DESIGNING COMPUTER GAMES

TO FACILITATE LEARNING

by

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Abstract

The aim of this thesis was to explore the design of interactive computer learning environments. The particular learning domain selected was Newtonian dynamics. Newtonian dynamics was chosen because it is an important area of physics with which many students have difficulty and because controlling Newtonian motion takes advantage of the computer's graphics and interactive capabilities. The learning environment involved games which simulated the motion of a spaceship on a display screen. The purpose of the games was to focus the students' attention on various aspects of the implications of Newton's laws.

Playing the games did improve the students ability to solve Newtonian dynamics problems. It is hypothesized that the games facilitated understanding because the microworld embodies Newton's laws in a way that links everyday beliefs about force and motion to formal physics knowledge, because it provides feedback as to how everyday beliefs fail, and because the games focus students' attention on areas where their knowledge needs revising.

The design of the games and microworld was based on an analysis of why students have so much difficulty with Newtonian dynamics. This was done by taking protocols of students solving basic force and motion problems. The results revealed that the students possessed many kinds of knowledge, such as beliefs derived from living in a world with friction and prior experiences with addition, which interfered with their ability to understand Newtonian dynamics. The games and microworld were then redesigned to more effectively help the students revise their misconceptions and to draw upon aspects of their knowledge which would help them to better understand Newtonian dynamics. Finally, general principles of designing interactive computer learning environments were induced from this design process.

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Chapter I

Introduction

The development of powerful and yet inexpensive micro-computers has created the potential for designing stimulating, new learning environments. The challenge is to combine the interactive capabilities of micro-computers with research on learning and cognition to produce computer environments that help people to learn.

This thesis focuses on designing computer games to facilitate learning. In order to study the design process, a specific learning domain, Newtonian dynamics,[1] was selected.

The goal was to design a Newtonian computer microworld and an accompanying sequence of games formulated within the microworld which would help students understand Newtonian dynamics. To achieve this, the kinds of difficulties that students encounter when solving force and motion problems were located and the cognitive origins of these difficulties were analyzed. This information was used to design games which focused students' attention on overcoming their sources of difficulty.

A further goal was to abstract some general principles of designing computer learning environments which could then be applied to other domains.

1. Newtonian dynamics is part of elementary physics. It is the subset of Newtonian mechanics concerned with determining how forces affect the motion of objects.

1.1 Taking Into Account Prior Knowledge.

Students' attempts to understand Newtonian dynamics provide an excellent illustration of how one's already existing knowledge influences the assimilation of new knowledge.

The curriculum for teaching this topic does not appear to be difficult. There are only a few laws and formulas. Furthermore, students are given the simple procedure of vector addition as a way of computing the effects of forces on motion. Yet, this study will show that students have difficulty solving even very simple force and motion problems. Their justifications for incorrect answers indicate that their misunderstandings come from many sources. These include inaccurate beliefs about motion which have been derived from living in a world with friction and erroneous interpretations of vector addition which have been derived from knowledge of scalar arithmetic. The results emphasize the need, when designing learning environments, to take into account not just what the students need to know but, also, what they already know.

1.2 The Microworld

The learning environment utilized to help students overcome their difficulties with Newtonian dynamics is a Newtonian computer microworld. The term microworld, in this context, refers to an environment governed by a small subset of the numerous laws which govern the everyday world. In this particular instance, it is governed by Newton's first two laws of motion:

(1) in the absence of forces, objects at rest stay at rest and objects in motion maintain uniform motion in a straight line,

(2) a force applied to an object alters the motion of the object according to the formula $F = ma$ (force equals mass times acceleration).

The computer microworld embodies these two laws by simulating the motion of an object on a display screen. The student can control the motion of the object by applying impulse forces to it in various directions. The impulse forces, which are implemented by pressing buttons on a keyboard, affect the motion of the object in accordance with Newton's laws.

