velocity profile is shown in Fig 2-4a and the corresponding position is shown in Fig 2-4b. The desired position function, θ_i, is the desired trajectory for the motor.

Once the desired position is known at all times, the controller can read the actual position, θ , and compare it to the desired position. The difference between the two positions is called the position error. It is desirable to keep the position error to a minimum at all times. This guarantees that the motor follows the desired trajectory closely.

In order to close the position loop, the position error is

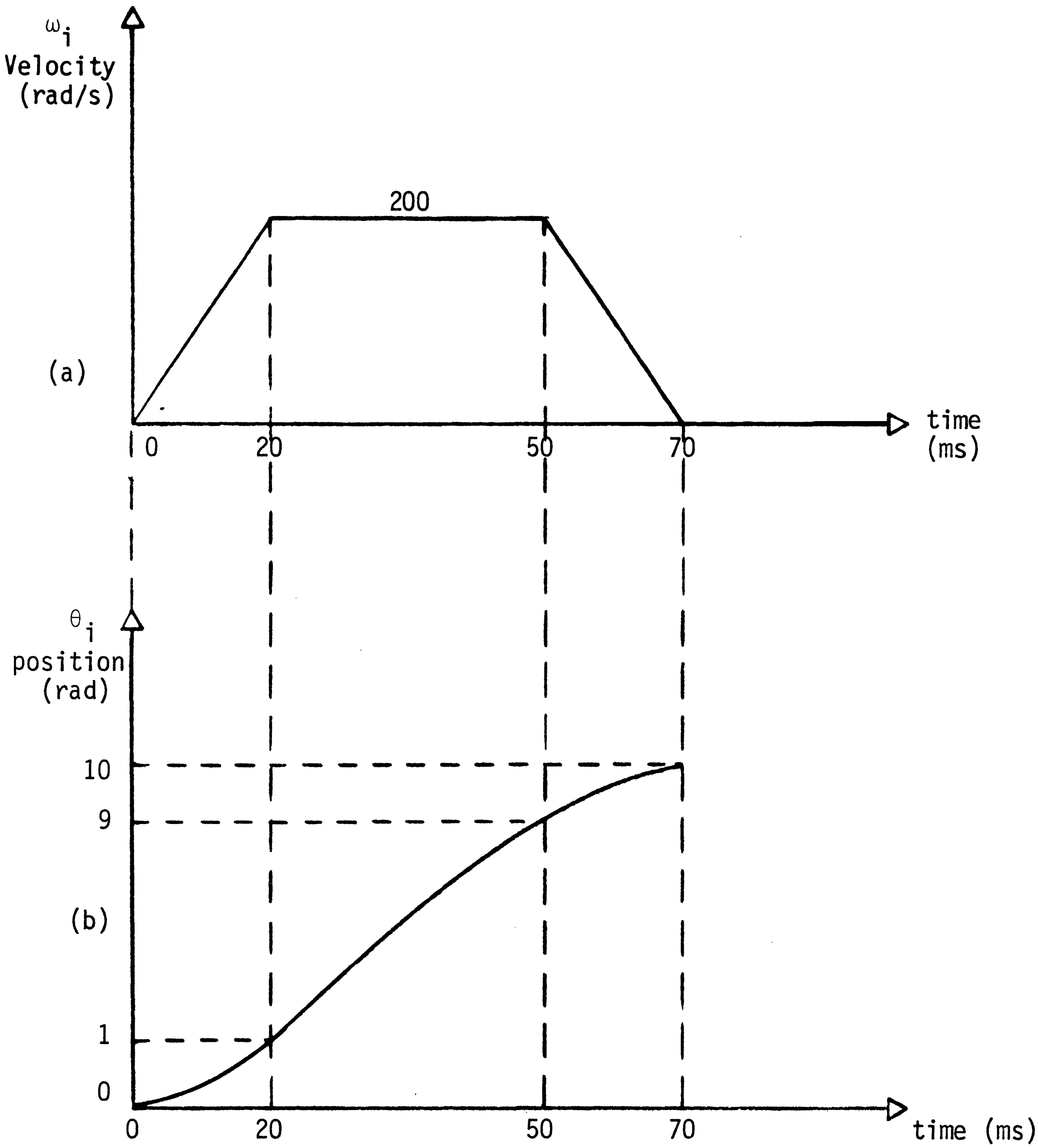

Fig. 2.4 The desired velocity ω_i (a) and the corresponding position θ_i (b). Note that θ_i is the integral of ω_i.

amplified and filtered. The transfer function of the digital filter is denoted by D(z). The method for generating D(z) is discussed in detail in Chapters 3 and 4.

In order for the MP to perform the required controller functions, it needs to have several features. The most important feature is the ability to multiply. This is required, if the motor has to perform the digital filtering within a reasonable time (less than 1 ms).

Another feature of the MP is whether it is an 8-bit or 16-bit device. For most motion control systems, an 8-bit MP is sufficient. However, if the system requires some advanced algorithms, such as adaptive control or high order filters, a 16-bit MP is necessary.

The requirements for memory, ROM and RAM depends on the system complexity. A basic control system can be implemented with 1 K of ROM and 64 bytes of RAM. Additional requirements need more memory.

Position feedback

The most common device for position feedback is the position encoder. The reason for the popularity of the encoder is its relatively low price. In order to decode the motor motion in the two directions, it is required to use an encoder with two channels in quadrature. The two signals, denoted as channels A and B are illustrated in Fig. 2-5. The two channels are shifted by 90 electrical degrees. As a result, if the motor moves forward, Channel A leads Channel B, and vice versa when the motion is in reverse. An encoder is characterized by the line density, N . both channels A and B generate N pulses per revolution. Therefore, if N = 360, one period of Channel A equals one mechanical degree.

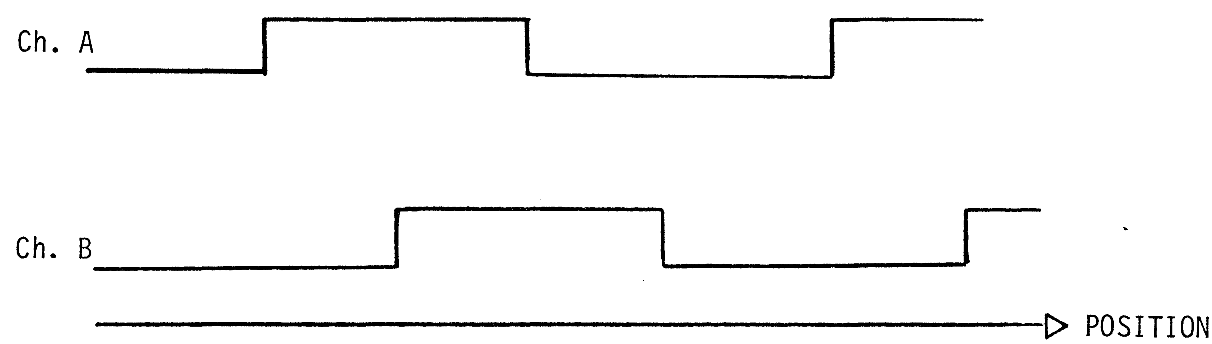

Fig. 2.5 Encoder signals

Since channels A and B are shifted by one-quarter of a cycle, it is possible to divide each encoder cycle into four quarters, these smaller increments are called quadrature counts. When the encoder has N lines per revolution, it is possible to increase its resolution to 4N quadrature counts/revolution. Since the increase in effective counts improves the system performance, it is advised to use this method of increased resolution. In the following discussions, it is assumed that the increased resolution is always used. In order to feed the position signal back to the MP, it is necessary to decode the position signal. This is done by the position decoder.

The position decoder includes two parts, the direction decoder and an up-down counter. The elements of the position decoder are shown in Fig. 2-6.

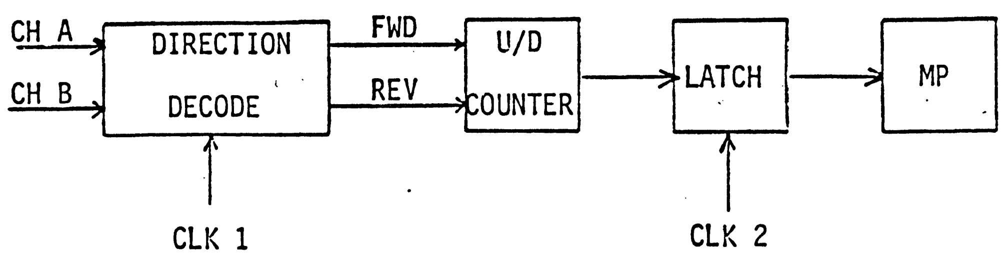

Fig. 2.6 Position decoder

The most effective method to decode the direction of the motion is by a ROM-based circuit, as shown in Fig 2-7.

The two encoder signals A and B are applied to D flip flop to generate the current value and the previous values of A and B. The combination of the four variables define an address of the PROM. The data of the PROM is shown in Table 2-1.

To illustrate the operation of the system, consider Fig 2-8. When the motion is FWD, Channel A leads Channel B. The signals New A and old A are shown in the diagram.

Similarly the signals new B and old B are illustrated. The

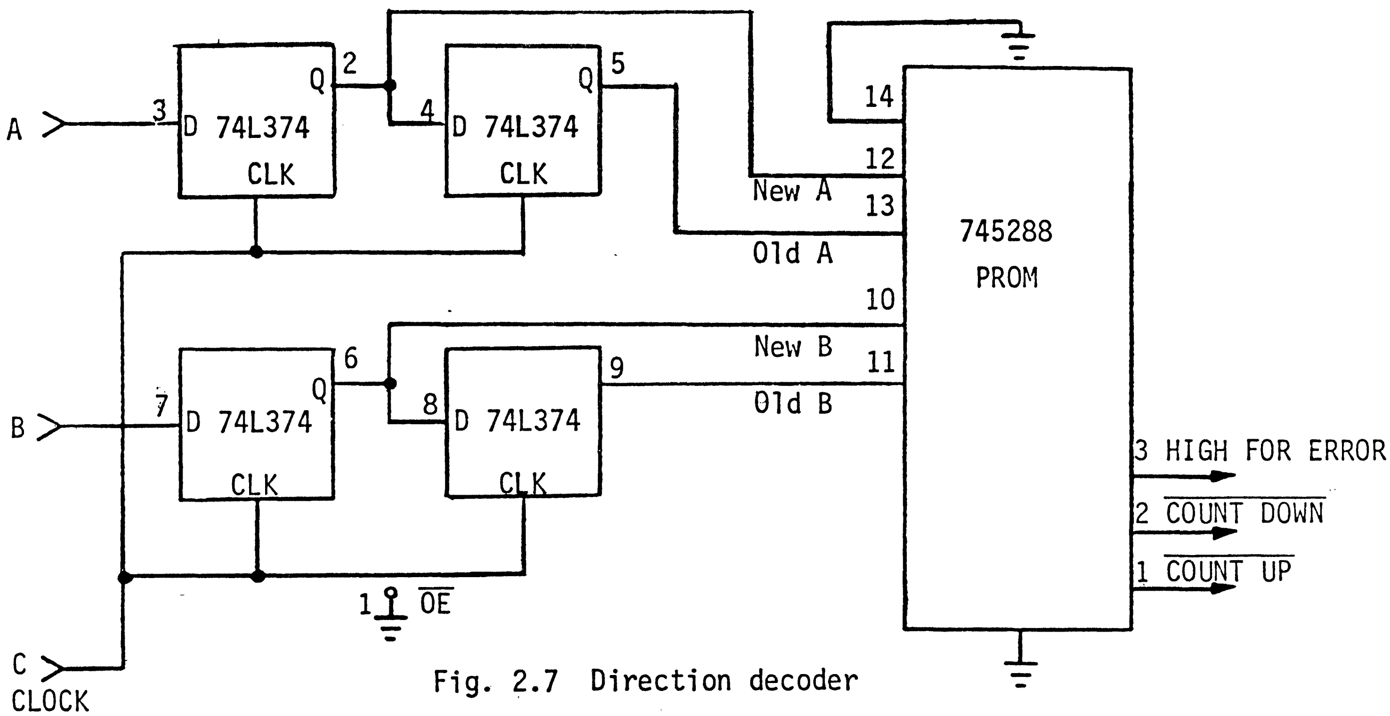

Fig. 2.7 Direction decoder

ADDRESS	DATA	INTERPRETATION
0	.03	NO COUNT
1	01	DOWN
2	02	UP
3	03	NO COUNT
4	02	UP
5	FF	ERROR
6	FF	ERROR
7	01	DOWN
8	01	DOWN
9	FF	ERROR
A	FF	ERROR
B	02	UP
C	03	NO COUNT
D	02	UP
E	01	DOWN
F	03	NO COUNT

Table II.1 PROM Program

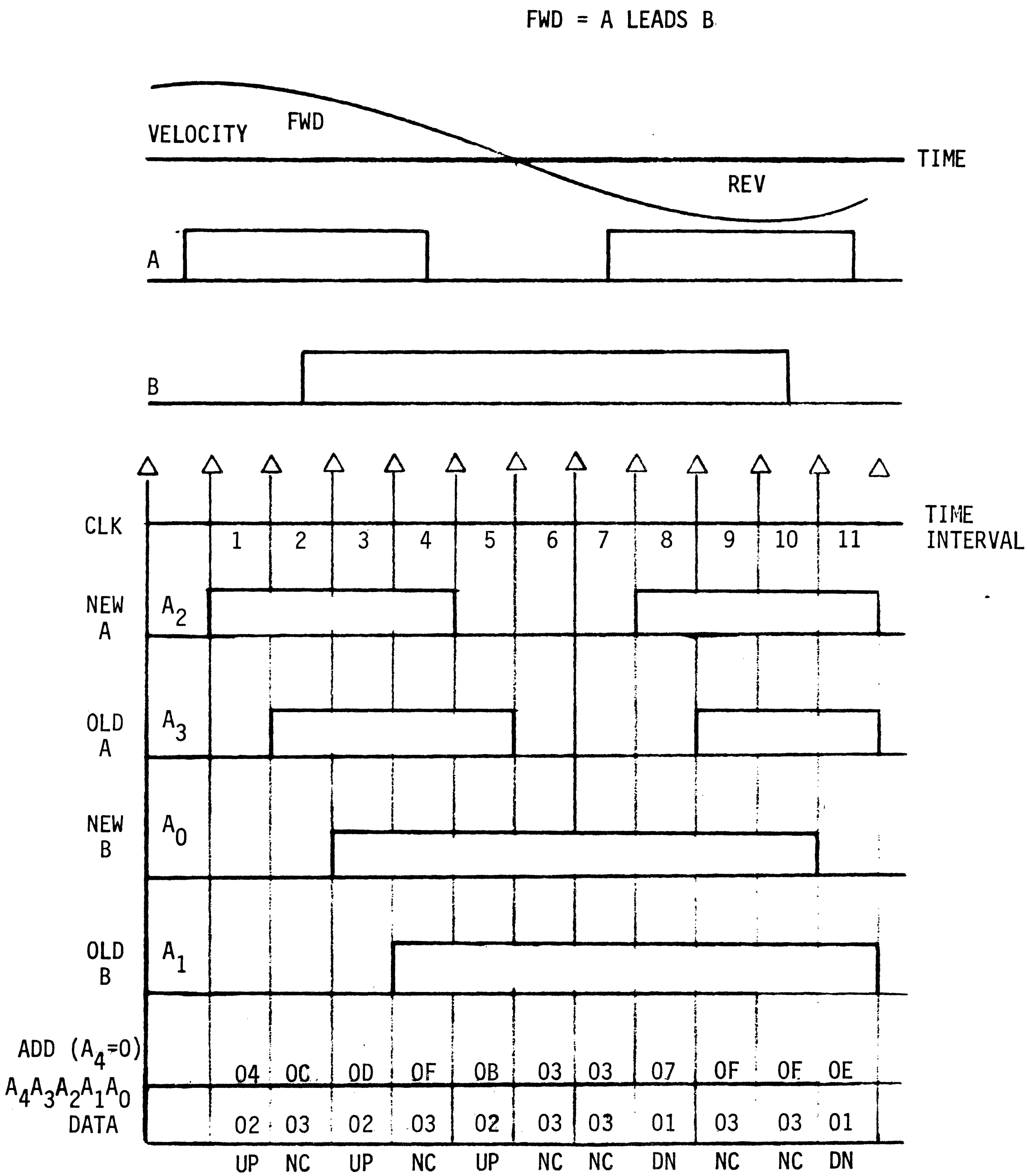

Fig. 2-8 Direction decoding signals

address of the PROM is defined by 5 bits $A_4\ A_3\ A_2\ A_1\ A_0$. Where

$A_4 = 0$

$A_3 =$ old A

$A_2 =$ new A

$A_1 =$ old B

$A_0 =$ new B.

For example, during the time interval 1, the values of $A_4\ A_3\ A_2\ A_1\ A_0$ are 00100. This defines an address 04, and at that address the data is "count up." This is in response to the leading edge of Channel A.

The output of the PROM are applied to an up-down counter, usually of 8 bits. This counters output is then applied to a latch before it is read by the MP.

The position decoder circuit is also available as an IC, GL-1000. For more information, see the product catalog at the end of the book.

The output of the counter is read by the MP and the position is decoded.

In order to discuss the position decoding process in specific terms assume that the up-down counter has 8 bits. As a consequence, the counter can count between 0 to 255 and it can overflow.

Suppose now that the MP reads the counter every T seconds. A typical value for T is 1 ms. Also assume that the maximum motor velocity is f revolutions/s, and that the encoder line density is N lines per revolution.

The maximum rotation of the motor in T seconds is fT revolutions, and since each revolution equals 4N quadrature counts, the total change in the counter is limited to 4NfT.

Next suppose that the limit 4NfT is less than 128. This condition is required for correct encoder reading.

To illustrate the principle of the position decoding, suppose that the counter output was read as 20, 40, 240 in 3 consecutive readings. During the first interval, the change from 20 to 40 could be caused by a FWD motion of 20 counts, or by a REV motion of 236 counts. Since the total change is limited to 128 counts, only the possibility of 20 counts FWD holds. For similar reasons, the motion during the second interval is in the REV direction by 56 counts.

The algorithm for the position decoding by the MP is as follows: Let the position feedback and the counter output, during the k^{th} sample, be P(k) and C(k) respectively. Define the variable

$$D(k) = C(k) - C(k-1)$$

D(k) is the change in the counter value. D(k) is always a number between 0 and 255.

The algorithm for position decoding is

$$P(k) = \begin{cases} P(k-1) + D(k) & \text{if } 0 \leq D(k) < 128 \\ P(k-1) + D(k) - 256 & \text{if } 128 \leq D(k) \leq 255 \end{cases}$$

where P(k) is the position feedback sequence.

The requirement that the total counter change per sample, 4NfT, must be less than 128 is not very restrictive. For example, consider a system where N = 500 lines/rev and the maximum motor velocity is 3000 rpm, or f = 50 rev/s. Further, assume that the sample time is T= 1 ms. The resulting maximum change is

$$4NfT = 4 \cdot 500 \cdot 50 \cdot 10^{-3} = 100 < 128 .$$

When a higher speed is required, it is necessary to reduce the sample time or to use a counter with more than 8 bits.

DAC

The DAC performs the function of converting the digital output of the MP to an analog signal that can be used to drive the amplifier and the motor. The most common way of performing this output function is by a DAC integrated circuit. An alternative method for the DAC function is the generation of a pulse-width-modulated (PWM) signal. Such a signal may be switched between two values, and as such, it can be generated by the MP, or by some digital components.

A PWM signal switches at a fixed frequency and varies the width of the pulse according to the digital motor command. For example, when the motor command is zero, the duty cycle of the PWM signal is 50%. As the motor command increases toward 127, the duty cycle increases gradually toward 100%. Similarly, when the motor command is negative the pulse width becomes narrower, and when the motor command is -128, the pulse width is zero. Typical waveforms of a PWM signal are shown in Fig 2-9.

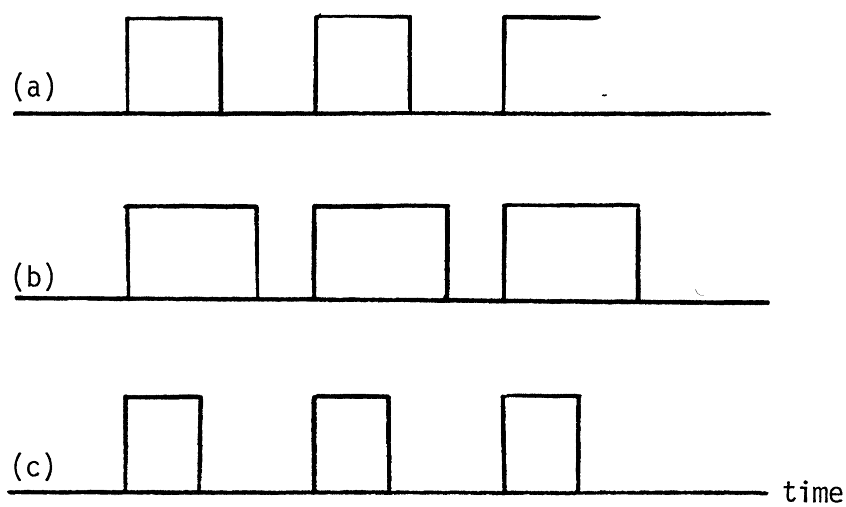

Fig. 2.9 Pulse-width-modulated (PWM) signals. (a) shows a 50% duty cycle when the motor command (MC) is zero. (b) and (c) correspond to positive and negative values of MC respectively.