1.3 Games as a Mechanism for Focusing Attention.

Within the context of such a microworld, one could either set the learners free to explore as they choose or one could give them some activities to pursue. Giving the learners specific goals to achieve has one distinct advantage: the goals can be used to help the student to discover the implications of Newton's laws.

To illustrate, setting a goal, such as hitting a target or navigating a maze, creates a game-like challenge. While trying to achieve the goal of the game, the students' attention will be focused on specific aspects of Newtonian dynamics. By changing the goal, their focus of attention can be changed. Further, one can use this focusing mechanism to get people to pay attention to things they might otherwise ignore. For example, many students in this study had incorrect ideas concerning how forces would affect the speed of motion. By designing games that focus on achieving certain speeds, their attention can be drawn to the inaccuracies and inconsistencies of their beliefs.

1.4 Why Choose Newtonian Dynamics?

Newtonian dynamics was chosen as the domain for exploring the design of computer learning environments for several reasons.

One reason is that Newtonian dynamics can take advantage of the computer's graphics capability. The motion of objects can be portrayed very effectively on a display screen. Further, the computer provides a way of representing theoretically ideal motion that is easily modified to encompass more advanced physics. For example, inverse square forces like gravity can be added, or collisions could be included in the behavior of the computer microworld.

Another reason is that prior research by Clement [1979], diSessa [in Papert et al. 1979], and Viennot [1979] indicates that students have very basic misconceptions which interfere with their understanding of Newton's laws of motion. Since Newton's laws are fundamental to much of physics, it is important that the students have some way of overcoming these misconceptions.

1.5 Everyday Experience as a Source of Confusion.

It is not surprising that people have difficulty with Newtonian dynamics since most real world situations do not appear to satisfy Newton's laws. The real world is confusingly complex. It includes friction and gravity and has non rigid bodies that do not correspond to the point masses of the formal physics.

As a result of such sources of confusion, many ancient thinkers had erroneous ideas concerning forces and motion. For example, the great philosopher Aristotle [1941] believed that the natural state of earthly objects was stopped and Philo [Jammer, 1957] thought that angels were responsible for the motion of celestial bodies. It was not until Newton came along in the seventeenth century, trying to explain the motion of the planets, that these laws of motion were discovered.

The implications of Newton's laws are thus very counterintuitive for someone brought up in a world where unseen frictional forces are operating. For example, the first law states that when an impulse force, such as a kick, is applied to an object, the object will keep moving forever at a constant velocity as long as no other forces act upon the object. However, objects in a world with friction do not keep going forever at a constant velocity, rather, they keep slowing down until they stop. Further, when an object has a constant velocity, one assumes that there must be a continually acting force, such as an engine, producing the constant velocity. Clement [1979] has observed that students erroneously extend the belief that *constant motion implies a constant force* into frictionless situations.

The implications of Newton's second law are equally counterintuitive. For instance, when a force, like a kick, is applied to an object which is already moving, the resulting velocity is related to the original velocity of the object, the mass of the object, and the size and direction of the force being applied. However, in a world where friction operates, objects are usually stopped when a force is applied to them. This means that the original velocity of the object does not have to be taken into account because it is zero. This leads to the induction of the belief that *an object should move in the direction that it is kicked.* DiSessa [in Papert et al. 1979] has described how people incorrectly employ this belief to situations where the object is moving, not stopped, when the kick is applied.

Both the belief that *an object should move in the direction that it is kicked* and the belief that *a constant velocity implies there must be a constant force producing that velocity* could be described as beliefs which fail to take into account the momentum of the object. It is suggested that people ignore momentum because they live in a world with friction and friction eventually negates previously applied forces.

One potential solution to this difficulty is to provide students with experiences in a frictionless environment. The computer microworld, embodying Newton's first two laws, is just such a frictionless world. The movement of the Newtonian object within the computer microworld represents the effects of forces on motion in a theoretically ideal form. Initially, there are no extraneous complications, such as friction, to distract and confuse the learner. There is just Newtonian motion in a pure and simplified form. This should provide students with the kind of experience which would permit the induction of the correct beliefs about force and motion.