Fig. 2-9 PWM Signals

a. MC = 0

b. MC $>$ 0

c. MC $<$ 0

In order to convert the PWM signal to an analog signal it is necessary to filter it. A typical low-pass filter is shown in Fig 2-10. It can filter a PWM signal which switches between 0 and 5V. The filter output is in the range of ±10V.

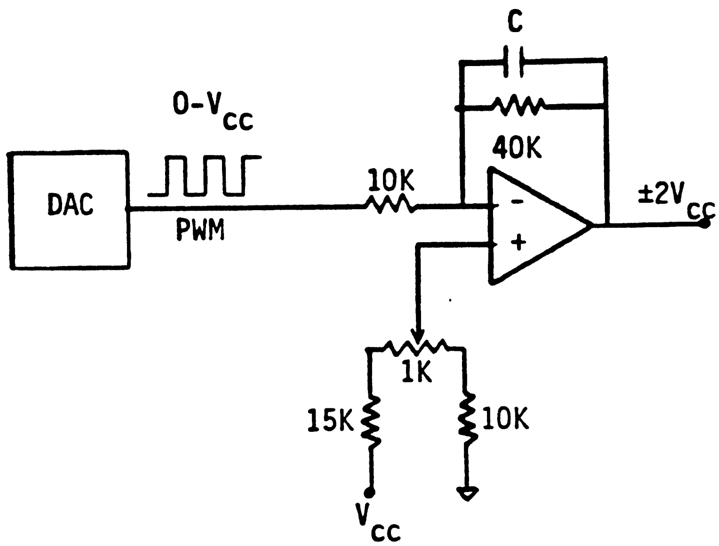

Fig. 2.10 Low-pass filter

The generation of the PWM signal can be done in the MP, or it can be done by external components. A typical circuit for the generation of the PWM signal externally is shown in Fig 2-11. Suppose that the counter is an 8-bit counter which continuously counts up. As a result, its output is a sawtooth between 0 and 255. The motor command is stored by the MP in the latch. The comparator compares the latch output to that of the counter. As long as the latch output is larger, the comparator output is high. For example, if the latch output is 128, the PWM is on 50% of the time. When the latch output increases, the duty cycle of the PWM increases proportionately.

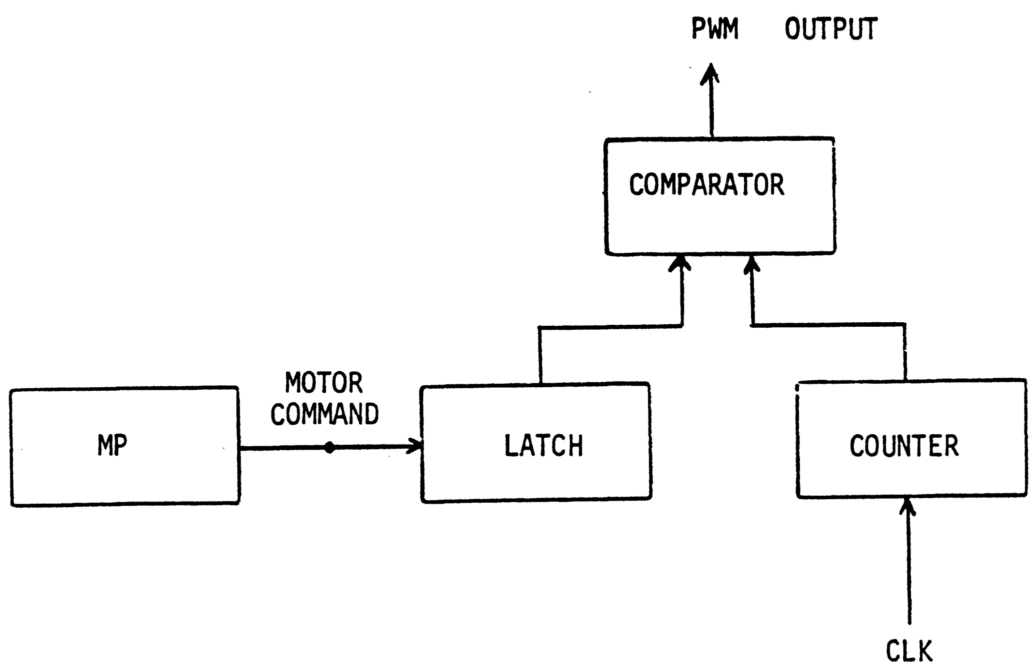

Fig. 2.11 Externally - generated PWM signal.

A PWM generating circuit, such as the one shown in Fig. 2-12, is included in the integrated circuit GL-1000, which is described at the end of the book.

Motor and Driver

A MP-based position control system can control motors of various types. The motor may be a brush-type dc motor, a brushless dc motor, an ac motor, a step motor or a hydraulic motor. The only requirement on the motor and the driver is that the driver should respond to an analog command. In order to be specific, the following discussions assume that the motor is a dc motor, unless specified otherwise. Although, most of the discussions apply equally to the other motor types.

<u>Velocity Feedback</u>

The most common velocity sensor is the tachometer. The tachometer generates an analog signal that is proportional to the velocity. The continuous velocity signal helps stabilize the system when the motor rotates at low speed, resulting in a smoother velocity.

The use of the tachometer, however, requires an additional component. This increases the cost of the system and requires additional space. Therefore, an alternative source for the velocity feedback is highly desirable.

Such an alternative source may be the velocity decoder integrated circuit SGS-L290 (second source SG-290). This device requires an encoder with analog outputs (sinusoidal or triangular) and it derives the velocity signal from the encoder waveform. The functional block diagram of the velocity decoder is shown in Fig. 2-12.

The operation of the velocity decoder is illustrated by Fig 2-13. The encoder signals are denoted by x and y. The two signals are differentiated by the external RC circuit. Later, the output signal is generated as

$$\text{output} = \frac{dx}{dt} \quad \text{sign } y - \frac{dy}{dt} \quad \text{sign } x \, .$$

The graphs of Fig. 2-13 illustrate the operation.

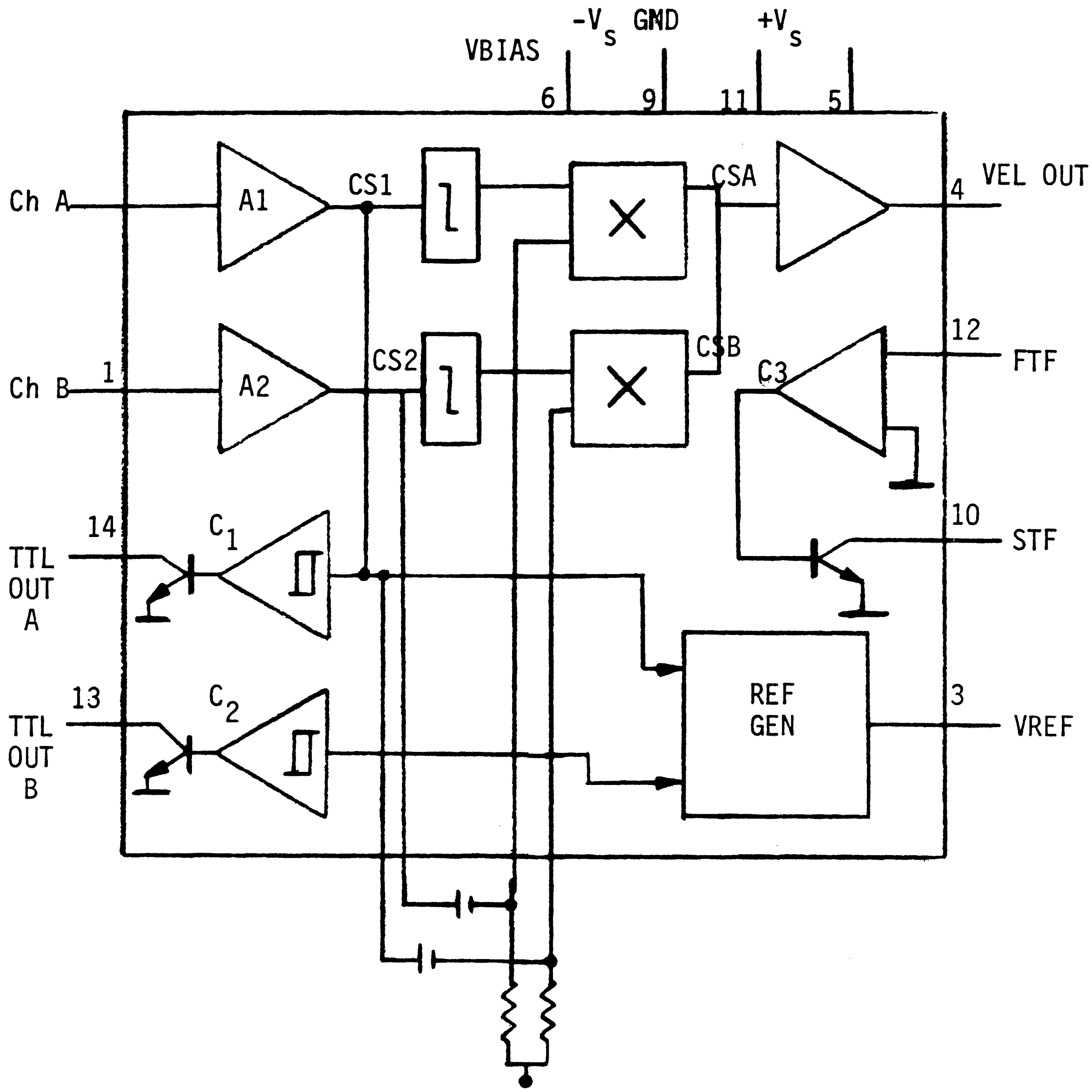

Fig. 2.12 Block diagram of the velocity decoder

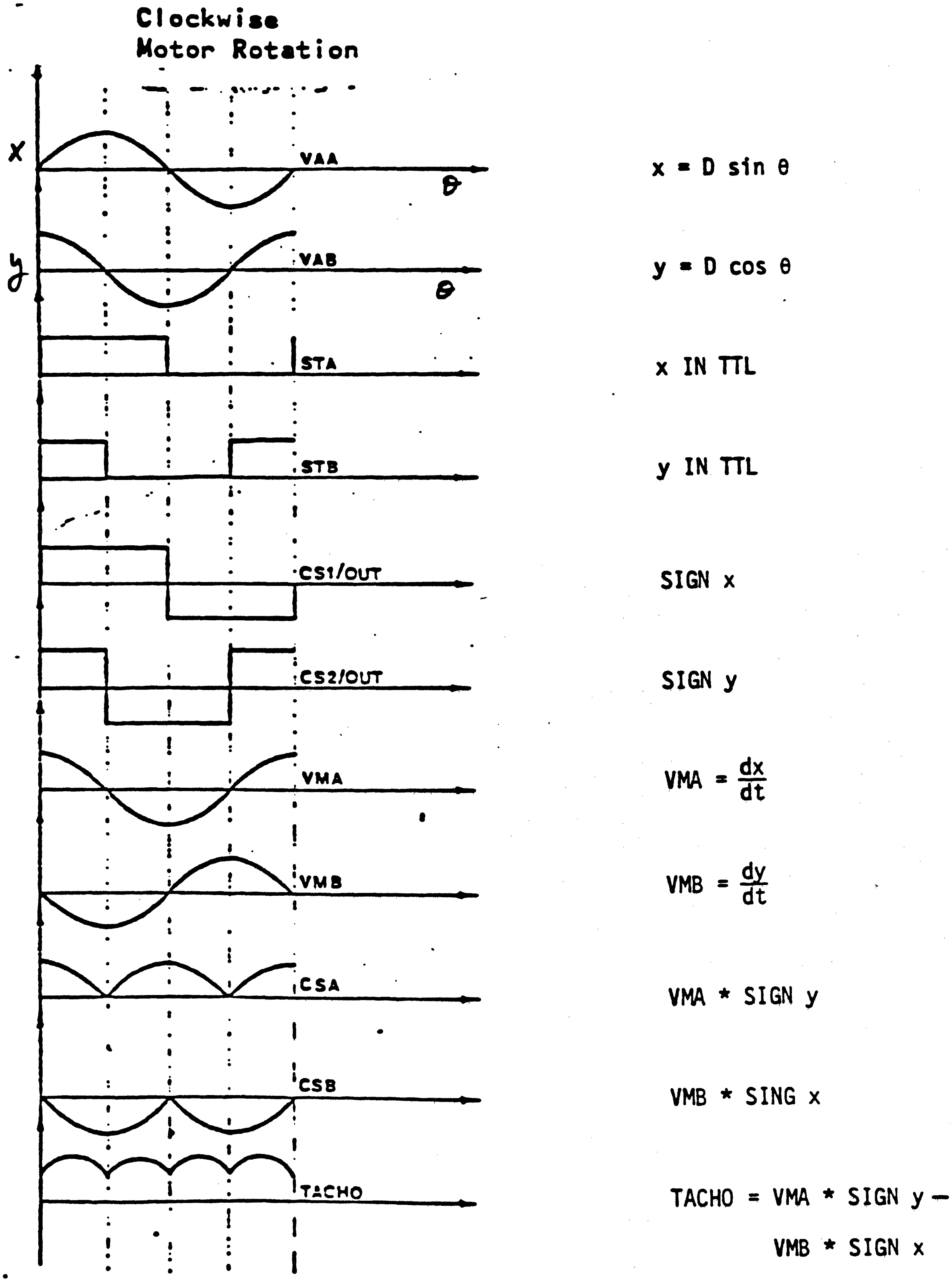

Fig. 2.13 Waveforms of the velocity decoder

III. Modeling Digital Position Control Systems

A digital position control system is represented by the block diagram of Fig 3-1.

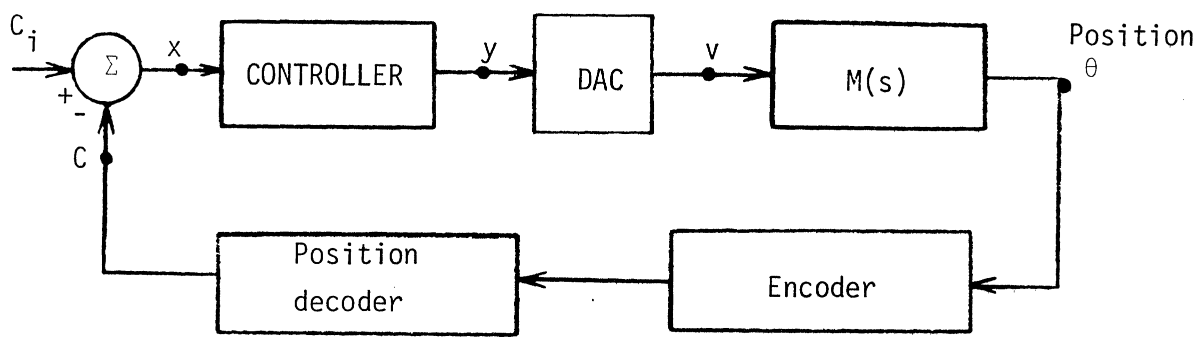

Fig. 3.1 Elements of the position control system

The desired position, expressed in encoder quadrature counts is C_i. This position is compared with the actual feedback, C, and the position error, X, is determined. The MP amplifies the position error, X, and it filters it. The output of the filter is then applied to the DAC which generates the motor command V. The motor and the drive are modeled together by the combined transfer function M(s). This is the transfer function between the motor command, V, and the angular position of the motor, θ. The motor angular position is sensed by the encoder which generates two signals in quadrature, channels A and B. These two signals are then applied to the position decoder, which generates the position feedback, C.

In order to clarify the discussion, the system parameters are listed below with a brief definition, and with their units.

C_i [quadrature counts]--desired position

C [quadrature counts]--feedback position

X [quadrature counts]--position error

Y [MP counts]--MP output

V [volt]--motor command

θ [radians]--motor angular position

Next we develop models for the system elements.

Motor-Driver

The motor under consideration may be of different types. The most common motors are the dc motors, both the brush type and the brushless type.

When a dc motor is driven by a voltage amplifier, the transfer function is

$$M(s) = \frac{K_m}{s(sT_m+1)(sT_e+1)} \left[\frac{\text{rad}}{\text{volt}} \right] \qquad (3\text{-}1)$$

The term s in the denominator indicates integration due to the fact that the position is fed back. The two time constants, T_m and T_e are the mechanical and electrical time constants of the motor. When the electrical time constant is short, in comparison with the system response time, it can be ignored. In that case, M(s) can be simplified to form:

$$M(s) = \frac{K_m}{s(sT_m+1)} \left[\frac{rad}{volt} \right] \tag{3-2}$$

When the motor is driven by an amplifier with current feedback, which acts as a current source, the corresponding transfer function becomes.

$$M(s) = \frac{K_m}{s^2} \left[\frac{rad}{volt} \right] \tag{3-3}$$

More information on the modeling of motors and amplifiers can be found in Ref [M.1-M.3].

DAC

The DAC generates a voltage V, which is proportional to Y, the MP output.

For example, if the DAC has 8-bits and its output is in the range of ±10V, the DAC gain is 20 volts for 256 counts. In general, if the DAC output voltage is within $\pm V_m$, and the DAC has n bits, its gain K_d is

$$K_d = \frac{2V_m}{2^n} \left[\frac{\text{volt}}{\text{count}} \right] \tag{3-4}$$

Encoder and Position Decoder

The encoder and the position decoder generate a count, C, which is proportional to the motor angular position θ. Let the encoder line density be N lines per revolution. Due to the quadrature sensing, the position decoder generates 4N counts per revolution. This corresponds to feedback gain, K_p, of

$$K_p = \frac{4N}{2\pi} \left[\frac{\text{count}}{\text{rad}} \right] \tag{3-5}$$

Controller

The controller function is performed by the MP. The MP reads the feedback signal, C, every T seconds, and compares it with the desired command, C_i, to form the position error

$$X(kT) = C_i(kT) - C(kT) \tag{3-6}$$

The controllers then filter X(kT) to generate Y(kT). The output Y(kT) is then applied to the DAC periodically. Since the output is held constant for a complete period, T, the output of the DAC is related to y(kT) by

$$V(t) = K_d\ y(kT) \qquad \text{for} \quad kT \le t < (k+1)T \qquad (3\text{-}7)$$

The effect where the signal is kept constant for a sampling period is called sample-and-hold or zero-order-hold (ZOH). The ZOH is modeled as a separate element, although it is performed by the MP.

Next we need to model the digital filter inside the MP. Digital filter algorithms are represented by difference equations. For example, consider the following difference equation

$$Y(kT) = 3X(kT) - 2X[(k-1)T] \qquad (3\text{-}8)$$

Since the difference equation is repeated at times that are multiples of T, it is not necessary to write T in every equation. Instead, we may write Eq (3-8) in the form

$$Y(k) = 3X(k) - 2X(k-1) \qquad (3\text{-}9)$$

In order to model Eq (3-9) as a transfer function we use the Z transform. The user may find information about the Z transform in Appendix A or in Ref [D.1] and [D.2]. At this point it is sufficient if we list one property of the Z transform. This property is known as the shifted-sequence property. It states: If the sequence f(k) transforms into the function F(z), then the shifted sequence, f(k-m), transforms into $z^{-m}F(z)$.

The shifted sequence property can be used to transform Eq (3-9). The sequences Y(k) and X(k) transform into Y(z) and X(z) respectively. The shifted sequence, X(k-1), transforms into $z^{-1}X(z)$. As a consequence, Eq (3-9) transforms into:

$$Y(z) = 3X(z) - 2z^{-1}X(z) \qquad (3\text{-}10)$$

Eq (3-10) allows us to find the ration between Y(z) and X(z). This ratio, D(z), is called the digital transfer function.

$$D(z) = \frac{Y(z)}{X(z)} = 3 - 2z^{-1} = \frac{3z-2}{z} \qquad (3\text{-}11)$$

The above procedure illustrates the method of generating a digital transfer function for any digital-filter algorithm.

Since all of the system elements are modeled separately, we can model the system by a block diagram. This is shown in Fig 3-2.

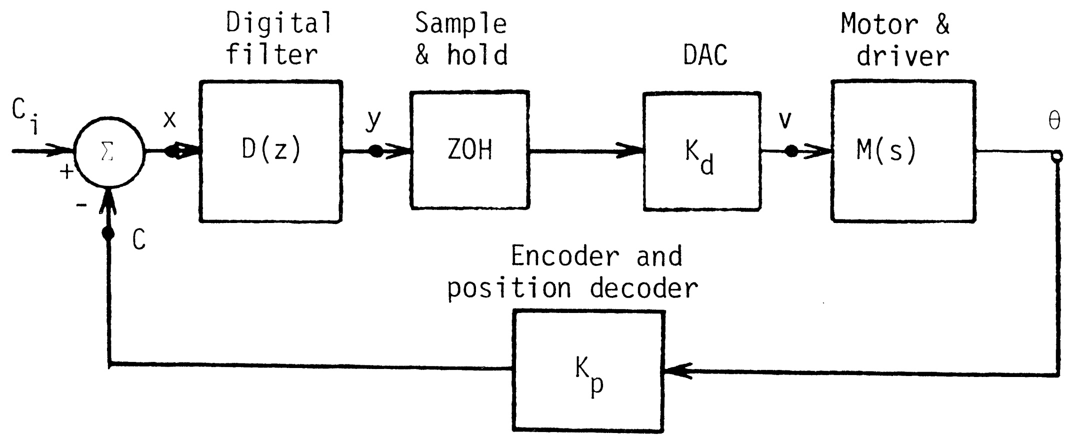

Fig. 3.2 Modeling the system elements

In order to illustrate the modeling process, consider the following example.

Example 3-1

A digital position control system, as illustrated by Figs 3-1 and 3-2 has the following parameters: The motor is a dc motor, which is driven by a voltage-source-amplifier. The gain of the motor-amplifier is such that one volt of input, V=1, results in a motor velocity of 10 rad/s. The mechanical time constant of the motor and the load is 100 ms, and the electrical time constant is negligible. The 8-bit DAC has an output of ±10 volts and the encoder has N=500 lines per revolution.

The controller sampling period is T=1 ms, and the digital filter algorithm is

$$Y(k) = 25X(k) - 16X(k-1) + 0.5Y(k-1) \qquad (3-12)$$

The information given above is sufficient for developing a model for the system. We start with the motor-amplifier combination. The model is as given by Eq (3-2) where

$$K_m = 10 \quad \frac{rad/s}{volt}$$

and

$$T_m = 0.1 \text{ s}$$

This leads to

$$M(s) = \frac{10}{s(1+0.1s)} \quad [\ \frac{rad}{volt}\ ] \qquad (3\text{-}13)$$

The model of the DAC can be found from Eq. (3-4)

$$K_d = \frac{20}{256} = 0.078 \ [\ \frac{volt}{count}\ ] \qquad (3\text{-}14)$$

The position feedback gain, K_p, is found from Eq. (3-5)

$$K_p = \frac{4 \cdot 500}{2\pi} = 318 \left[\frac{\text{count}}{\text{rad}} \right] \tag{3-15}$$

In order to model the digital filter, apply the z transform to Eq. (3-12)

$$Y(z) = 25X(z) - 16z^{-1}X(z) + 0.5z^{-1}Y(z) \tag{3-16}$$

This can be written as

$$(1-0.5z^{-1})\ Y(z) = (25-16z^{-1})\ X(z) \tag{3-17}$$

or

$$(z-0.5)\ Y(z) = (25z-16)\ X(z) \tag{3-18}$$

The resulting transfer function is

$$D(z) = \frac{Y(z)}{X(z)} = \frac{25z - 16}{z - 0.5} \tag{3-19}$$

The system model can now be represented by the diagram of Fig 3-3.

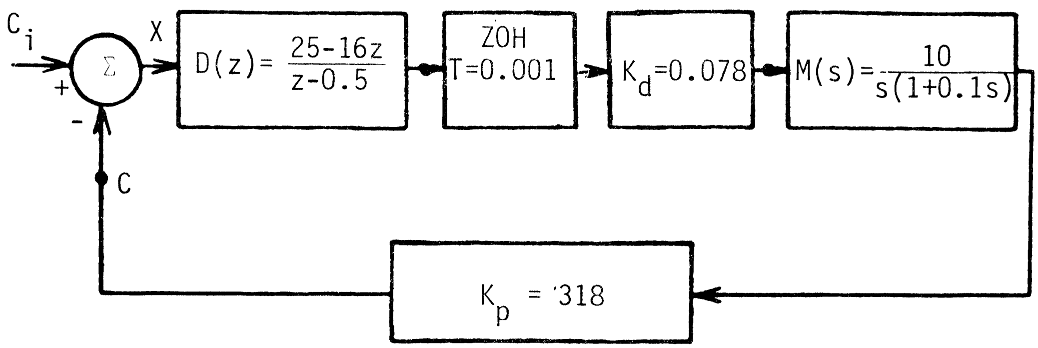

Fig. 3.3 Modeling the system of Ex. 3.1.

The model developed in this chapter includes a digital transfer function, D(z), and a continuous transfer function, M(s). The analysis method for such a system is shown in the following chapter.

IV. Analysis of Digital Control Systems

The discussion of Chapter III illustrated how the elements of a digital position control system can be modeled. The models of these elements are shown in Fig. 3.2.

The elements of the digital control system can be divided into three types: The first type is the digital controller which is modeled by the digital transfer function, D(z). The second type is the ZOH, and the third type are those elements which are modeled by continuous transfer functions, or gains. To simplify the discussion, we combine all of the elements of the third type into one transfer function, H(s)

$$H(s) = K_d K_p M(s) \qquad (4-1)$$

Accordingly, the digital position control system can be represented by three elements, as shown in Fig. 4-1.

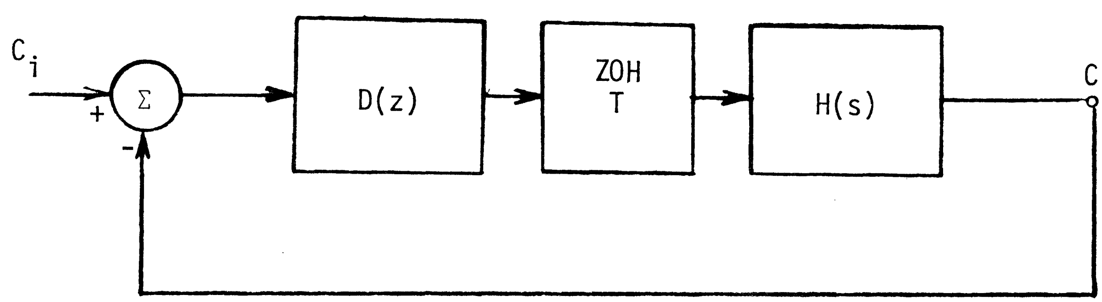

Fig. 4.1 Position control system represented as 3 elements.

In order to analyze the system, it is necessary to develop models of the same forms for all of the system elements. This can be done by two methods: The digital analysis method and the continuous approximation method. The two methods are described in this chapter.

Digital Analysis

The first step in the digital analysis is to develop a digital model for the ZOH and the continuous element H(s). The digital model for both H(s) and the ZOH is denoted by E(z). It is given by

$$E(z) = (1-z^{-1})Z\left[\frac{H(s)}{s}\right] \qquad (4\text{-}2)$$

The operation Z[ ] indicates finding the transform pair of the function in paranthesis. Such transform pairs are given in z-transform tables, such as Table 4.1. Additional information can be found in Ref D1-D4. The process of modeling the elements is illustrated by the following examples.

Example 4.1

Consider a continuous transfer function H(s) where

$$H(s) = \frac{1}{s} \tag{4-3}$$

The function H(s) is preceded by a ZOH with a sampling period of T = 0.001 s. Find the digital transfer function, E(z), of H(s) and the ZOH.

The first step is to determine

$$\frac{H(s)}{s} = \frac{1}{s^2} \tag{4-4}$$

Next we find the transform pair

$$Z[\ \frac{1}{s^2}\ ] = \frac{Tz}{(z-1)^2} \tag{4-5}$$

Finally, E(z) is

$$E(z) = (1-z^{-1})\frac{Tz}{(z-1)^2} = \frac{T}{z-1} \tag{4-6}$$

Example 4.2

Consider the system discussed in Example 3.1 and illustrated by Fig. 3-3. Find the digital transfer function of all of the continuous elements and the ZOH.

First note that H(s) is

$$H(s) = K_d K_p M(s) = \frac{248}{s(1+0.1s)} \tag{4-7}$$

Therefore

$$\frac{H(s)}{s} = \frac{248}{s^2(1+0.1s)} = \frac{2480}{s^2(s+10)} \tag{4-8}$$

The Z transform table gives the following relationship

$$\frac{a}{s^2(s+a)} \leftrightarrow \frac{z[(aT-1+e^{-aT})z - (aTe^{-aT}-1+e^{-aT})]}{a(z-1)^2(z-e^{-aT})} \tag{4-9}$$

Here $a = 10$

$T = 0.001$

Therefore, the transform pair is

$$\frac{2480}{s^2(s+10)} \leftrightarrow \frac{248z(4.98z + 4.96) \cdot 10^{-3}}{10(z-1)^2(z-0.99)} \tag{4-10}$$

$$E(z) = (1-z^{-1})Z\left[\frac{H(s)}{s}\right]$$

$$= \frac{1.24 \cdot 10^{-3}(z+1)}{(z-1)(z-0.99)} \tag{4-11}$$

Stability Analysis

Once the model for all the loop elements was established, it is possible to analyze the system behavior. The position control system depicted by Fig. 4-1 can now be represented by the simplified model of Fig. 4-2.

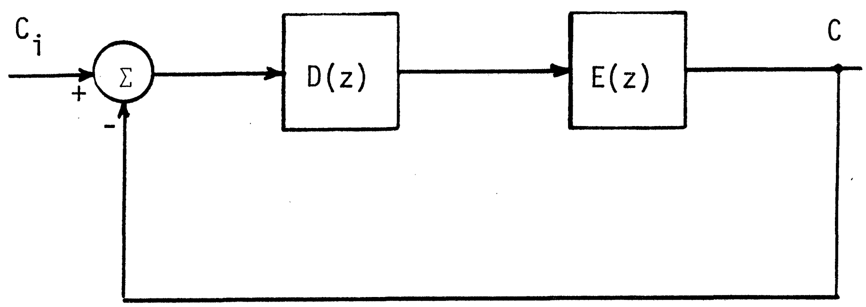

Fig. 4.2 Simplified digital model

The closed loop transfer function of the system is

$$\frac{C(z)}{C_i(z)} = \frac{D(z)E(z)}{1 + D(z)E(z)} \qquad (4\text{-}12)$$

The transient response of the system and its stability depend on the system characteristic equation

$$1 + D(z)E(z) = 0 \qquad (4\text{-}13)$$

The roots of the characteristic equation, which are called the poles of the closed-loop system, determine the nature of the system response.

When a digital system has a pole at $z=z_1$, the transient response that corresponds to that pole is of the form

$$f(k) = z_1{}^k \qquad k = 0,1,2... \qquad (4\text{-}14)$$

In order for the system response to be stable, all the transient responses must decay and approach zero. The sequence f(k), as represented by Eq (4-14) will approach zero if the magnitude of the complex number z_1 is less than one.

For this reason, in order for the system to be stable, all of the elements of its transient response must approach zero, and this requires that all of the poles of the system must have a magnitude that is less than one.

$$|z_i| < 1 \qquad i = 1,2, ... \qquad (4\text{-}15)$$

Equivalently, if the system poles are graphically represented in the complex plane, the stability condition requires all the system poles to be within the unit circle.

The concept of the stability is further illustrated by Example 4.3.

<u>Example 4.3</u>

Consider a digital control system of the form shown in Fig. 4-2 and let the open loop transfer function be

$$D(z)E(z) = \frac{K}{z(z-1)} \tag{4-16}$$

Find the values of K which result in a stable system.

The poles of the transfer function are the roots of the equation

$$1 + D(z)E(z) = 0 \tag{4-17}$$

or

$$1 + \frac{K}{z(z-1)} = 0 \tag{4-18}$$

This can be written as

$$z(z-1) + K = 0 \tag{4-19}$$

or

$$z^2 - z + K = 0 \tag{4-20}$$

The solutions of Eq. (4-20) are

$$z_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - K} \quad (4\text{-}21)$$

The roots $z_{1,2}$ depend on the parameter K. Several values of K and the corresponding solutions are shown in Table 4.2. The solutions z_1 and z_2 are next plotted in the Z domain, the resulting curves are shown in Fig 4-3.

The plot of the roots of the characteristic equation (4.13) as a function of the gain parameter, K is known as the root locus diagram. This graphical tool is very useful in determining the system stability. More on this subject can be found in Ref [C1]-[C3].

The root locus diagram shows clearly that if K is negative, at least one of the roots is outside the unit circle, and therefore, the system is unstable. For gain values between 0 and 1 both z_1 and z_2 are inside the unit circle, and then for K = 1, the system becomes unstable again. In conclusion, the system is stable when the gain K is in the range

$$0 < K < 1 \quad (4\text{-}22)$$

In general, when the characteristic equation is of a higher order, it is possible to find the roots by a numerical procedure. The system is stable if the magnitude of all the

K	z_1	z_2
0	1	0
0.25	0.5	0.5
0.5	0.5+j0.5	0.5-j0.5
0.75	0.5+j0.7	0.5-j0.7
1.0	0.5+j0.866	0.5-j0.866
1.25	0.5+j	0.5-j
-0.25	1.2	-0.2
-0.5	1.366	-0.366
-0.75	1.5	-0.5

Table 4.2 Roots of Eq.(4.20)

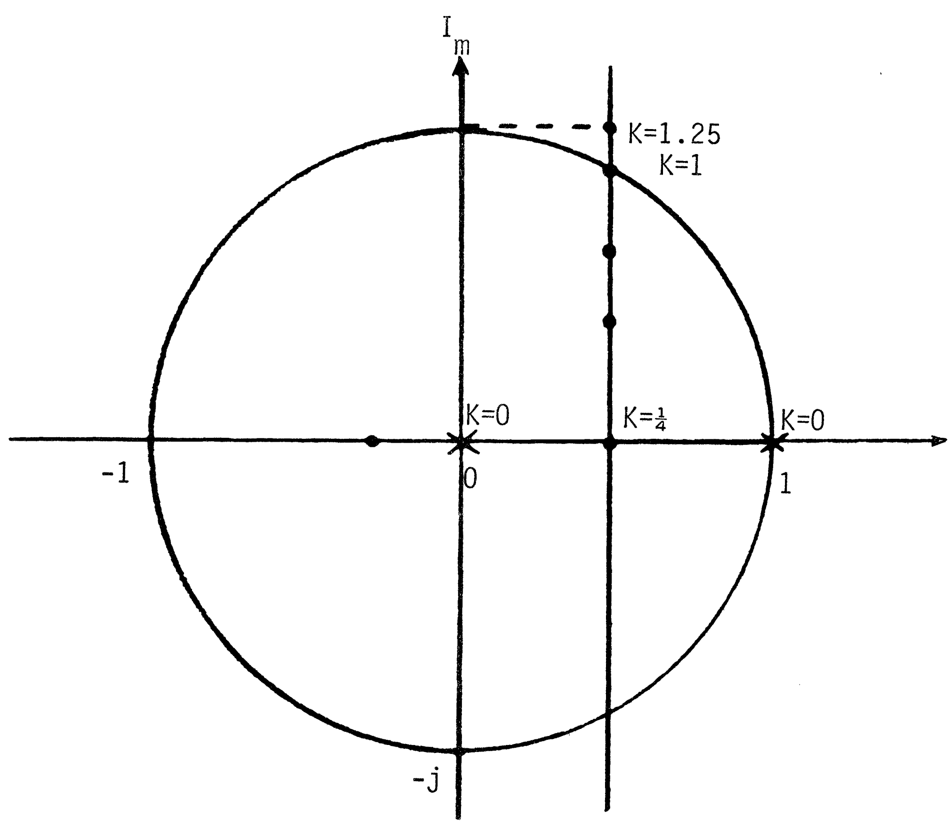

Fig. 4.3 Root locus diagram for Ex. 4.3.

roots is less than one. This procedure is illustrated by the following example.

Example 4.4

The open loop transfer function of a digital system is

$$D(z)E(z) = \frac{0.2}{z(z^2-0.6z+0.1)} \tag{4-23}$$

Is the system stable?

The characteristic equation is

$$1 + \frac{0.2}{z(z^2-0.6z+0.1)} = 0 \tag{4-24}$$

This can be written as

$$z^3 - 0.6z^2 + 0.1z + 0.2 = 0 \tag{4-25}$$

The roots of Eq. (4-25) are found numerically

$$\begin{aligned} z_1 &= 0.4 \\ z_2 &= -0.5 + j0.5 \\ z_3 &= -0.5 - j0.5 \end{aligned} \tag{4-26}$$

The magnitudes of z_1, z_2 and z_3 are 0.4 and 0.707 respectively.

They are all less than 1.0. Therefore, the system is stable.

Analysis by Continuous Approximation

The control system can be analyzed by the continuous approximation method. In order to use this method, we need to develop a continuous model for all of the system elements.

A digital system with transfer function D(z) can be modeled as a continuous transfer function G(s). To determine G(s) we substitute in D(z)

$$z = e^{sT} \tag{4-27}$$

As a consequence

$$G(s) = D(z)\Big|_{z = e^{sT}} \tag{4-28}$$

The function G(s), as given by Eq. (4-28), is difficult to deal with, due to the exponential terms. A more simple expression for G(s) can be obtain by the approximation

$$z = \frac{1 + sT/2}{1 - sT/2} \tag{4-29}$$

The resulting function

$$G(s) = D(z)\Big|_{z = \frac{1+sT/2}{1-sT/2}} \qquad (4\text{-}30)$$

is a continuous approximation for D(z).

The second element that needs to be modeled is the ZOH. This element can be modeled as a pure delay of half of a sampling period. The resulting transfer function is

$$F(s) = e^{-sT/2} \qquad (4\text{-}31)$$

Eqs. (4-30) and (4-31) allow us to represent all of the loop elements by continuous transfer functions. This permits the use of continuous control analysis.

This analysis procedure is illustrated by the following example.

Example 4.5

Consider the system discussed in Example 4.2. The system is represented by the following block diagram.

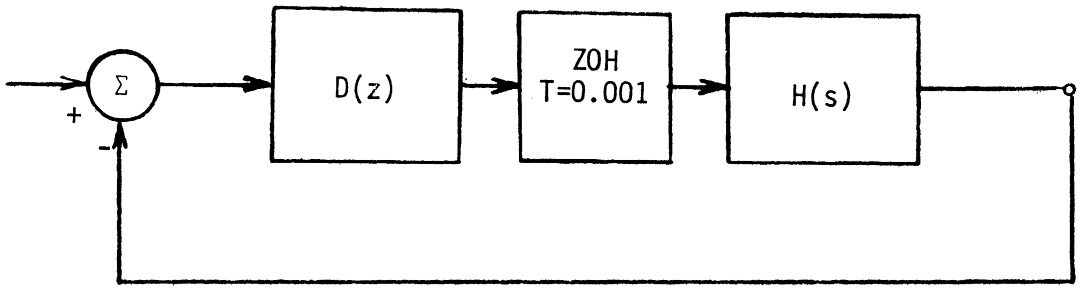

Fig. 4.4 System block diagram

Suppose that the digital controller, D(z) is

$$D(z) = \frac{130(z-0.8)}{(z-0.2)} \tag{4-32}$$

The function H(s) is given by Eq. (4.7).

$$H(s) = \frac{248}{s(1+0.1s)} \tag{4-33}$$

and the sampling time is T = 0.001 s. The system stability will be analyzed by the continuous approximation method.

The first step is to develop a continuous model for D(z). According to Eq. (4.29), z is replaced by

$$z = \frac{1+0.0005s}{1-0.0005s} = \frac{2000+s}{2000-s} \tag{4-34}$$

Combining (4-32) with (4-340 results in:

$$G(s) = \frac{s+222}{s+1333} \tag{4-35}$$

The ZOH is modeled as a delay

$$F(s) = e^{-sT/2} = e^{-0.0005s} \tag{4-36}$$

The open loop transfer function, A(s) is the product of the terms in Eqs. (4-33), (4-35) and (4-36)

$$A(s) = \frac{4.83 \cdot 10^5 \, e^{-0.0005s}(s+222)}{s(s+10)(s+1333)} \tag{4-37}$$

The first step in the stability analysis is to find the crossover frequency, the frequency at which the open loop gain equals one.

Require

$$|A(j\omega_c)| = 1 \tag{4-38}$$

or

$$|A(j\omega_c)| = \left|\frac{4.83 \cdot 10^3(j\omega_c+222)}{j\omega_c(j\omega_c+10)(j\omega_c+1333)}\right| = 1 \tag{4-39}$$

The solution of (4-39) is

$$\omega_c = 400 \text{ rad/s} \tag{4-40}$$

The phase shift of $A(j\omega_c)$ is

$$\phi = \arg[A(j\omega_c)] = \tan^{-1}\frac{\omega_c}{222} - 0.0005\omega_c \cdot \frac{360°}{2\pi} - 90° - \tan^{-1}\frac{\omega_c}{10} - \tan^{-1}\frac{\omega_c}{1333} \tag{4-41}$$

For $\omega_c = 400$, Eq. (4-41) becomes

$$\phi = -146° \tag{4-42}$$

Since ϕ is greater than -180 , the system is stable. The phase margin, θ_m , is

$$\theta_m = 180 + \phi = 34° \tag{4-43}$$

V. Designing Position Control Systems

In the previous chapter it was shown that the position control system includes elements of three types: Digital, ZOH and continuous. The system elements are shown in the block diagram of Fig. 5-1.

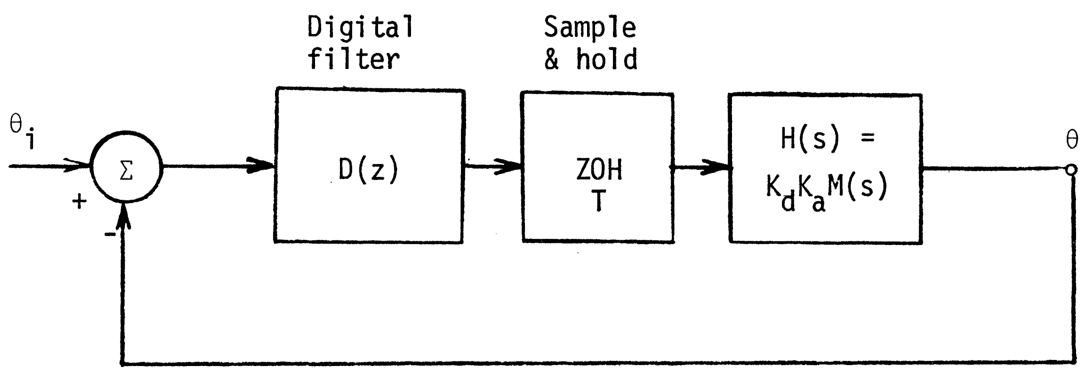

Fig. 5.1 Block diagram representation of the position control system

The digital filter is represented by the transfer function D(z), the continuous elements are combined into H(s), and the ZOH represents the sample-and-hold nature of the motor command. The objective of this chapter is to show how to design D(z) and how to implement it by a microprocessor.