Providing students with frictionless environments is not a new idea. Frictionless pucks have been used in physics classes to demonstrate Newton's laws for decades. However, the tasks given to the students within the computer microworld go beyond frictionless puck demonstrations in many important respects discussed in the following sections.

1.6 Why The Microworld Should Facilitate Learning.

Beyond the potential for deriving correct beliefs is the potential for modifying erroneous beliefs. The students will approach the microworld with expectations, such as, *things move in the direction that you kick them*, which they have derived from living in a world with friction. Their expectation as to the effect of an impulse will be violated indicating to the students that there is something wrong with their beliefs. Further, the microworld will give them feedback as to how their idea failed. The games should thus facilitate the process of knowledge evolution since they encourage the students to think about how to achieve a given effect, try it out, get feedback, and then modify their approach.

This raises the important issue of the role of feedback in knowledge evolution. One of the advantages of working in the computer microworld as opposed to textbook problem solving is the amount and character of the feedback. The computer microworld provides immediate feedback as to whether or not a given strategy will work. In addition, one gets information as to how one's strategy fails. The computer microworld thus provides a quality of feedback not found in ordinary problem solving.

Another feature of this microworld is that the students can utilize general problem solving heuristics to successfully achieve goals. For example, suppose students are trying to hit a target and they fire an impulse in a direction that they thought would make the object turn the amount needed in order to intersect the target. However, the object turns in the right direction but not as much as they thought it would, so they fire another impulse in the same direction and that completes the desired turn. The general heuristic used is *if one almost works but not quite, try two.* The students can thus achieve some success in this domain without having completely understood the physics involved. This is important since students find the behavior of the Newtonian object counterintuitive. If the games required an a priori knowledge of the physics, then the students would find the games dull if they already knew the physics and impossible if they did not. In either case they are not likely continue playing the games. However, if the students can attain some success via some other means, they will be more motivated to keep on exploring the domain and not give up in frustration.

One might be tempted to conjecture that achieving success, without knowing the physics involved, would be nonproductive. However, the results of this research suggest that this is not the case. The students did learn about Newtonian dynamics from playing the games even though they often relied upon a combination of intuitive beliefs, feedback, and general heuristics to succeed. The success of the heuristics kept the students motivated and helped them to discover phenomena relevant to Newton's laws of motion.

An additional potential of this way of representing Newton's laws is that it will encourage the learners make connections between their intuitive beliefs about the effects of forces on motion and their knowledge of formal physics encompassing Newton's laws and vector addition. The microworld, even though it has no friction, does share features of the everyday world: the object moves and one can apply a force to it. Thus the learners' beliefs about force and motion derived from everyday experience will be invoked. However, the expectations associated with these beliefs will be violated. This should then invoke ideas derived form physics class as an explanation for the behavior of the Newtonian object. The students' beliefs about force and motion will then be modified, the formal physics and the everyday world thus having been linked.

Further, the fact that the games are like situations that happen in the real world should help the students learn about the application of physics principles to real world situations. It should help develop knowledge about when Newton's laws are relevant to controlling and explaining the motion of objects that they have to deal with in their everyday lives.[1]

1.7 Games as a Form of Problem Solving.

Such games are a form of problem solving where the student has to figure out how to achieve a given goal such as hitting a target or reducing an object's speed. This differs from most textbook problems where the student is asked, for example, to predict the acceleration given the mass and the force. Textbook problems are more often of the form *what would happen if...* as opposed to *how do you achieve ....* The former could be described as predicting the consequences of an activity, whereas, the latter involves determining the precursor or antecedents to an activity.