The following discussion describes two design methods for the digital compensation. We start with the design by equivalent continuous filters, and later we present the combination method.

Design by Equivalent Continuous Filters

The design procedure starts with the model of the system and the design requirements. The model of the system elements is included in the continuous transfer function H(s) of Fig. 5-1. The design objective is to select D(z) such that the resulting system will have a crossover frequency ω_c, and a stability phase margin θ_m.

The crossover frequency is the frequency at which the open-loop transfer function has a gain of one. This frequency is nearly the same as the closed-loop bandwidth.

The design procedure starts by the determination of the phase shift of H(s) at the frequency ω_c.

$$\phi_1 = \arg[H(j\omega_c)] \tag{5-1}$$

The next step is to assume a value for the sampling time, T. It is desirable to keep T as small as possible. A typical value for T is between 0.5 and 1.0 ms.

The ZOH is modeled as a delay of T/2 sec.

$$F(s) - e^{-sT/2} \tag{5-2}$$

The phase shift of F(s) at ω_c is

$$\phi_2 = \arg[F(j\omega_c)] = -\frac{\omega_c T}{2} \text{ rad.} \quad (5\text{-}3)$$

The total phase shift, ϕ, is equal

$$\phi = \phi_1 + \phi_2 = \arg[H(j\omega_c)] + \arg[F(j\omega_c)] \quad (5\text{-}4)$$

In order to achieve the desired phase margin, the digital compensation must generate a phase lead of ϕ_3. This can be found from:

$$\phi_m = 180° + \phi_3 + \phi \quad (5\text{-}5)$$

The phase lead is generated by a lead compensation. This compensation is initially designed as a continuous filter G(s).

$$G(s) = K\frac{s+\omega_1}{s+\omega_2} \quad ; \quad \omega_2 > \omega_1 \quad (5\text{-}6)$$

For maximum phase lead, the frequencies ω_1 and ω_2 are such that

$$\omega_c = \sqrt{\omega_1\omega_2} \quad (5\text{-}7)$$

The ratio between the frequencies ω_1 and ω_2 is denoted by a and is called the lead span.

$$\omega_2 = a\omega_1 \quad (5\text{-}8)$$

The lead span is selected according to the desired phase lead. Fig. 5-2 shows the relation between ϕ_3 and a. Finally, the gain of the filter, K, is selected so that the open loop gain equals one at the crossover frequency. This requires:

$$|G(j\omega_c)H(j\omega_c)| = 1 \qquad (5\text{-}9)$$

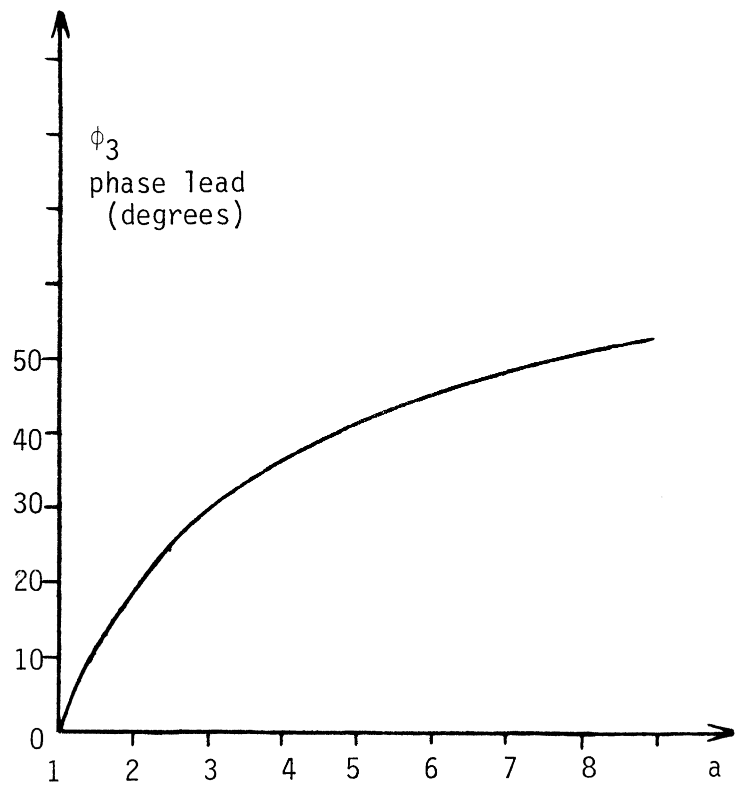

Fig. 5.2 Phase lead vs. span.

Now that we selected G(s), we need to find a digital filter, D(z), with an equivalent transfer function. To find the digital filter, D(z), replace s by

$$s = \frac{2}{T}\,\frac{z-1}{z+1} \qquad (5\text{-}10)$$

This results in

$$D(z) = G(s)\Big|_{s = \frac{2}{T}\frac{z-1}{z+1}} \qquad (5\text{-}11)$$

D(z) has an equivalent transfer function to that of G(s), and therefore, it generates the desired phase lead. The design procedure is illustrated by an example.

Example 5.1

A position control system uses a dc motor and an amplifier with the following parameters:

Torque constant: K_t = 10 oz-in/A = 0.0706 Nm/A

Resistance: R = 1.4 Ω

Inductance: L = 0

Total moment of inertia: J = 0.1 oz-in-s^2 =

= 7.06 · 10^{-4} Kgm^2

Amplifier gain: K_a = 5V/V

The transfer function of the motor is

$$\frac{\omega}{V}(s) = \frac{1/K_t}{sT_m+1} \qquad (5\text{-}12)$$

Where the mechanical time constant, T_m, is

$$T_m = \frac{RJ}{K_t^2} \tag{5-13}$$

For the given example, the motor transfer function between the input voltage and the output velocity is

$$\frac{\omega}{V} = \frac{14}{0.2s+1}$$

Note that the motor position is fed back. The motor position, θ, is the integral of the velocity, ω.

$$\frac{\theta}{\omega}(s) = \frac{1}{s}$$

Since the amplifier gain is

$$K_a = 5$$

The combined motor-amplifier transfer function, M(s), is

$$M(s) = \frac{70}{s(0.2s+1)}$$

The DAC has 8 bits and an output of ±10V. Therefore, its gain, K_d, is 10V per 128 counts.

$$K_d = \frac{10}{128}$$

The position is sensed by an encoder with a line density of N =

500 lines/rev. When the encoder has two channels in quadrature, it is possible to increase the resolution to 4N. In this case the effective resolution is 2000 quadrature counts per revolution. This corresponds to a feedback gain of

$$K_p = \frac{2000}{2\pi} = 318 \frac{\text{counts}}{\text{rad.}}$$

The sampling time is assumed to be 1 ms. Therefore, the model of the ZOH is

$$F(s) = e^{-0.0005\ s}$$

At this point, it is best to describe all of the system elements by a block diagram. This is done in Fig. 5-3.

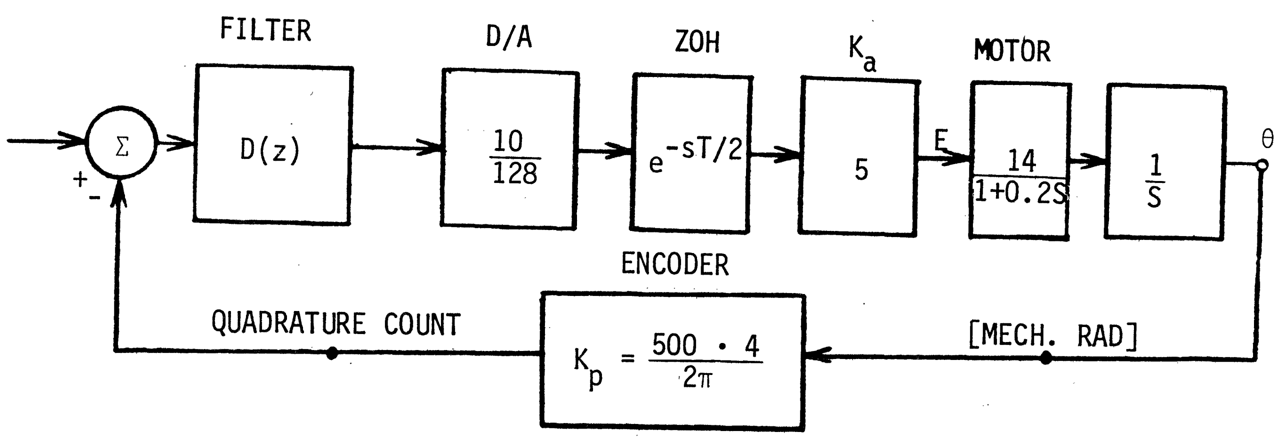

Fig. 5.3 System elements.

The design objective is to achieve a bandwidth of 20 Hz, or

$$\omega_c = 125 \text{ rad/s}$$

and a phase margin of

$$\theta_m = 45°$$

We start the design by combining all of the continuous elements of the system in one function, H(s).

$$H(s) = K_d\, M(s)\, K_p = \frac{1740}{s(0.2s+1)}$$

The phase shifts ϕ_1 and ϕ_2 are found from Eqs. (5-1) and (5-3).

$$\phi_1 = \arg[H(j\omega_c)]$$

Here

$$\phi_1 = \arg\left[\frac{1740}{j125(j0.2 \cdot 125+1)}\right] =$$

$$= -90 \;\; - \tan^{-1} 25 = -\ 178°$$

Similarly

$$\phi_2 = \arg[F(j\omega_c)] = -0.06 \text{ rad} = -3°.$$

The resulting total phase shift, ϕ , is

$$\phi = \phi_1 + \phi_2 = -181°$$

Since the objective is to have a phase margin of 45 , the phase lead, according to Eq. (5-5), must be

$$\phi_3 = 46°$$

The lead span, a, which results in 46 is found from Fig. 5-2.

$$a = 6.25$$

Next, we have to find ω_1 and ω_2 such that

$$\omega_2 = 6.25\ \omega_1$$
$$\omega_c = \sqrt{\omega_1 \omega_2}$$
$$\omega_c = 125$$

The solutions for ω_1 and ω_2 are

$$\omega_1 = 50 \quad \text{rad/s}$$

$$\omega_2 = 312 \quad \text{rad/s}$$

To find the compensation gain, note that the open loop transfer function is G(s)F(s)H(s). Since F(s) has a unit gain, require,

$$|G(j\omega_c)H(j\omega_c)| = \left|K \frac{j125+50}{j125+312} \frac{1740}{j125(0.2j125+1)}\right| = 1$$

This results in

$$K = 4.5$$

and therefore

$$G(s) = 4.5 \frac{s+50}{s+312}$$

Finally, to find d(z), follow Eqs. (5-9) and (5-10)

$$s = \frac{2}{T}\frac{z-1}{z+1} = \frac{2000(z-1)}{(z+1)}$$

$$D(z) = \frac{2000\frac{(z-1)}{(z+1)} + 50}{2000\frac{(z-1)}{(z+1)} + 312} = 4.0\ \frac{z-0.95}{z-0.73}$$

The implementation of the filter algorithm is discussed later in this chapter.

The design method by equivalent continuous filters is simple and straightforward. It appeals to the designer because it relies on continuous control theory, and does not require any knowledge of digital control. It has a disadvantage however. This method limits the choices of D(z), to those functions that have continuous equivalent filters. This limitation becomes important in control system with high bandwidth. To overcome this limitation, we describe a second design method, the combination method, which combines the continuous and digital design methods.

Design by the Combination Method

This design method starts with the same procedure as the previous design method. We start with the system that is represented by Fig. 1. We specify the desired crossover frequency and phase margin, and proceed to determine the desired phase lead, as given by Eq. (5-5).

The difference between the design methods is in the selection of the digital filter, D(z). Here, we start directly with a digital filter of the form:

$$D(z) = K \frac{z-A}{z+B} \tag{5-14}$$

and we proceed to select A, B and K to give the required phase lead and crossover frequency.

To analyze the phase lead, rewrite Eq. (5-14) as

$$D(z) = k \frac{z-A}{z} \frac{z}{z+B} \tag{5-15}$$

Note that D(z) consists of a gain term, K, a zero element (z-A)/z and a pole element z/(z + B). We substitute for z

$$z = e^{j\omega T}$$

The resulting function is a complex function of , and its phase shift is readily found.

The phase shift of the zero term, ϕ_z, is given by

$$\phi_z = \arg[\frac{e^{j\omega T} - A}{e^{j\omega T}}] \qquad (5\text{-}16)$$

Eq. (5-16) can be interpreted by Fig. (5-4). The denominator, z = $e^{j\omega T}$, has an argument of ωT radians. The numerator, on the other hand, has a larger argument. Therefore, the total argument, ϕ_z , is positive. The dependence of ϕ_z on the frequency can be interpreted by the movement of the point c along the unit circle.

Next define the magnitude of the zero term

$$m_z = \frac{e^{j\omega T} - A}{e^{j\omega T}} \qquad (5.17)$$

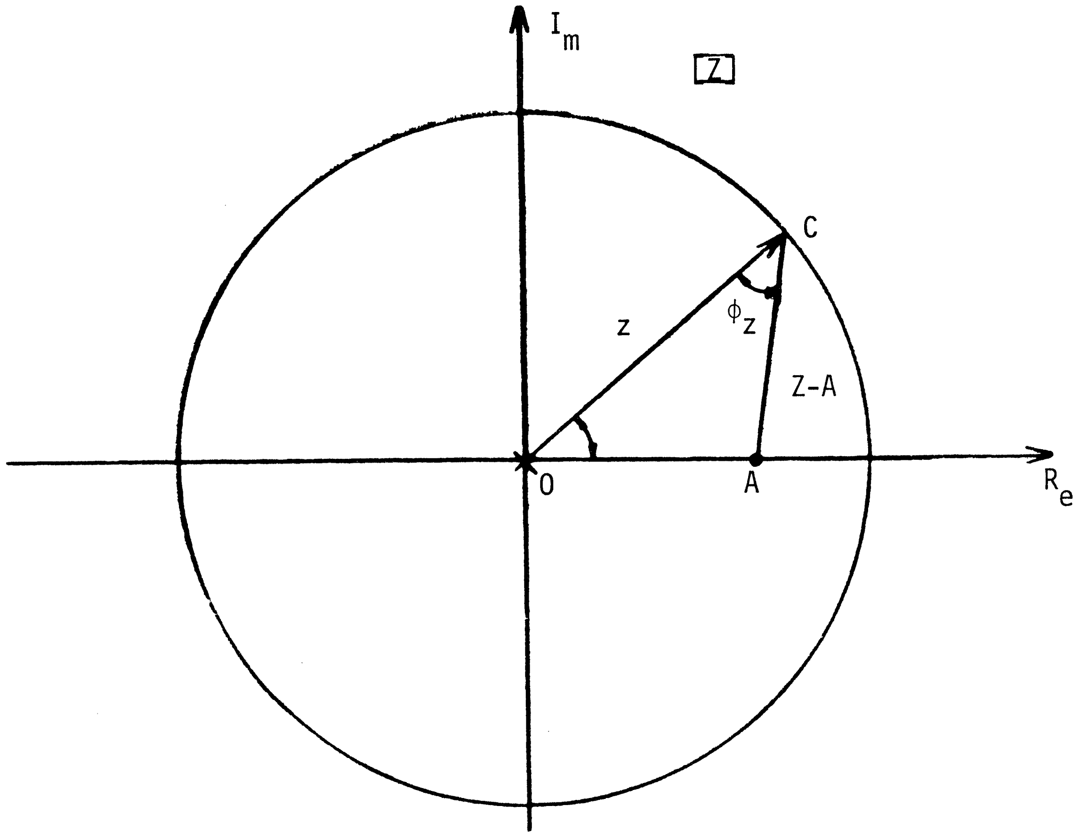

Fig. 5.4 Phase lead of a zero term

$$D(z) = \frac{Z-A}{Z}$$

The magnitude m_z can be interpreted as the ratio of the two vectors. At low frequencies, the ratio is less than one, but it increases with larger frequencies. The values of ϕ_z and m_z were calculated and presented by Fig. 5-5, which effectively shows the "Bode" plots of the zero term.

In a similar manner we can define the phase and magnitude of the pole term as

$$\phi_p = \arg\left[\frac{e^{j\omega T}}{e^{j\omega T}+B} \right] \qquad (5\text{-}18)$$

and

$$m_p = \left| \frac{e^{j\omega T}}{e^{j\omega T}+B} \right| \qquad (5\text{-}19)$$

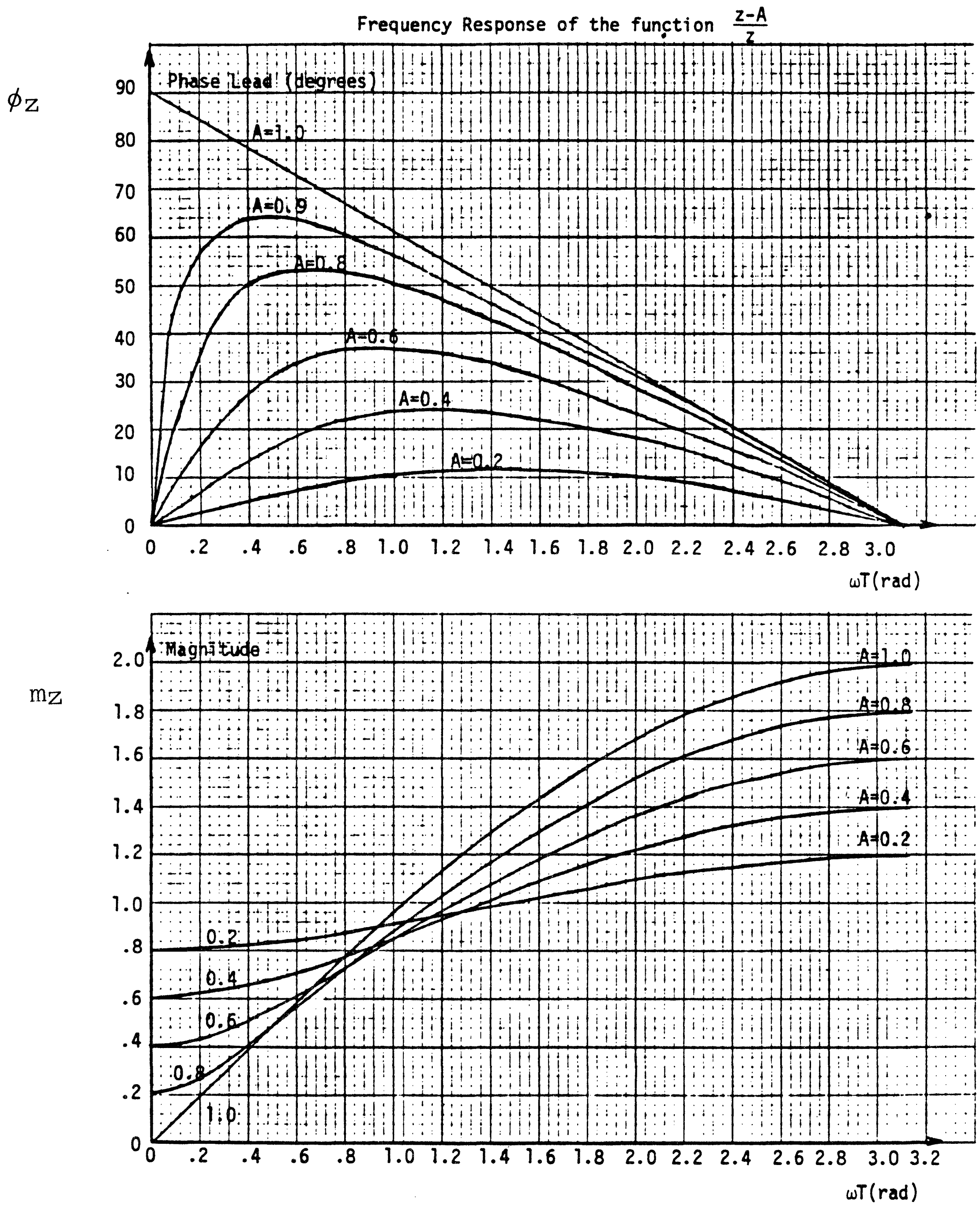

Fig. 5.5 Phase lead and magnitude of a zero term

Here again, ϕ_p maybe interpreted as the phase in Fig. 5.6, and the magnitude, m_z, is the ratio of the two vectors. The phase and magnitude of the pole terms are shown in Fig. 5.7.

Fig. 5.5 and 5.7 allow us to choose the values of A and B, to meet the phase lead requirements. The filter gain, K, is selected so that the loop gain is one at the crossover frequency.

This design method is illustrated by Example 5.2.