Furthermore, the textbook problems are usually quantitative in nature, requiring one to compute some value from the appropriate formula. In contrast, the games are qualitative involving knowledge of general changes in direction or speed caused by given impulses. Students need to invoke their knowledge about what happens rather than their knowledge of formulas. This study indicated that it is this knowledge about what happens, the qualitative implications of Newton's laws, that is the source of the students' difficulty. Most of them achieved good grades on physics tests where they had to plug numbers into formulas. However, when it came to answering basic questions about forces and motion, the students had many misconceptions.

In order to determine what the individual games should focus on so that they can help students understand Newtonian dynamics, one has to trace the sources of these misconceptions. To facilitate this

1. The reader is referred to diSessa [1981] for arguments relating to the need to link the formal physics to common-sense and intuitive knowledge.

process, protocols of students solving basic force and motion problems were taken. Special attention was paid to the errors they made and to the justifications which they gave for their answers.

1.8 Methodology

The aim was thus to design a sequence of games set in the context of a Newtonian computer microworld which would help students to understand Newtonian dynamics. The games were used as a means of focusing the students' attention on various aspects of the implications of Newton's laws. The criteria for deciding what a given game in the sequence should focus on was based on an analysis of the errors that students made while solving force and motion problems.

The design process was an iterative one. Preliminary work suggested what properties of Newtonian dynamics the students needed to realize in order to overcome their difficulties. Games were created on the basis of this preliminary analysis. Also, a questionnaire consisting of basic force and motion problems was designed to provide more data on the sources of difficulty and to measure the effectiveness of the games. The questionnaire was then administered on two separate occasions to two groups of students who had studied Newtonian dynamics. One group played with the computer games after being given the questionnaire the first time, the other group did not. This data was then analyzed to determine the effectiveness of the games and to determine what difficulties the students had. The potential origins of these difficulties were then explored and the results were used to suggest how the microworld and the game sequence could be improved. Finally, general principles of designing interactive learning environments were abstracted from this design process.

1.9 Results

Trying to achieve goals, such as controlling the direction of motion, within the interactive Newtonian computer microworld did help students to solve Newtonian dynamics problems. This result is taken as a positive indication of the potential of this approach.

The games provided an environment where the students could debug their knowledge that relates to describing how forces affect motion. This occurred by facilitating the use of general problem solving heuristics and by providing feedback.

The kinds of difficulties which students exhibited when solving Newtonian dynamics problems went beyond the ignore momentum difficulties described previously. In fact, under many conditions, most of the students could qualitatively predict the effect that a force would have on altering the direction of motion. However, they often failed to take into account the current speed of motion as a factor in quantifying the change in the direction of motion. Further, many of the students had misconceptions concerning how applying forces would affect the speed of motion.

The students' justifications for their answers indicated that the sources of such difficulties are past experiences which interfere with learning Newtonian dynamics. Further, their past experiences which cause problems are many and diverse. For instance they include the fact that everyday experiences controlling motion are complicated by unseen frictional forces. This leads to the derivation of erroneous beliefs concerning how forces relate to the motion of objects. Also, the fact that the students originally learned about addition in a scalar, not vector, context causes difficulty. When the students are exposed to vector addition, they mistakenly assume that it has the same properties as the familiar domain of scalar arithmetic. However, there are differences between scalar and vector arithmetic and not realizing these differences leads to errors. Finally, the students' past experiences which cause difficulty include the fact that the vectors are represented geometrically by arrows. Arrows are a familiar representational device. Prior to their use in the vector context, they have always represented direction, not speed and direction. Further, there is confusion as to what the length of a vector represents. Its use varies depending on the context: it can represent displacement, size of a force, speed of motion, and so on.

When designing an environment to help students understand Newtonian dynamics, one has to take into account these sources of difficulty and create tasks which will help students to realize their misconceptions and help them evolve towards a more useful and consistent way of thinking about forces and their effects upon motion. To achieve this, some general principles were utilized when designing the games and microworld. These principles are briefly summarized as follows:

- focus the students on aspects of their knowledge that need revising.
- encourage better ways of representing and thinking about the domain.
- facilitate the use of problem solving heuristics.
- encourage the application of relevant knowledge from other domains.
- represent the phenomena of the domain clearly.
- eliminate irrelevant complexities from the computer microworld.