Example 5.2

Consider the system of Example 1, and suppose that the system requirements for bandwidth are increased to 80 Hz, or

$$\omega_c = 500 \text{ rad/s}.$$

The phase shifts, ϕ_1 and ϕ_2 are:

$$\phi_1 = \arg\,[H(j\omega_c)] =$$

$$= \arg\left[\frac{1740}{j500(0.2j500+1)}\right] = -179°$$

$$\phi_2 = -\frac{\omega_c T}{2} = -0.25 \text{ rad} = -14°$$

Therefore, the total phase shift is

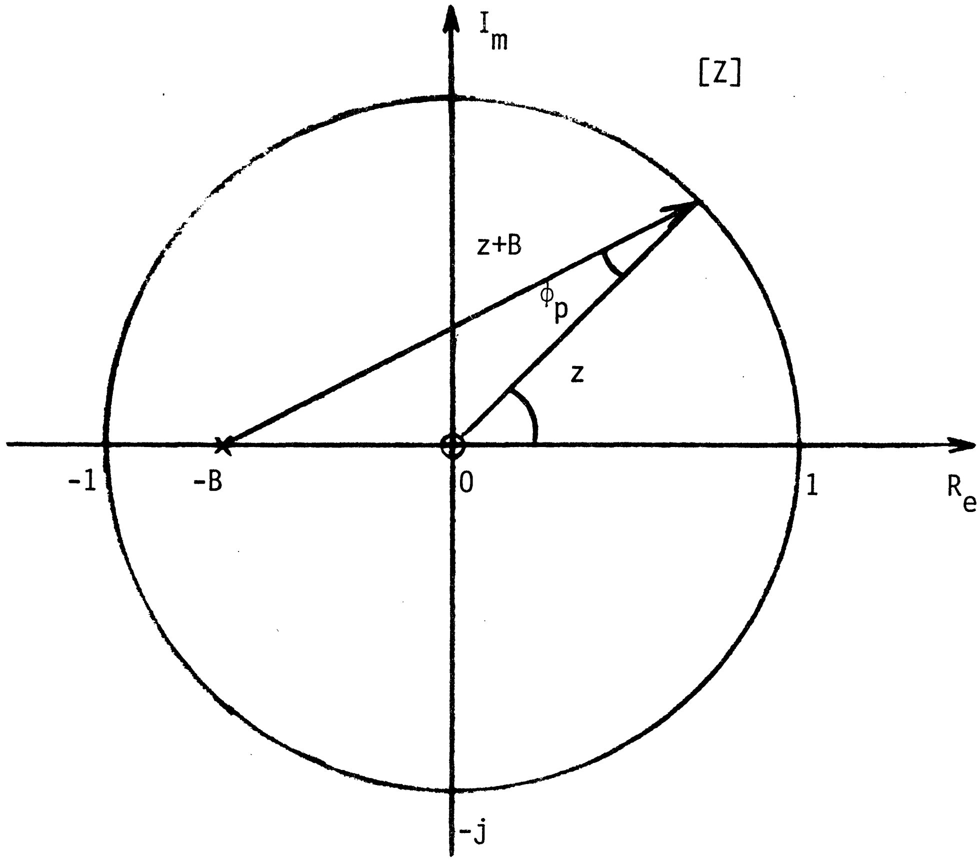

Fig. 5.6 Phase lead of a pole term

$$D(z) = \frac{z}{z+B}$$

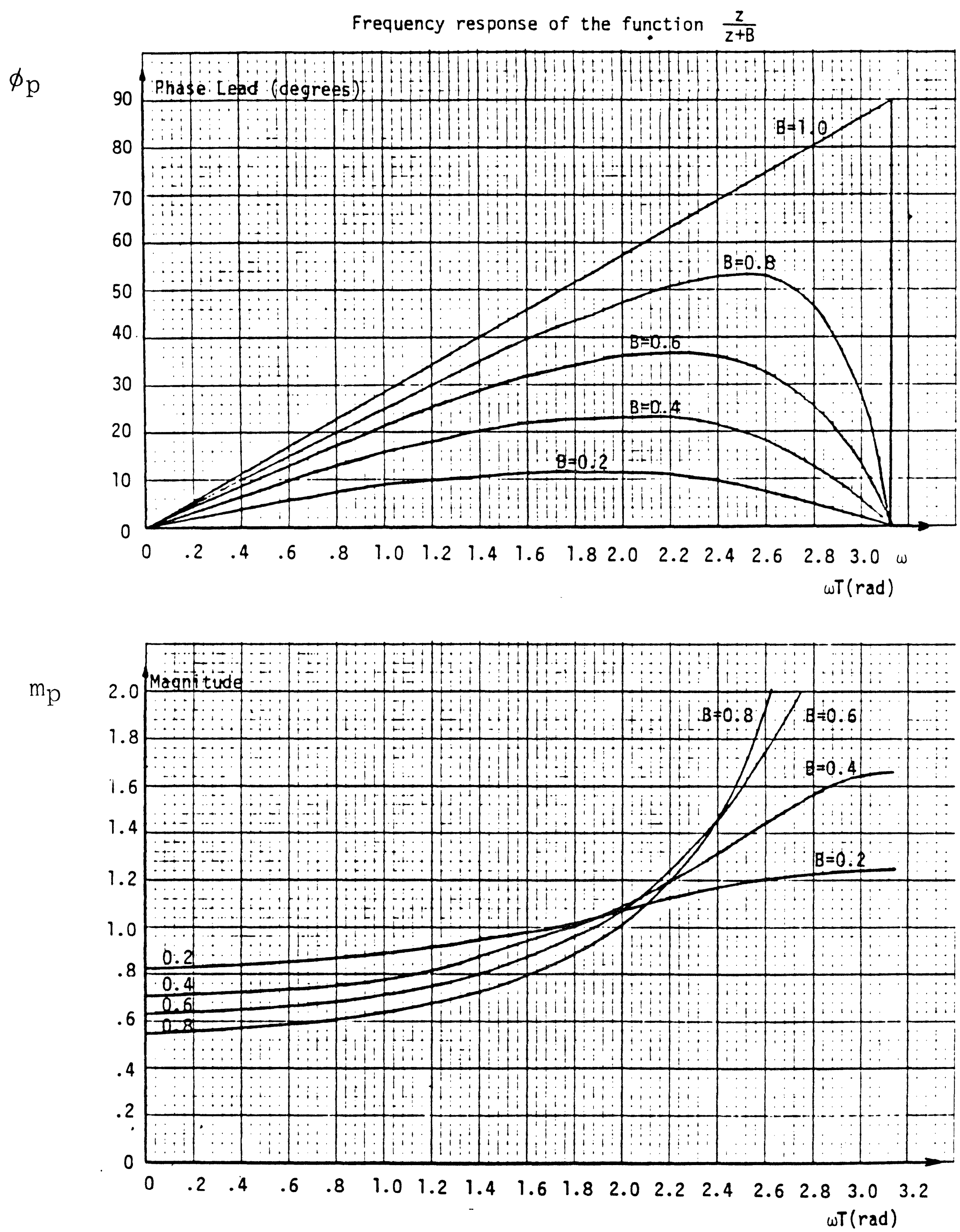

Fig. 5.7 Phase lead and magnitude of a pole term.

$$\phi = \phi_1 + \phi_2 = -193°$$

In order to obtain a 45° phase margin, we need a phase lead of

$$\phi_3 = 58°$$

The required phase lead, ϕ_3, can be generated as the sum of ϕ_z and ϕ_p.

Starting with ϕ_p, note that for the given values,

$$\omega_c T = 5000 \cdot 0.001 = 0.5 \text{ rad.}$$

The phase lead ϕ_p can be found from Fig. 5.8. Note that the maximum value of ϕ_p is 14° when B = 1.0. By selecting B = 0.4, we achieve ϕ_p = 8°. Note further, that the magnitude, m_p equals 0.72. To proceed with the design we need the phase lead, ϕ_z, to be

$$\phi_z = \phi_3 - \phi_p = 58° - 8° = 50°$$

Fig. 5.9 shows that the choice of A = 0.8 gives a phase lead of 52°, which is sufficient.

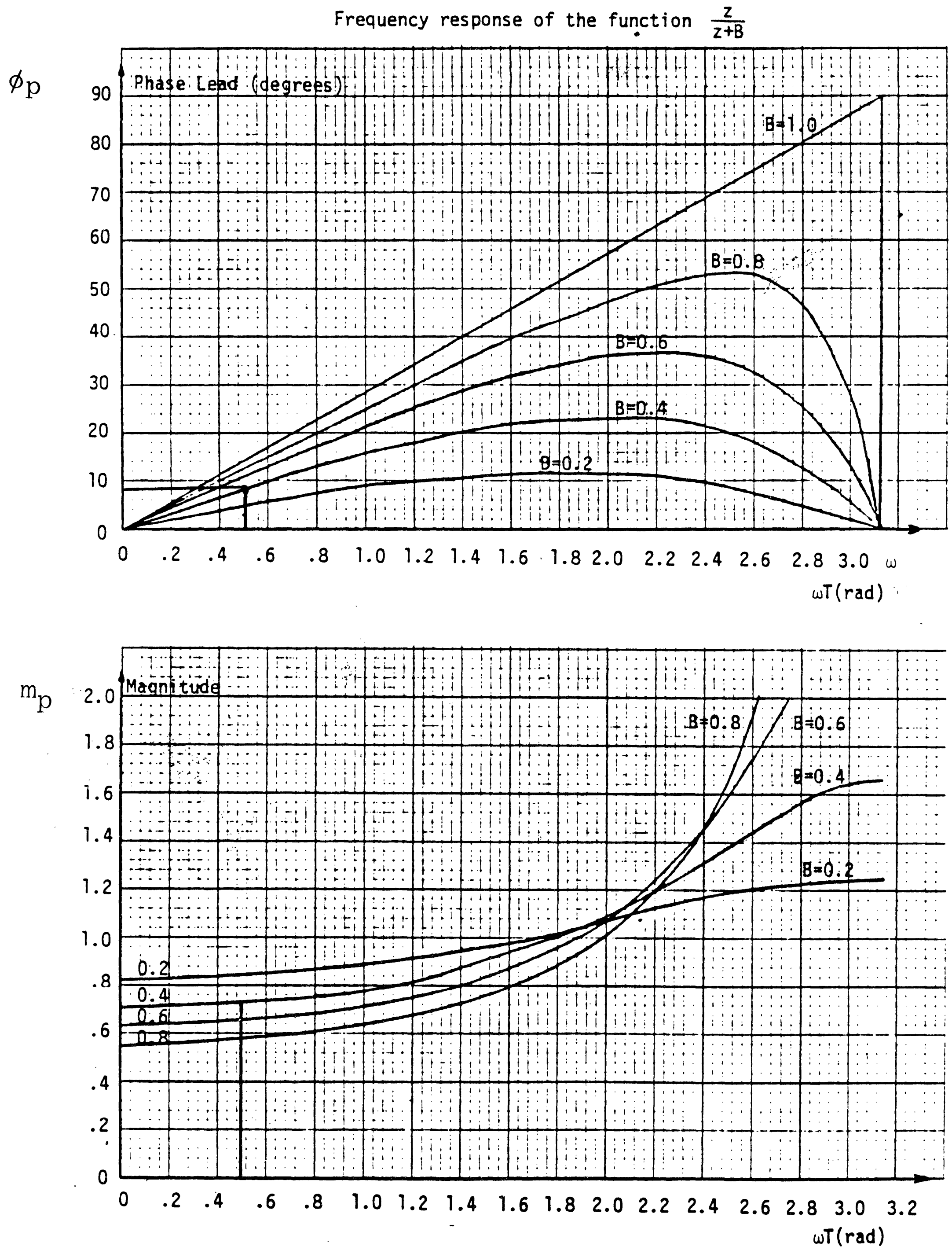

Fig. 5.8 The determination of phase lead and magnitude for B = 0.4

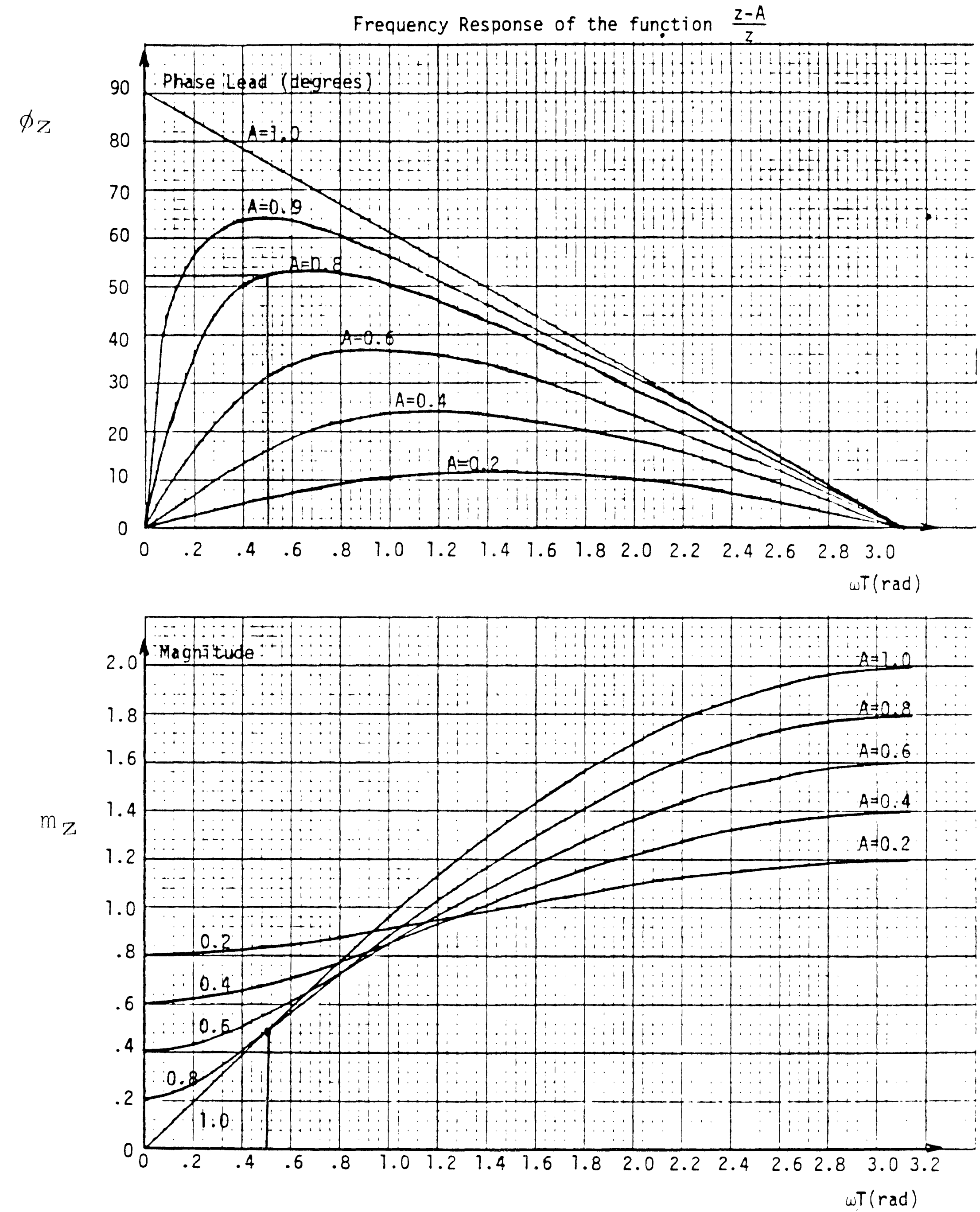

Fig. 5.9 The determination of the phase lead and magnitude for A=0.8

Furthermore, the magnitude, m_z, equals $m_z = 0.45$.

The choice of A = 0.8 and B = 0.4 provides the required phase lead. To obtain the specified crossover frequency, the loop gain at the crossover frequency, must be equal to one.

The magnitude of H(s) equals

$$|H(j\omega_c)| = \left| \frac{1740}{j500(0.2j500+1)} \right| = 0.035$$

The magnitude of the ZOH is one, and the digital filter gain equals $Km_z m_p$. In order to get a unity loop gain require,

$$K\ m_z\ m_p |H(j\omega_c)| = 1$$

This leads to

$$K \cdot 0.45 \cdot 0.72 \cdot 0.035 = 1$$

$$K = 88$$

and the digital filter is

$$D(z) = 88\ \frac{z-0.8}{z+0.4}$$

The two design methods described above sugget a method for designing the digital filter D(z). Before we discuss the implementation, we need to consider the position accuracy. This is discussed in the following section.

System Accurcy

The accuracy of the position control system is determined by the loop gain and the load torque, which includes the friction.

In order to determine the relationship between the system parameters, consider the block diagram representation of Fig. 5-10. Since the accuracy is determined when the motor has stopped, the back emf of the motor, which is proportional to the velocity, is ignored. This simplifies the analysis.

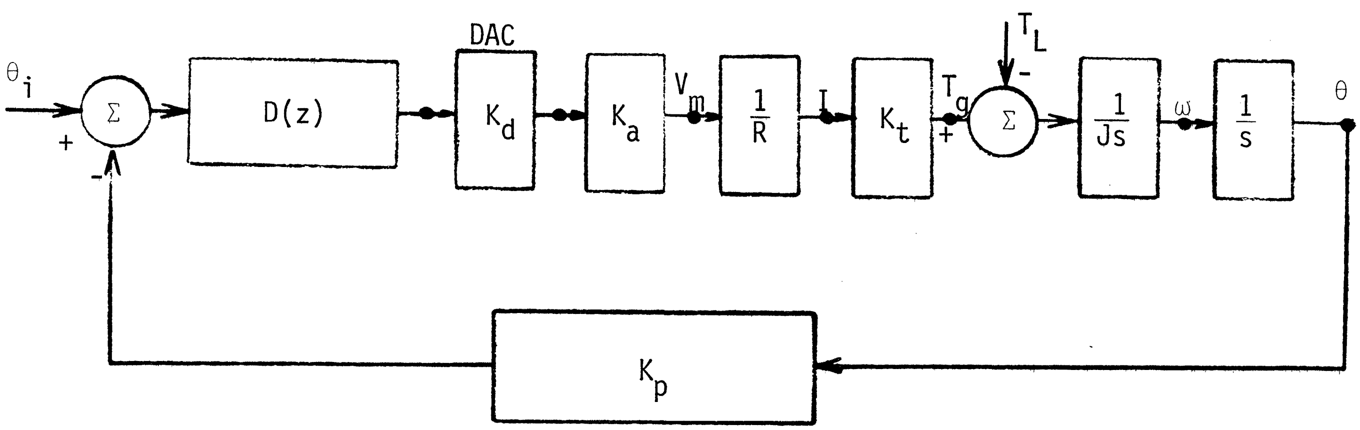

Fig. 5.10 System elements

Suppose that the load torque is T_L. This torque may consist of friction, gravitational torque or any other source. In order to overcome the load, the system must generate an equal torque T_g

$$T_g = T_L \tag{5.20}$$

This requires a current

$$I = \frac{T_g}{K_t} \tag{5.21}$$

The required motor voltage is

$$V_m = RI = \frac{RT_L}{K_t} \tag{5.22}$$

and the input to the power amplifier is

$$V_1 = \frac{V_m}{K_a} = \frac{RT_L}{K_t K_a} \tag{5.23}$$

The output of the microprocessor is a number N_c. This is applied to the DAC to generate V_1. Therefore

$$N_c = \frac{V_1}{K_d} = \frac{RT_L}{K_t K_a K_d} \tag{5.24}$$

In order for the controller to output a number N_c, it will generate a position error of N_e counts. The two numbers are related by the dc gain, K_o, of the digital filter

$$N_c = K_o N_e \tag{5.25}$$

The dc gain of any filter D(z), is found by substituting Z = 1 in D(z).

$$K_o = D(z)\Big|_{z=1} \qquad (5.26)$$

When Eq. (5.25) and (5.24) are combined, the resulting position error is

$$N_e = \frac{RT_L}{K_t K_a K_d K_o} \qquad (5.27)$$

The procedure for analyzing the system accuracy is illustrated by Example 5.3.

Example 5.3

Consider the system designed in Example 5.1. Let the friction load be 5 oz-in, and determine the system position accuracy.

We start with Eq. (5.21). A special caution should be taken to verify that the units of torque of K_t and T_L are the same. Here

$$I = \frac{T_L}{K_t} = \frac{5}{10} = 0.5A$$

the required motor voltage is

$$V_m = RI = 0.7 \text{ V}$$

The amplifier input is

$$V_1 = \frac{V_m}{K_a} = 0.14\text{V}$$

The input to the DAC, N_c, is

$$N_c = \frac{V_1}{K_d} = \frac{0.14}{0.078} = 1.8 \text{ counts}$$

The digital filter was chosen as

$$D(z) = 4\,\frac{z-0.95}{z-0.73}$$

The dc gain of D(z) is

$$K_o = D(z)\Big|_{z=1} = 0.74$$

This results in

$$N_e = \frac{N_c}{K_o} = \frac{1.8}{0.74} = 2.4 \text{ counts.}$$

Clearly both N_c and N_e must be integers. As a consequence, the position error will be between 2 and 3 quadrature counts. Since the system has a resolution of 2000 counts per revlution, the position error equals

$$\Delta\theta = \frac{2.4 \cdot 360^\circ}{2000} = 0.43^\circ$$

If a higher precision is required, it can be achieved in several methods. The most simple method is to add a lag compensation. This compensation is a low-pass filter which increases the dc gain. The lag compensation can be implemented by the microprocessor, or by an external circuit. The main disadvantage of this method is that it can make the system conditionally stable, and may result in low-frequency oscillations when the system saturates.

An alternative approach is to use a feedforward control that compensates for the friction, without affecting the system stability.

Implementation

The implementation procedure of a digital transfer function is best illustrated by an example.

Let the desired transfer function be

$$D(z) = \frac{Y(z)}{X(z)} = K\,\frac{z-A}{z+B} \qquad (5.28)$$

Where x is the position error and y is the motor command. The first step is to rewrite (5.28) as

$$(z+B)\ Y(z) = (Kz-KA) \qquad (5.29)$$

Next, divide Eq. (5.29) by the highest power of z.

$$(1+Bz^{-1})\ Y(z) = (K-KAz^{-1})\ X(z) \qquad (5.30)$$

and

$$Y(z) + Bz^{-1}Y(z) = KX(z) - KAz^{-1}X(z) \qquad (5.31)$$

now apply the inverse Z transform to Eq. (5.31).

$$y(k) + B\ y(k-1) = K\ x(k) - KA\ x(k-1) \qquad (5.32)$$

or

$$y(k) = Kx(k) - KA\ x(k-1) - B\ y(k-1) \qquad (5.33)$$

Eq. (5.33) describes the algorithm that needs to be implemented by the microprocessor. Note that the output, y(k), is a linear combination of the current position error, x(k), the previous position error, x(k-1), and the previous output, y(k-1).

A special care should be taken to minimize the delay between the time that the position feedback is read and the time that the motor command is applied. All the computation that can be done before hand, should be done before the feedback is read.

VI. DESIGNING MOTION CONTROL SYSTEMS WITH GENERAL-PURPOSE CONTROLLERS

General-purpose controllers (GPC) control the motor position by closing the position loop and by adding the required compensation for stabilizing the system.

The use of GPC is the most effective design method when a small number of systems needs to be produced. Another effective use of the GPC is in prototype development. A GPC allows the designer to control the motion within hours, rather than months.

The procedure for designing with GPC is presented in the following sections. In order to provide a specific discussion, we describe the design procedure using Galil's DHC 100. However, most of the discussion applies to any GPC.

The design procedure is devided into steps, as discussed below.

Step 1--Getting the system components.

The elements of the motion control system are shown in Fig. 6.1.

The motor can be of various types. It can be a dc motor, an ac motor or even a hydraulic motor. The only requirement is that the motor driver will respond to an analog command signal in the range of ±10 V.

The position sensor can be of any type that interfaces with the controller. The most common types of sensors are incremental encoders and resolvers.

At this step of the design, the driver is connected to the motor and the position feedback is applied to the GPC. In addition, the communication between the GPC and the host computer, or a terminal, needs to be established. At the end of this phase, the system should be as shown in Fig. 6.2.

Before we close the position loop, we need to verify that the polarity of the feedback is correct. To test the feedback polarity, we suggest the following procedure.

Apply a small positive voltage, v_1, to the driver. As a result, the motor will move. The resulting motion should cause the motor command signal, MC, to become negative, when the polarity is correct. If the MC becomes positive, reverse the motor leads or the feedback sensor polarity.

Step 2--Closing the loop.

In order to close the position loop with no oscillations, it is advised to start by setting the gain and bandwidth of the GPC to a minimum. Once the loop is closed, the gain and bandwidth can be increased gradually, to provide the best response.

The digital filter in the DMC 100 controller is of the form,

$$D(z) = K \frac{z-A}{z-B} \qquad (6.1)$$

The coefficients of D(z) are controlled as expressed by the parameters GN, ZR, and PL. The relationship between the filter parameters are

Laplace Trans- form F(s)	Time function f(t)	z-transform F(z)
1	$\delta(t)$	1
e^{-kTs}	$\delta(t-kT)$	z^{-k}
$\frac{1}{s}$	$u_s(t)$	$\frac{z}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
$\frac{2}{s^3}$	t^2	$\frac{T^2z(z+1)}{(z-1)^3}$
$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-at}}{(z-e^{-at})^2}$
$\frac{a}{s(s+a)}$	$1-e^{-at}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT}}$
$\frac{a}{s^2(s+a)}$	$t-\frac{1}{a}(1-e^{-at})$	$\frac{z[Az-aTe^{-aT}+1-e^{-aT}]}{a(z-1)^2(z-e^{-aT})}$ $A = (aT-1+e^{-aT})$

Table 4.1 Laplace Z Transform Pairs

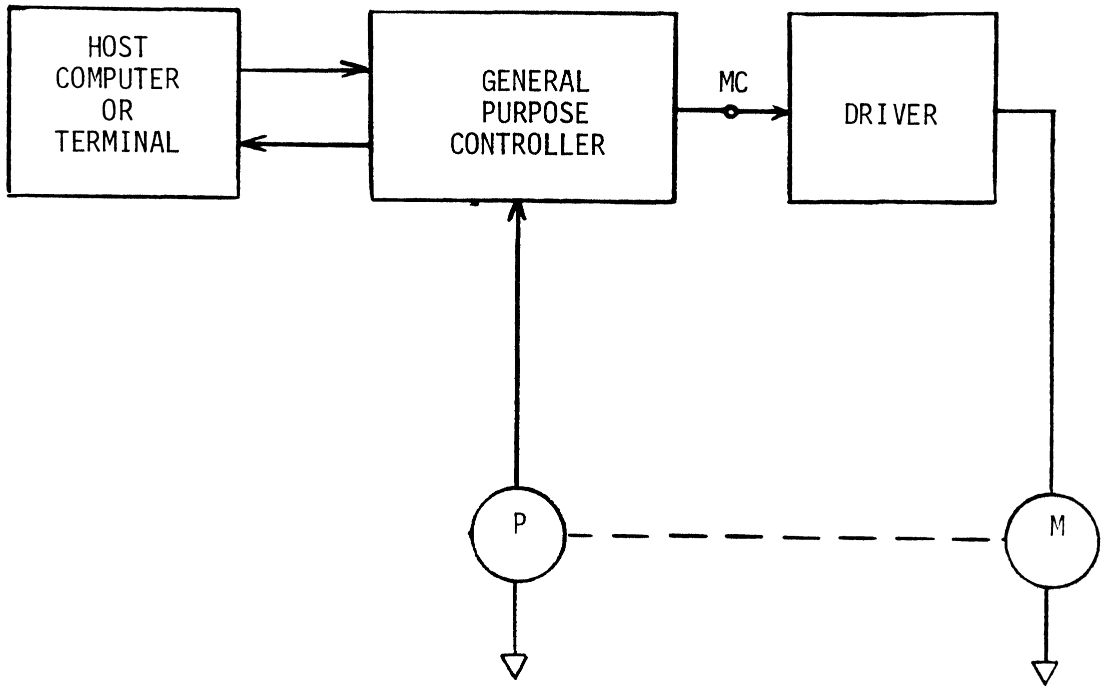

Fig. 6.1 Elements of motion control system.

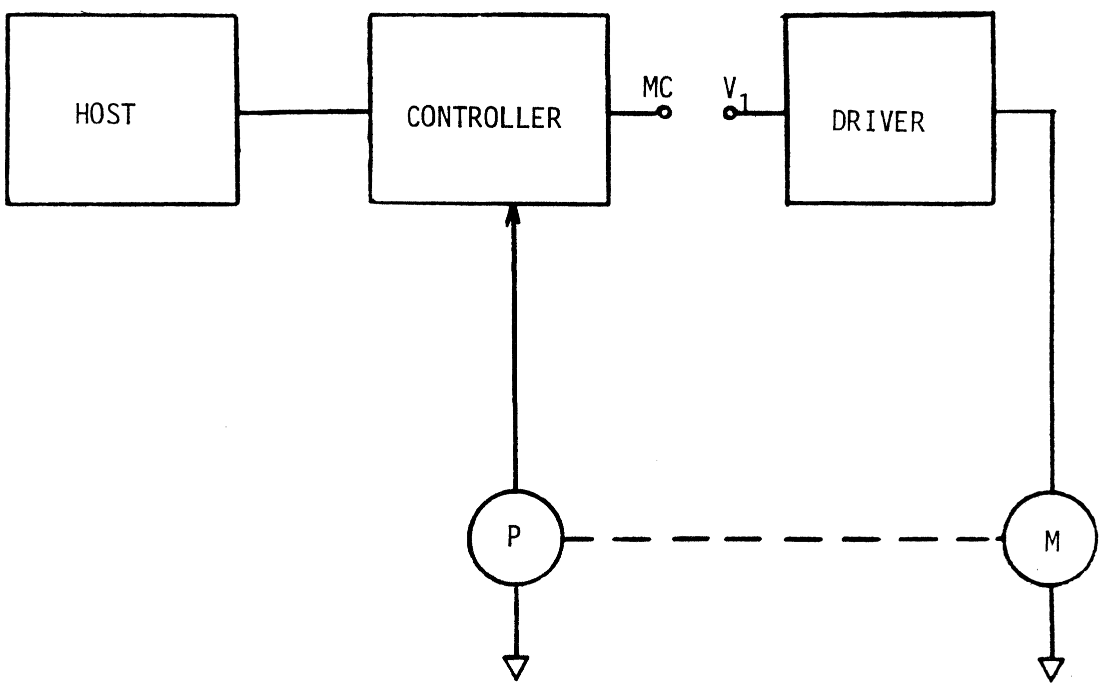

Fig. 6.2 System elements in Step 1.

$$\left.\begin{aligned} GN &= K \\ ZR &= 256A \\ PL &= 256B \end{aligned}\right\} \qquad (6.2)$$

Accordingly, the digital filter is

$$D(z) = GN\frac{z-(ZR/256)}{z-(PL/256)} \qquad (6.3)$$

In order to start with minimum gain and bandwidth, select the parameters as:

$$\left.\begin{aligned} GN &= 1 \\ ZR &= 255 \\ PL &= 0 \end{aligned}\right\} \qquad (6.3)$$

After closing the loop, the gain can be increased, and the zero can be decreased. The procedure is described in the following section.

If the motor runs away from the starting position, it indicates that the polarity of the feedback is wrong. To correct for that, reverse the motor leads or the encoder phases.

Step 3--System compensation

The objective of the compensation phase is to select the coefficients of the digital filter to produce a system with fast and stable response, and high accuracy.

We start with developing the tools for evaluating the system performance. To observe the system step response, the motor has to be commanded to step periodically. The response can be observed by attaching an analog velocity or position sensors. It can also be monitored by observing the motor command signal.

The system accuracy can be determined by measuring the position deadband due to the friction. To measure that, turn the motor away from the desired position and let go. The motor will return to the desired position. By repeating the process in both direction, and monitoring the actual position, we can find the motor accuracy.

The closed loop system should be free of limit cycles. These are motor oscillations around the commanded position. Limit cycles often produce an audible noise. They can also be measured by checking the motor position, or by observing the motor command.

Once we have established the tools for evaluating the system performance, we can proceed to select the filter coefficients.

One possible design method is to follow the design procedure, which is described in Chapter V. However, that method requires a knowledge of the system model, and that is not always given. A GPC offers the advantage, that the filter coefficients can be adjusted experimentally for best response of a given system.

The experimental compensation method generates a filter with lead compensation. The designer selects the desired crossover frequency, ω_c. Accordingly, the frequency of the zero is

$$\omega_1 = 0.4\ \omega_c \tag{6.4}$$

and the frequency of the pole is

$$\omega_2 = 2.5\ \omega_c \tag{6.5}$$

Once the pole and the zero are selected, the gain is increased gradually, for optimum performance. The process may be repeated several times with different values of ω_c, for best system response. The experimental compensation procedure is summarized below.

1. Step 1--Select the crossover frequency, ω_c. It is best to start with a low value, of 5 Hz, and to increase that gradually in consequent iterations.
2. Step 2--Select the filter coefficients. Use Table 6.1 to find the values of the pole and zero which correspond to the desired ω_c.
3. Step 3--Gain adjustment. Start with minimum gain and gradually increase it. Stop when the system develops a limit cycle at the stop position.

At the end of step 3, determine the system accuracy and note the speed of response. Next, increase ω_c, and repeat the design procedure for optimal response.

Table 6.1--Filter Coefficients

Crossover frequency ω_c [Hz]	Sample time T=0.5 ms		Sample time T=1 ms	
	ZR	PL	ZR	PL
5	254	246	253	237
10	253	237	250	219
20	250	219	243	171
50	240	173	226	117
100	226	117	199	53
200	199	53	155	11

Once the compensation process is complete, the resulting system has the best dynamic response. At this point, we are ready to design the required motion.

Step 4--Designing the required motion

Designing with GPC allow the user to specify the required distance, the velocity and the acceleration rates. The GPC generates a motion with trapezoidal velocity profile, as shown in Fig. 6.3.

Most GPC provide additional features which simplify the design of the required motion. These features include:

1. End-of-travel limit switches.
2. Initialization--find the "home" position.
3. Change the velocity "on the fly."

4. Change the filter coefficients "on the fly."
5. Allow a certain deadband in the position control.
6. Interrogate the system to find the position, and the position error at all times.

These features are helpful in designing the system, as they allow the GPC to perform more functions, thereby freeing the host computer to do other tasks.

While the design procedure may differ according to the type of the GPC, there is one potential problem to all systems. This is the possibility of overdriving. The problem is described below.

When a motor is commanded to follow a given velocity profile, such as the one shown in Fig. 6.3, the actual motor position lag slightly behind the desired position. The difference between the desired position and the actual position, called the position error, is typically a few encoder counts.

Sometimes the system is commanded to accelerate or slew at a rate that is beyond the capabilities of the motor and the amplifier. In those cases, the motor lags behind the desired command. The position error grows to levels that cause system overshoot, as shown in Fig. 6.4. When that occurs, the motor and amplifier capabilities, should be increased, to match the increased requirements.

To detect the overdrive condition, the designer may monitor the velocity profile. When overshoot of large magnitude occurs, it may indicate overdrive condition. An alternative method is

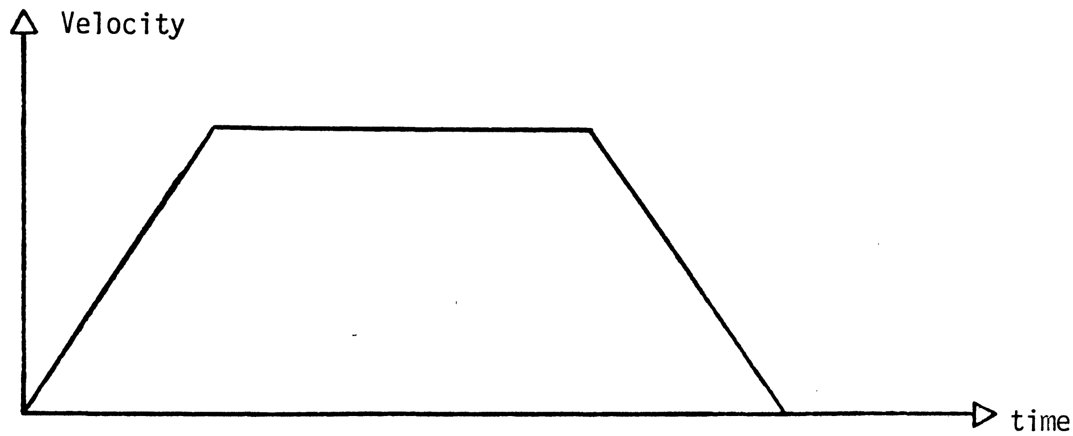

Fig. 6.3 Desired motor velocity

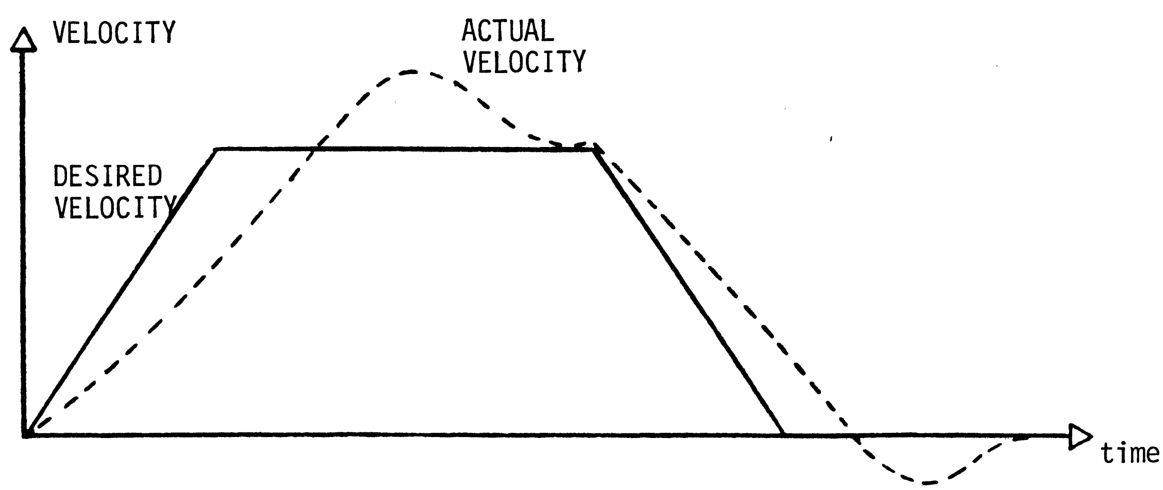

Fig. 6.4 Motor velocity under overdrive conditions.

to monitor the position error while the motor is moving. The DMC 100 provides that information in response to the command TE (Tell error). When that error becomes unusually large, it indicates an overdrive condition.

VII. VELOCITY CONTROL SYSTEMS

7.1 Introduction

Velocity control systems control the speed of the motor. They can be designed to generate a constant velocity or a variable velocity according to the system requirements.

The best performance and speed accuracy can be achieved by position control systems. A position control system, such as the one described in Chapters IV and V follows a given position command. To achieve a desired velocity, generate a command position that is the integral of the desired velocity.

Position control systems result in velocity control systems with zero errors. They can also be designed to combine velocity and position controls.

An alternative method is to control the velocity directly by the MP. This design is much more simple. Another advantage of velocity control systems is that they require position sensor with lower resolution. This results in lower system cost.

The design of velocity control systems is described below.

7.2 System Operation

The velocity control system consists of a controller, driver, motor and a feedback sensor. These are ilustrated by Fig. 7.1.

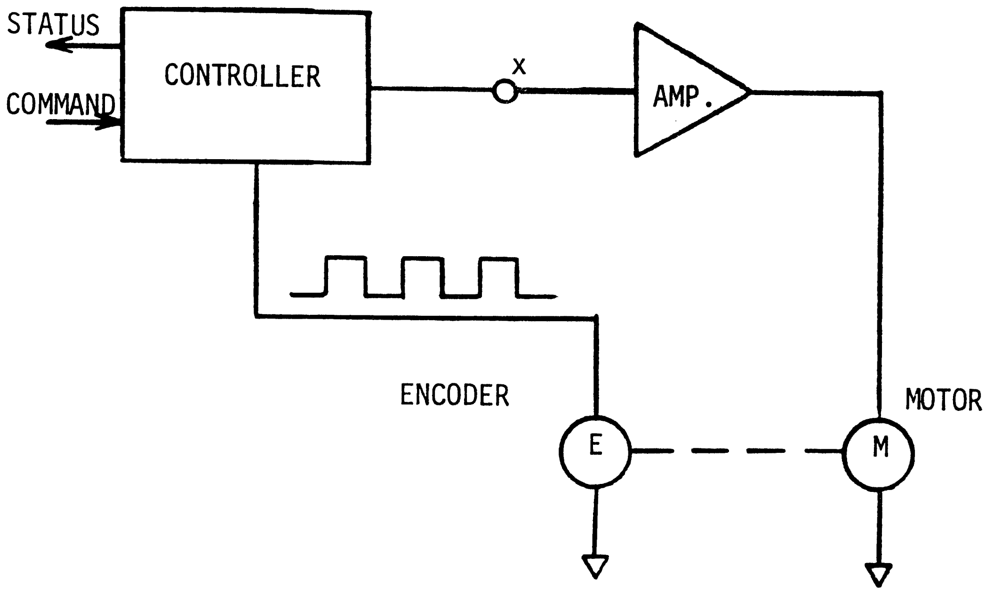

Fig. 7.1 Velocity control system.

The velocity control method can be done by two approaches: velocity loop and period loop.

The velocity loop method measures the period of the encoder signal, T. It then derives the velocity $\bar{\omega}$ from the period, T. The feedback velocity $\bar{\omega}$ is then compared with the desired velocity ω_0 to form the velocity error $(\omega_0 - \bar{\omega})$. The motor command in this case, X, is given by

$$X = K_V[\omega_0 - \omega] \qquad (7.1)$$

The period loop, on the other hand, measures the encoder period T and compares that with the desired period T. The motor command is proportional to the period error $(T-T_o$

$$X = K_1(T-T_o) \qquad (7.2)$$

Althrough the two methods are very similar, there is some subtle difference between them. This is discussed in the following section.

7.3 Dynamic Modeling

In order to analyze the velocity control system, we need to develop a mathematical model for it. Special attention needs to be given to the digital nature of the controller and the feedback sensor.

The motor an the amplifier can typically be modeled as a second order system with two time constants, the mechanical and the electrical time constants.