1.10 Summary

The aim of this thesis was to explore the design of interactive computer learning environments. The particular learning domain selected was Newtonian dynamics. Newtonian dynamics was chosen because it is an important area of physics which many students have difficulty with and because controlling Newtonian motion takes advantage of the computer's graphics and interactive capabilities. The learning environment involved games which simulated the motion of a spaceship on a display screen. The purpose of the games was to focus the students' attention on various aspects of the implications of Newton's laws.

Playing the games did improve the students ability to solve Newtonian dynamics problems. It is hypothesized that the games facilitated understanding because the microworld embodies Newton's laws in a way that links everyday beliefs about force and motion to formal physics knowledge, because it provides feedback as to how everyday beliefs fail, and because the games focus students' attention on areas where their knowledge needs revising.

The design of the games and microworld was based on an analysis of why students have so much difficulty with Newtonian dynamics. This was done by taking protocols of students solving basic force and motion problems. The results revealed that the students possessed many kinds of knowledge, such as beliefs derived from living in a world with friction and prior experiences with addition, which interfered with their ability to understand Newtonian dynamics. The games and microworld were then redesigned to more effectively help the students revise their misconceptions and to draw upon aspects of their knowledge which would help them to better understand Newtonian dynamics. Finally, general principles of designing interactive computer learning environments were induced from this design process.

Chapter II

The Pilot Study

One aim of this thesis was to design a Newtonian computer microworld and sequence of problem solving activities formulated within the microworld which would help students to understand Newtonian dynamics. A major difficulty with this goal is that the same sequence of problem solving activities may not be helpful to all students. This is especially likely to be true for students with differing amounts of formal physics education.[1] What is a challenge to a physics naive[2] student may be trivial for a more advanced student and, similarly, what interests an advanced student may be meaningless to a more naive person. For this reason, the decision was made to design the games for students at a certain level with respect to their formal physics education. After the sequence of games has been tried out with such a subpopulation of physics students, the issue of whether the same sequence of games can help most students at a given level or whether one has to customize the game sequence to fit each student's idiosyncratic difficulties can more appropriately be discussed.

One purpose of the pilot research was thus to determine what level of students would be most helped by interacting with such a Newtonian computer microworld. Two further objectives were to gain preliminary evidence as to:

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1. The amount of formal physics education is the most obvious individual difference that will affect the usefulness of the games to a particular student. There are undoubtedly other individual differences that will emerge as affecting the suitability of a given game for a given person.
2. The expression "physics naive" will be used to refer to a person with no formal physics education.

- how people interact with such a microworld,
- what difficulties people have with understanding Newtonian dynamics.

These results were then utilized when designing the games.

2.1 Observing Subjects Interacting with a Newtonian Computer Microworld.

A variety of people who either visited or worked at the M.I.T. LOGO Laboratory were observed while playing a computer game implemented in the Newtonian microworld.

2.1.1 Subjects

These people included:

- 2 secretaries - no physics background,
- a foreign visitor - no physics background,
- a six year old child - no physics background,
- a psychology graduate student - no physics background.
- a teenager - had some physics in school,
- 2 school teachers - both had one year of college physics,
- 3 A.I. graduate students - all had several years of college physics,

2.1.2 Description of the game of Target

The computer game that these subjects played was called Target. This game was created by diSessa and Watt in 1978 and was implemented in the Newtonian microworld. It involves trying to direct the motion of a triangular object, which diSessa called a dynaturtle, so that it hits a circular target (see figure 1).

At the beginning of each game of Target, the dynaturtle is displayed in the same starting position, in the stopped state, and has a heading of zero. The circular target is positioned at 45 degrees with respect to the dynaturtle. Initially the person cannot aim directly at the target since the dynaturtle can only be turned in 30 degree increments. That is, only headings of zero, thirty, sixty degrees, etc., are possible (as illustrated in figure 2). When the dynaturtle hits the target, the speed is printed out. The aim of the game is to hit the target with as low a speed as possible.