The resulting transfer function is

$$\frac{\omega}{X}(s) = \frac{K_m}{(sT_m+1)(sT_e+1)} \qquad (7.3)$$

To model the velocity feedback device, note that the feedback velocity is derived as the reciprocal of the encoder period. This implies that the period must be completed before the velocity feedback is derived. The delay associated with the completion of the period equals half a period, T/2. Therefore, we can model the feedback sensor by

$$H(s) = \frac{\overline{\omega}}{\omega} = e^{-sT/2} \tag{7.4}$$

Since the feedback period, T, and the nominal period T_o, are nearly the same, we may approximate H(s) by

$$H(s) = e^{-sT_o/2} \tag{7.5}$$

To model the controller, note that the motor command, X, is proportional to the velocity error

$$X= K_V(\omega_o-\overline{\omega}) \tag{7.6}$$

Furthermore, note that the motor command is updated periodically, upon the completion of a encoder period. During the following period the motor command X is held constant. This effect of sample-and-hold can be modeled as a delay of $T_o/2$.

$$F(s) = e^{-sT_o/2} \tag{7.6}$$

The elements of the system model are illustated by Fig. 7.2.

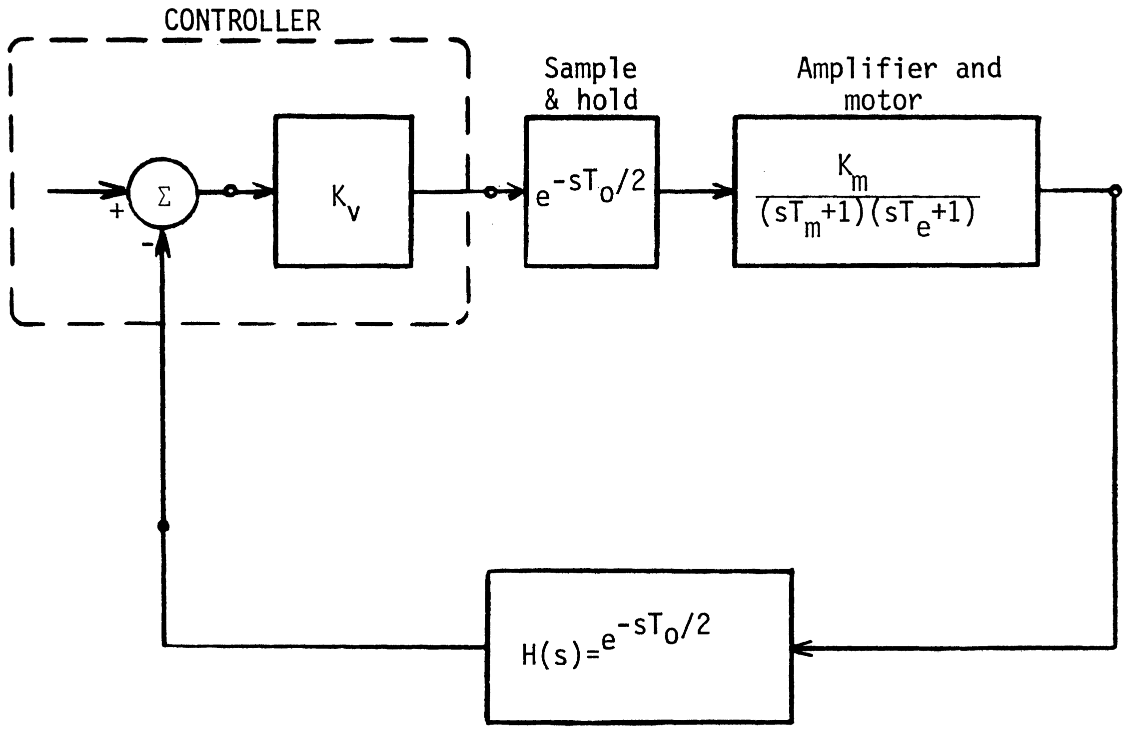

Fig. 7.2 Linearized model for control system

The model of velocity control systems utilizing the "period loop approach" differs in modeling the controller. Here the motor command is

$$X = K_1(T-T_o) \tag{7.7}$$

Assume that the feedback sensor generates N pulses per revolution, the relationship between the velocity , and the period T is

$$N\,\bar{\omega}\,T = 2\pi \tag{7.8}$$

Where $\overline{\omega}$ and T are expressed in rad/s and s respectively. Note that Eq. 7.8 applies also to the desired values.

$$N \omega_o T_o = 2\pi \tag{7.9}$$

Next, we can combine Eq. (7.8) and (7.9) with (7.7). This results in

$$\begin{aligned} X &= K_1\left(\frac{2\pi}{N\overline{\omega}} - \frac{2\pi}{N\omega_o}\right) \\ &= \frac{2\pi K_1}{N}\left(\frac{1}{\overline{\omega}} - \frac{1}{\omega_o}\right) \end{aligned} \tag{7.10}$$

Eq. (7.10) can be written as

$$X = \frac{2\pi K_1}{N} \frac{\omega_o - \overline{\omega}}{\omega_o \overline{\omega}} \tag{7.11}$$

and when $\overline{\omega}$ is nearly the same as ω_o, we can approximate (7.11) by

$$X = \frac{2\pi K_1}{N\omega_o^2} (\omega - \overline{\omega}) \tag{7.12}$$

Note that Eq. (7.12) is equivalent to Eq. (7.6) with the velocity gain being

$$K_V = \frac{2\pi K_1}{N\omega_o^2} \tag{7.13}$$

Namely, in both methods of the velocity loop and the period loop, the motor command is proportional to the velocity error.

The subtle difference between the two methods is that the velocity loop gain in Eq. (7.13), is a function of the velocity. As the velocity decreases, the effective loop gain increases.

This feature is undesirable, as it results in different dynamic characteristics at different speeds. Therefore, when the control system operates at different velocity levels, the period loop method is inadequate, and the velocity loop approach should be followed.

Next we proceed to analyze the system on the basis of the dynamic model.

7.4 Analysis and Design

The dynamic model of the velocity control system is shown in Fig. 7.2. The transfer function of the system is

$$\frac{\omega}{\omega_o} = \frac{K_V FG}{1+K_V FGH} \qquad (7.14)$$

Note that the dynamic behavior and the loop stability are determined by the open loop transfer function L(s).

$$L(s) = K_V FGH = \frac{K_V K_m e^{-sT_o}}{(sT_m+1)(sT_e+1)} \qquad (7.15)$$

The stability of the system can be analyzed by the Nyquist criterion. The most simple method to analyze the stability of systems with time delay is to determine the crossover frequency at which the phase margin is zero, and then to find the corresponding gain.

Similarly, the design procedure starts with the crossover frequency that produces the desirable phase margin and then proceeds to determine the corresponding gain. The analysis and design procedures are illustrated by the following example.

Example 7.1

Consider a velocity control system where the motor is a dc motor with the parameters:

Torque constant: K_t = 2 oz-in/A = 0.014 Nm/A

Resistance: R = 2Ω

Inductance: L = 3 mH

Moment of inertia: J = 0.002 oz-in-s^2

= $1.4 \cdot 10^{-5}$ Kgm^2

Amplifier gain: $K_a = 10$

For the given system the mechanical time constant is

$$T_m = \frac{RJ}{K_t^2} = 0.14s$$

The electrical time constant is

$$T_e = \frac{L}{R} = 0.0015s$$

and the constant K_m is

$$K_m = K_a/K_t = 142$$

Therefore, the combined model for the amplifier and the motor is

$$G(s) = \frac{142}{(1+0.14s)(1+0.0015s)}$$

The objective is to design a velocity control system to control the motor velocity at 3600 rpm. The feedback sensor is an index pulse which occurs once per revolution. The desired velocity is

$$\omega_o = 3600 \text{ rpm} = 377 \text{ rad/s}$$

Therefore, the desired period is

$$T_o = \frac{2\pi}{N\omega_o} = 0.0167s$$

The open loop transfer function of the system is given by Eq. (7.15).

$$L(s) = \frac{142\ K_V\ e^{-0.0167s}}{(1+0.14s)(1+0.0015s)}$$

To determine the system stability, note that phase margin, θ_m, is given by

$$\theta_m = 180 \;\; + \arg[L(j\omega_c)] \qquad (7.16)$$

Where ω_c is the crossover frequency of the system, where the open loop gain is one.

The arguement of $L(j\omega_c)$, as given by Eq. (7.15) is

$$\arg[L(J\omega_c)] = -\ \omega_c T_o \frac{360°}{2\pi} - \tan^{-1}\omega_c T_m - \tan^{-1}\omega_c T_e \qquad (7.17)$$

The system becomes unstable when the phase margin approaches zero.

For the system described in this example, the phase margin is

$$\theta_m = 180° + \arg[L(j\omega_c)] =$$

$$= 180° - \tan^{-1}(0.14\omega_c) - \tan^{-1}(0.0015\omega_c) - 0.0167\ \omega_c \frac{360°}{2} \qquad (7.18)$$

Equation (7.18) is evaluated numerically, and plotted in Fig. 7.3. It can be seen that when ω_c equals 91 rad/s, the phase margin is zero and the system is unstable.

The gain K_v, which will cause instability, is the one that corresponds to a crossover frequency $\omega_c = 91$. This can be found from Eq. (7.19).

$$|L(j\omega_c)| = 1 \qquad (7.19)$$

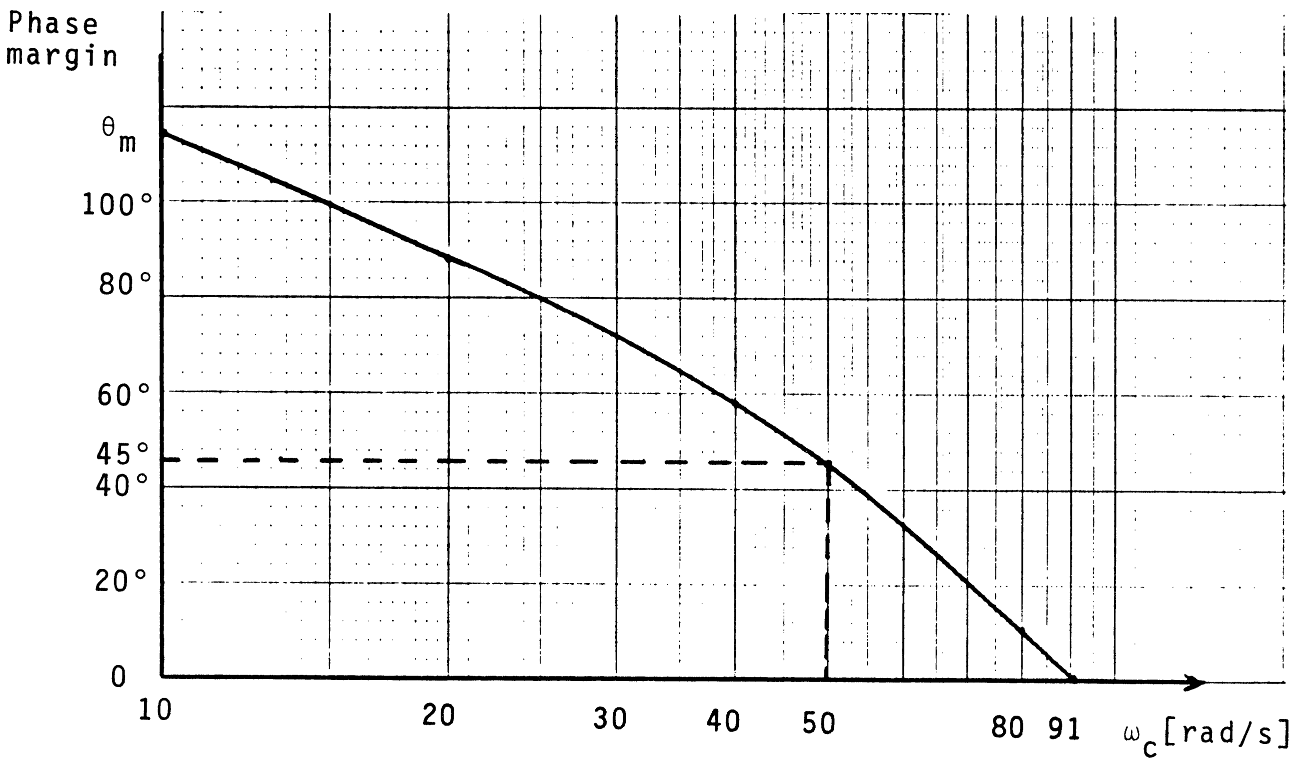

Fig. 7.3 Phase margin vs. the crossover frequency

which becomes here

$$\left| \frac{K_V K_m}{(j\omega_c T_m+1)(j\omega_c T_e+1)} \right| = 1 \qquad (7.20)$$

for $\omega_c = 91$, the corresponding value of K_v is

$$K_v = 0.09$$

We conclude that the system is stable as long as K_v is less than 0.09.

To insure adequate system response, we design the system with a certain phase margin. A common design parameter for the phase margin is 45. The graph of Fig. 7.3 shows that the phase margin of 45 can be achieved when the crossover frequency is ω_c = 50. The corresponding gain is found from Eq. (7.20) to be

$$K_v = 0.05$$

7.5 Velocity Variations

The velocity control system will experience velocity variations in response to changes in the system parameters. The most common speed variations are in response to load changes. These are described below.

Load Variations

The objective of the following analysis is to determine the velocity variations in response to a load change.

In order to analyze the velocity variations consider the block diagram representation of Fig. 7.4. This diagram describes the motor elements in detail. Note that the delay elements are ignored, as we are investigating the steady state variations.

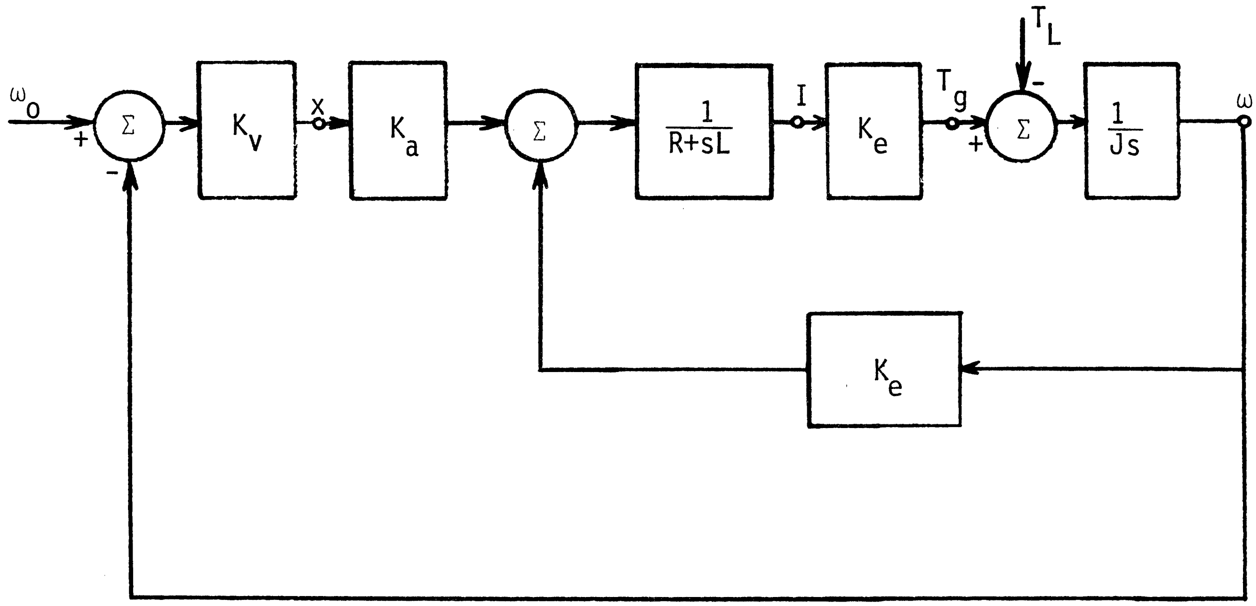

Fig. 7.4 Block diagram of velocity control system

The most simple method for analyzing the speed variation is to trace the effect of the change in parameters.

Note that a change in load by T_L, will cause a matching change in motor torque

$$\Delta Tg = \Delta T_L \tag{7.21}$$

This requires an increase in the current by

$$\Delta I = \frac{\Delta Tg}{K_t} = \frac{\Delta T_L}{K_t} \tag{7.22}$$

The change in current is caused by a corresponding change in motor voltage

$$\Delta V = R\Delta I = \frac{R\Delta T_L}{K_t} \tag{7.23}$$

Note that the effect of the inductance is ignored since the change is continuous. Similarly, the change in the back emf voltage is ignored as the speed change is small.

The motor command, X, will change by an amount

$$\Delta X = \frac{\Delta V}{K_a} = \frac{R\ \Delta T_L}{K_a K_t} \tag{7.24}$$

and the corresponding speed change is

$$\Delta\omega = \frac{\Delta X}{K_V} = \frac{R\ \Delta T_L}{K_V K_a K_t} \tag{7.25}$$

The analysis method is illustrated by the following example.

<u>Example 7.2--Load Variation</u>

Consider the system described in Example 7.1 and determine the speed variations in response to a load change of

$$T_L = 2 \text{ oz-in}$$

The required current is

$$\Delta I = \frac{\Delta T_L}{K_t} = 1A$$

The motor voltage is

$$V = RI = 2V$$

and the change in motor command is

$$\Delta X = \frac{\Delta V}{K_a} = 0.2V$$

Since the velocity gain is

$$K_V = 0.05$$

the speed change is

$$\Delta\omega = \frac{\Delta X}{K_V} = \frac{0.2}{0.05} = 4 \text{ rad/s}$$

Note that the speed change is approximately 1% of the nominal value.

Low Speed Instability

The dynamic behavior of digital control systems depends on the system velocity. When the system operates at lower speeds, the system phase margin decreases, and the loop may become unstable.

In order to understand this phenomenon, refer to Fig. 7.2. Note that at lower speeds the encoder period T_o increases. This results in a decrease in the phase margin, as shown in Eqs. (7.16) and (7.17), which may lead to instability.

The instability analysis is illustrated by the following example.

Example 6.3

Consider the system described by Example 7.1. This system is designed to operate at a velocity of 3600 rpm. At this velocity the system was designed to have a phase margin of 45 , and a crossover frequency of 50 rad/s. As the system velocity is reduced, the phase margin is decreased. Find the speed that causes the system to become unstable.

To determine the conditions for instability, rewrite Eq. (7.18) for a general period T_o

$$\theta_m = 180° - \tan^{-1}(0.14\omega_c) - \tan^{-1}(0.0015\omega_c) - \omega_c T_o \frac{360°}{2\pi} \quad (7.26)$$

Since $\omega_c = 50$, Eq. (7.26) can be written as

$$\theta_m = 180° - \tan^{-1}(7.0) - \tan^{-1}(0.075) - 2865\ T_o =$$

$$= 94° - 2865\ T_o \quad (7.27)$$

The system becomes unstable when $\theta_m = 0$. It occurs when

$$T_0 = 0.033 \text{ s}$$

This corresponds to a speed of

$$\omega_0 = 30.5 \text{ rev/s} = 1830 \text{ rpm}.$$

In other words, if this system is required to operate at speeds below 1830 rpm, it will be unstable.

Appendix A - Z Transform

DEFINITION

$$F(z) = Z\{f(k)\} = \sum_{k=0}^{\infty} f(k)z^{-k}$$

EXAMPLE: $f(k) = \{1, -1, 1, 0, 0 \ldots\}$

$F(z) = 1 - z^{-1} + z^{-2}$

SHIFTED SEQUENCE

$$Z\{f(k-m)\} = Z^{-m} F(z)$$

DIFFERENCE EQUATION

$$g(kT) = f[(k-1)T] - 0.5\ f[(k-2)T]$$

WRITE AS:

$$g(k) = f(k-1) - 0.5\ f(k-2)$$

APPLY Z TRANSFORM

$$G(z) = Z^{-1} F(z) - 0.5\ Z^{-2} F(z)$$

DERIVE THE TRANSFER FUNCTION

$$D(z) = \frac{G(z)}{F(z)} = \frac{Z^{-1} - 0.5\ Z^{-2}}{1} = \frac{Z - 0.5}{Z^2}$$

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PRODUCT DESCRIPTION

DMC 100

GL 100

GALIL MOTION CONTROL 1916-C Old Middlefield Way Mountain View, CA 94043 (415) 948-6551	DMC 100 DC MOTOR CONTROLLER

FEATURES

- Controls the motor position, velocity or torque.
- Generates a trapezoidal velocity profile with programmable acceleration and slew rates.
- Velocity level can be changed on the fly, permitting virtually any velocity profile.
- Speed range of 30,000:1.
- Generates an analog and pulse-width-modulated output command for motor control.
- Communicates via RS232, STD bus or local switches.
- No analog feedback is required. System stability is achieved by digital filtering.
- Can vary the coefficients of the digital filter. Ideal for robotics and automation.
- Fault detection--detects and reports faults that cause excessive position errors.

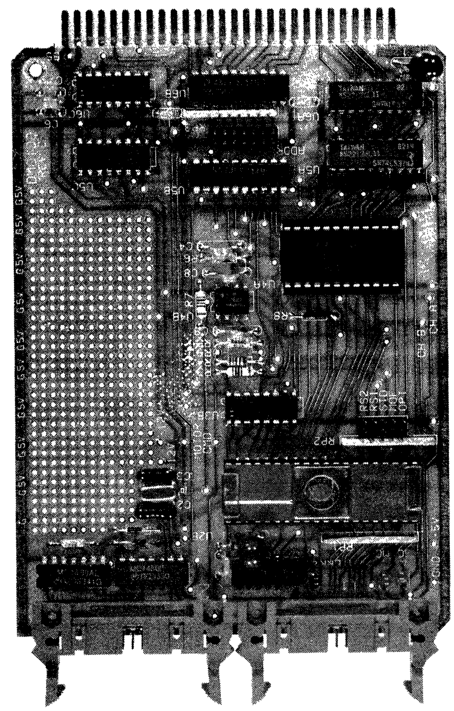

FIGURE 1 DMC 100

GENERAL DESCRIPTION

DMC 100 is a general-purpose motion controller for DC Motors. It is available as an STD Bus-compatible PC board. The feedback comes from an incremental encoder and no additional velocity feedback is required.

As a true general-purpose controller, DMC 100 can operate in numerous modes, and there is no limit on the size of the motor it can control. It can also receive commands from the STD Bus, RS232 or local switches.

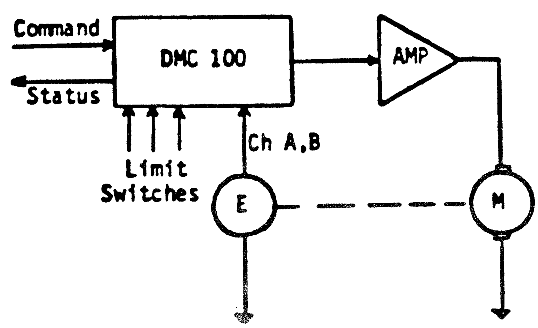

FIGURE 2 MOTION CONTROL SYSTEM ELEMENTS

OPERATION

DMC receives its feedback from an encoder or a resolver, as shown in Figure 2. It generates an output signal to drive the motor. For maximum versatility, the operation modes can be selected independently.

- Input modes: Local control by switches or remote control by RS232 or STD bus.
- Control modes: Position, velocity or torque control.
- Holding mode: Servo, deadband or shutoff mode.
- Start mode: Remote, local or combination.
- Termination mode: Remote, local or combination.
- Status reporting: Status lights, Input status, Report motion completion, fault condition.
- Digital filter: The gain, the zero, and the pole of the digital filter are programmable.

LOCAL SWITCHES

There are six local switches. These facilitate local control of the motor. The inputs to all the switches are pulled up with 1.0K resistors.

The local switches include:

Remote/Local*--Determines the control mode-local or remote. In local mode, DMC 100 ignores all remote commands.

Stop/Start*--Upon start command, the motor accelerates to the slew speed. When a stop is commanded, the motor decelerates to a stop.

Direction Switch--Determines the direction of the motion when the direction on switch or find edge modes are in use.

FWD Limit Switch*--When this switch is activated, no further motion in the FWD direction is permitted.

REV Limit Switch*--When this switch is activated, no further move in the REV direction is permitted.

Emergency Stop*--Stops any motion instantaneously.

INPUT/OUTPUT

- Data is transferred in ASCII.
- RS232--Can communicate at Baude rates of 300, 1200, and 9600.
- STD Bus interface--DMC 100 is configured as an I/O card. It uses two selectable addresses, for control and data transfer.
- Encoder feedback is TTL, two channels in quadrature. Encoder frequency must not exceed 60 KHz.
- Motor command is either an analog signal in the range of $\pm$10V, or a PWM signal with the magnitude and sign - TTL levels.
- Motion command complete*--open collector output.
- Sequence command complete*--open collector output.
- Excessive position error*--open collector output.

INSTRUCTION SET

Control Modes and Required Motion

PR($\pm$n)	Position mode, move a relative distance of n counts ($8x10^6$)**.
PA($\pm$n)	Position mode, move to absolute position n ($8x10^6$).
TQ(n)	Torque mode, set the torque level to n units (127).#
VM	Velocity mode.
SP(n)	Speed rate in counts/s (250,000).#
AC(n)	Acceleration rate in counts/s^2 ($1.3x10^8$).
DF	Required direction is FWD.
DR	Required direction is REV.
DS	Direction by local switch.
GN(n)	Compensation gain is n (255).#
ZR(n)	Compensation zero is n (255).#
PL(n)	Compensation pole is n (255).#
SM(n)	Sign magnitude mode (absolute value) when n=1, bipolar for n=0.

Holding Modes

SV	Servo mode.#
DB(n)	Deadband of ± n counts (127).#
MO	Motor shut-off.#

Start/Stop

BG	Begin motion.
ST	Stop motion--decelerate to a stop.#
AB	Abort motion--emergency stop.#
RP(n)	Repeat the motion n times. If n=0, repeat indefintely. (32,000).
RR(n)	Same as RP(n), with direction alternately reversed. (32,000).
WT(n)	A delay between moves in repeat sequence, n is in units of ms (32,000).#
SS(n)	Start on switch if n ≠ 0. Start the motion when the stop/start* switch goes low after the BG command.
ES(n)	End on switch if n ≠ 0. Stop the motion when the stop/start* switch goes high.
TO(n)	Time out--Stop after n milliseconds. If n=0, this command is inactive (4.10^6).
OE(n)	Shut off motor if position error >1024 counts and if n=1.#

Number System

DC	Input in decimal, output in Hex.#
HX	Input in Hex, output in Hex.#

Reporting

TP	Tell position (absolute).#
TE	Tell position error.#
TV	Tell velocity.#

TI Tell inputs and status#:

Bit 7 Executing sequence
6 Executing move
5 FWD limit switch*
4 REV limit switch*
3 Remote/local*
2 Stop/start* switch
1 Direction switch
0 Excessive position error

RD(n) Report H when the motion command is complete if $n \neq 0$.#

Interrogate

TQ ? Report torque command level.#

GN ? Report gain.#

ZR ? Report zero.#

PL ? Report pole.#

DB ? Report deadband.#

OF ? Report offset command level.#

Other

OF (±n) Offset motor command by n units (127).

RS Reset controller to default values.

DH Define home--Define current position as absolute zero.

FE Find edge--Search for a position where a transition occurs in the direction switch signal.

---

*Active Low.

**Values in parentheses indicate the maximum value of n.

#These instructions may be applied while the motor is moving.

ELECTRICAL SPECIFICATIONS

	MIN	TYP	MAX	UNIT	COMMENTS
Power Supply					
i_{supply}					
+ 5V		300	400	mA	
+ 12V		10	20	mA	
- 12V		10	20	mA	
Inputs					
Local Switches					1.0 K Pullup to +5V
Encoder			60	kHz	1 CMOS LOAD
STD Bus					1 LS TTL LOAD
Outputs					
Motion Complete and Error					LS05
STD Bus					LS374
Motor Command	-10		+10	Volts	TL082
i_{MC}			3	mA	
V_{OFFSET}			50	mV	

<u>CONNECTORS</u>

J1 GEN I/O (26 PIN RIBBON)

1	GND	2	SEQUENCE COMPLETE*
3	GND	4	MOTION COMPLETE*
5	GND	6	ERROR*
7	FWD	8	RVS LIMIT SWITCH*
9	REMOTE/LOCAL*	10	STOP/START*
11	GND	12	DIR RVS/FWD*
13	PWM	14	EMERGENCY STOP*
15	GND	16	SIGN
17		18	RESET*
19	+5V	20	PHASE A
21	GND	22	PHASE B
23	+12V	24	-12V
25	GND	26	MOTOR COMMAND

J2 GEN I/O (26 PIN RIBBON)

1		2	
3	RECEIVE	4	
5	TRANSMIT	6	
7		8	
9	CLR TO SEND	10	
11	DATA SET RDY	12	
13	SIGNAL GND	14	DATA TERM RDY
15	CARRIER DETECT	16	

J3 STD BUS (56 pin card edge)

1	+5V	2	+5V
3	GND	4	GND
5	N.C	6	NC
7	D3	8	D7
9	D2	10	D6
11	D1	12	D5
13	D0	14	D4
15	A7	16	NC
17	A6	18	NC
19	A5	20	NC
21	A4	22	NC
23	A3	24	NC
25	A2	26	NC
27	A1	28	NC
29	A0	30	NC
31	WR*	32	RD*
33	IORQ*	34	NC
35	IOEXP	36	NC
37	NC	38	NC
39	NC	40	NC
41	NC	42	NC
43	NC	44	NC
45	NC	46	NC
47	RESET*	48	NC
49	NC	50	NC
51	NC	52	NC
53	GND	54	GND
55	+12V	56	-12V

STD Bus Registers

ADDRESS	R/W	DESCRIPTION
N	R	Read data
N	W	Write data
N+1	R	Status D_0-data in output register* D_1-Ready to receive data D_2-Sequence complete* D_3-Excessive error*

*Active low

Galil Motion Control reserves the right to make changes in this data sheet without prior written notification.

GALIL MOTION CONTROL
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Rev. (2-84)

GALIL motion control
1916-C OLD MIDDLEFIELD WAY
MOUNTAIN VIEW, CA 94043
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GL 1000
MOTION CONTROL INTERFACE

FEATURES

- Decodes the position feedback from an incremental encoder, and generates an 8-bit counter output.
- Includes a special filter for encoder noise immunity.
- Index pulse can reset the counter to eliminate cumulative position errors.
- DAC generates a PWM motor command.

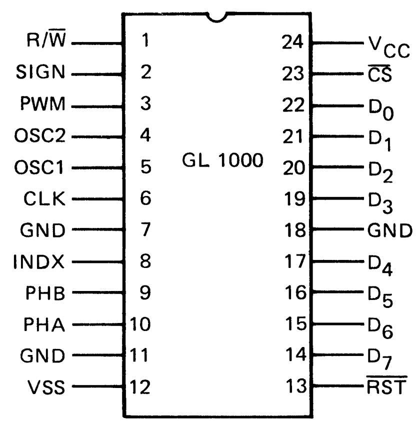

FIGURE 1. PIN DIAGRAM

DESCRIPTION

GH1000 interfaces between a microprocessor (MP) and a motion control system for dc motors. A block diagram of GL 1000 and the elements of the control system are shown in Figure 2.

GL 1000 performs two functions: DAC and position decoding. The two functions can be performed independently.

The DAC converts an 8-bit motor command from a microprocessor to a PWM signal which can be applied to the amplifier. The position decocer converts the signals of an incremental encoder to an 8-bit position feedback number that can be read by the MP. The functional elements of GL 1000 are shown in Figure 3.

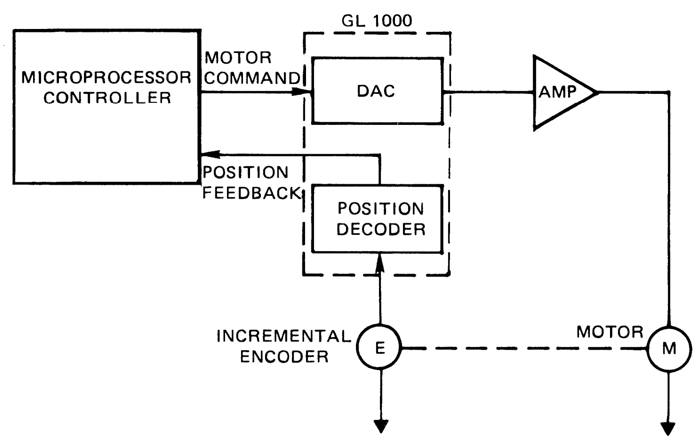

FIGURE 2. SYSTEM BLOCK DIAGRAM

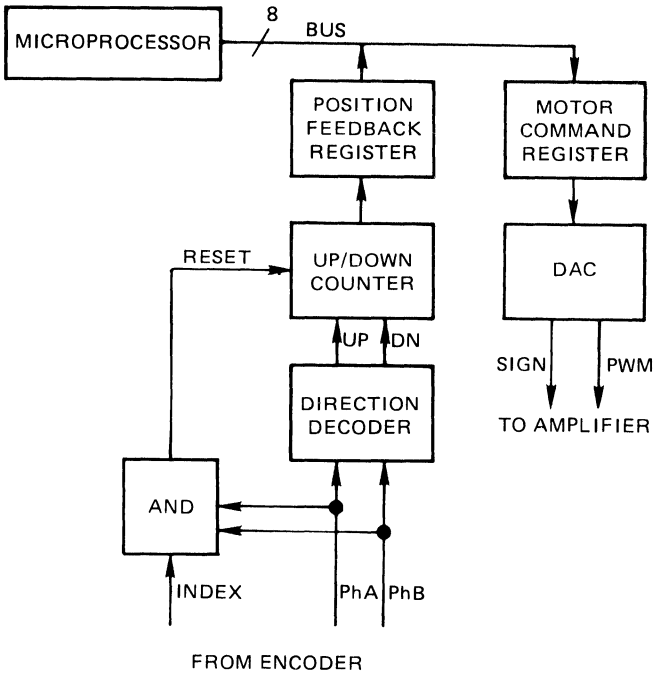

FIGURE 3. FUNCTIONAL BLOCK DIAGRAM

POSITION DECODER

The position decoder senses the direction of motion. Encoder transitions are counted by an 8-bit up-down counter. The unit of resolution of the counter is one quarter of an encoder cycle.

Figure 4 shows the relations between the encoder phases and the counter output. When the motion is forward, and phase A leads phase B, the counter counts up. When the direction of motion is reversed, the counter counts down; and when the index, and the two encoder phases are all high, the counter resets to FF.

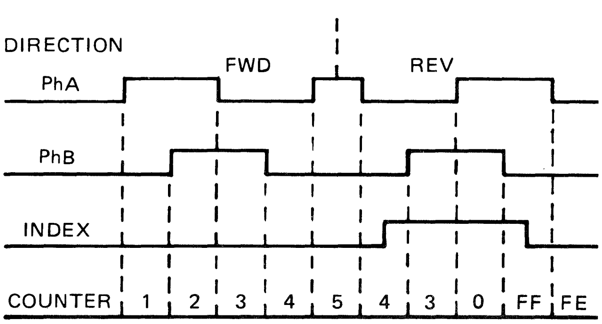

FIGURE 4. ENCODER PHASES AND COUNTER OUTPUT

CLOCK

GL 1000 can be driven by either an external or an internal oscillator. The clock frequency, f_c, must be limited to 5.25 MHz, in either case.

The circuit for connecting the oscillator is shown in Figure 5.

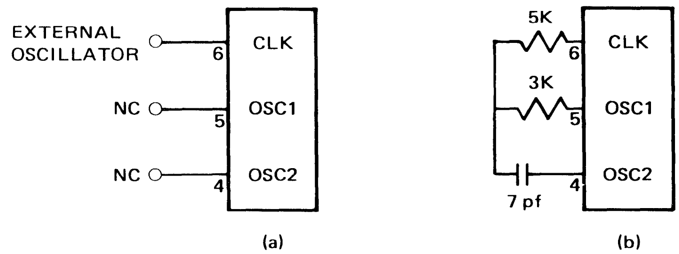

FIGURE 5. OSCILLATOR CIRCUIT DIAGRAM

TIMING

The timing constraints of the encoder signals are shown in Figure 6.

The on time, T_1, and the off time, T_2, must satisfy the conditions:

$$T_1 > 12\, T_c$$

$$T_2 > 12\, T_c,$$

where T_c is the clock period

$$T_c = \frac{1}{f_c}.$$

In order for the index to reset the counter, the index, PhA and PhB, must all be high for an interval T_3, where

$$T_3 > 4\, T_c\,.$$

The times T_2, T_2 and T_3 are shown in Figure 6.

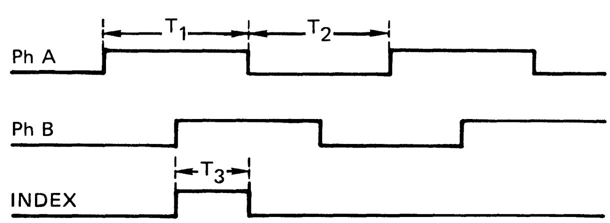

FIGURE 6. ENCODER TIME INTERVALS

DAC

The DAC converts the motor command to a PWM signal. The motor command is a number N between 0 and 255. The PWM signal has a switching frequency f_{PWM}, where

$$f_{PWM} = \frac{f_c}{256},$$

and a duty cycle of N/256, as shown in Figure 7.

The second output of the DAC is the sign bit. This equals the MSB of N.

The PWM signal can be applied directly to the power amplifier to control the motor. Alternatively, it can be filtered to generate an analog signal for amplifier control. A low-pass filter for the PWM signal is shown in Figure 8. The filter can be designed to give additional compensations if necessary.

READ/WRITE

In order to read the counter output, set R/$\overline{W}$ high and $\overline{CS}$ low. This places the output of the counter on the data bus. For details, note the timing diagram of Figure 9.

In order to guarantee that the data is stable while it is read, the counter stops counting while the signal $\overline{CS}$ is low. To avoid missing counts, limit the diration of $\overline{CS}$ to 500 ns, or lower the maximum encoder frequency.

In order to write to the motor command register, R/$\overline{W}$ must be low and the data must be stable for at least T_{set} before $\overline{CS}$ goes high. Data is latched on a rising edge of $\overline{CS}$.

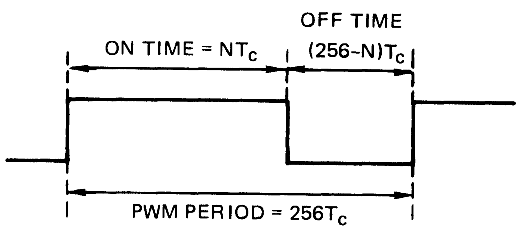

FIGURE 7. PWM SIGNAL

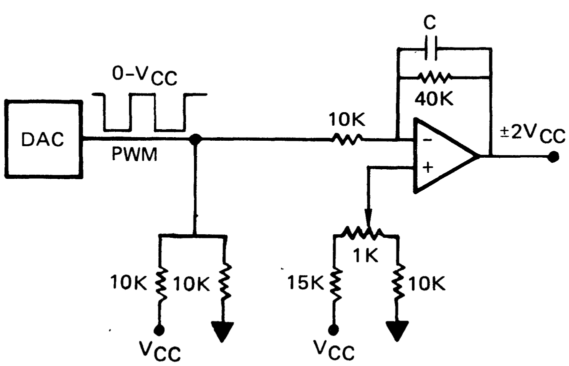

FIGURE 8. LOW PASS FILTER

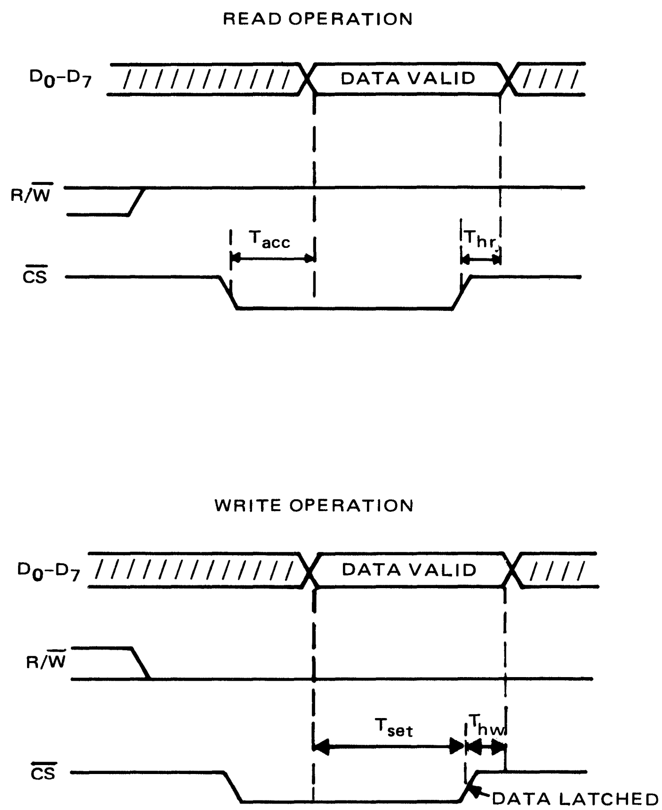

FIGURE 9. TIMING DIAGRAMS

TABLE 1. SPECIFICATION

Parameter	Definition	Min
T_c	Clock Period = $1/f_c$	190 ns
T_1	Encoder on time (Figure 6)	12 T_c
T_2	Encoder off time (Figure 6)	12 T_c
T_3	Index time (Figure 6)	4 T_c
T_{acc}	CS to data valid	150 ns
T_{set}	Data set up	100 ns
T_{hR}	Read hold time	20 ns
T_{hW}	Write hold time	10 ns

PIN DESCRIPTION

V_{CC} — 5V supply.

V_{SS} — Ground.

D7–D0 — Bidirectional data bus.
D7 - MSB.
D0 - LSB.

$\overline{\text{Reset}}$ — Input, asynchronous reset, active low. Upon reset, the outputs of GL 1000 are set at:

Sign = 1.

PWM = { Tristate when $\overline{\text{reset}}$ is low; 50% duty cycle (N = 128) when $\overline{\text{reset}}$ goes high.

Position register = 0

PhA — Input, Phase A of incremental encoder.

PhB — Input, Phase B of incremental encoder.

Index — Input, Index signal, resets the position counter when Index, Phase A and Phase B are all high.

R/$\overline{\text{W}}$ — Input, Read/write control.
1 reads the position feedback register.
0 writes to the motor command register.

$\overline{\text{CS}}$ — Input, chip select, active low. During a write operation, data from the position register is placed on the bus when the chip select is active.

CLK — Input, external clock input.

OSC1, OSC2 — Pins for self generating clock (See Figure 5).

Sign — Output, MSB of the DAC register equal 1 upon reset.

PWM — Output, Pulse width modulated output. Switching frequency equals $f_c/256$. Duty cycle equals N/256, where N is the value in the motor command register.
When $\overline{\text{reset}}$ is low, the PWM is in the tristate condition. Upon positive transition, PWM assumes 50% duty cycle.

GND — Ground.

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