The motion of the dynaturtle is controllable by applying kicks to it in any one of a fixed set of directions. The kicks are executed by pressing buttons on a key board:

Fig. 1.

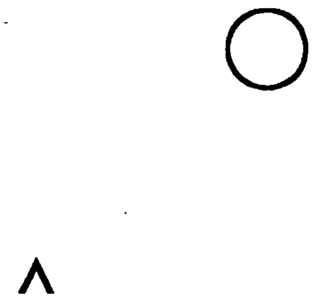
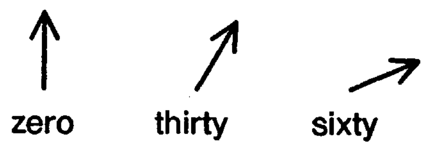


Fig. 2.



(1) pressing the buttons R and L on the keyboard reaim the dynaturtle by turning it to the right or left 30 degrees without affecting its motion,

(2) pressing K on the keyboard gives the dynaturtle a kick in the direction it is currently aiming. The effect of the kick is to add to the existing velocity of the dynaturtle a unit of speed in the direction the turtle is currently facing. The dynaturtle then moves with the resulting velocity.

For example, if the dynaturtle were stopped and one gave it a kick by pressing button K on the keyboard, the dynaturtle would move across the screen at a constant speed in the direction that it was kicked until it crashed into the edge of the screen. If one gave the dynaturtle another kick before it crashed into the edge of the screen, the velocity imparted by this second kick would be added to the dynaturtle's original velocity and the dynaturtle would then move with the new, resulting velocity in accordance with Newton's second law of motion.

The motion of the dynaturtle across the CRT display screen is achieved by a computer procedure entitled "LOOP". This procedure is constantly erasing and redisplaying the dynaturtle in a new location. The rapidity with which the computer does this creates the impression of smooth motion at a constant speed. The new location of the dynaturtle is computed by incrementing the dynaturtle's horizontal and vertical position on the screen according to the dynaturtle's current velocity. The LOOP procedure is also constantly testing to see if the player has given the dynaturtle a kick. If so, the dynaturtle's horizontal and vertical components of velocity are incremented appropriately. The effect of a kick is thus to instantaneously accelerate the dynaturtle.

2.1.3 Results.

The most striking results had to do with the interplay between the subjects' use of

(1) intuitive knowledge about how forces affect motion,

examples: (i) *objects go in the direction that you kick them*, and (ii) *if you kick an object, it will go for a ways and then stop*,

(2) formal physics knowledge,

examples: (i) *the kick adds a component of velocity to the original velocity*, and (ii) *in the absence of other forces, a kick will produce constant eternal motion*,

(3) knowledge from other domains,

examples: (i) timing: *if you get there late, leave earlier next time*, and (ii) cancellation: *an operation and its inverse neutralize each other's effect*,

(4) and general problem solving heuristics,

examples: (i) *if one works almost, but not enough, try two*, and (ii) *try experimenting with the operators to see what happens.*

For example, the six year old, who clearly had no formal physics background, was able to succeed very quickly at hitting the target. The fifth time he played the game he hit the target. He achieved this success by debugging his naive expectations as to what the effects of kicks should be. This was done through a combination of feedback from the microworld and what might be termed general problem solving heuristics or common sense reasoning. He appeared very systematic in his approach.

On the first trial he aimed slightly below the target (initially one could not aim directly at it) and gave the dynaturtle a kick. Then, when he was passing through a region where one could aim directly at the target, he aimed at the target and gave the dynaturtle a kick. This failed as shown in figure 3. This strategy implies that he had the expectation that the dynaturtle should always move in the direction that it is kicked, regardless of its current velocity.

He then tried the same thing again only this time he started by aiming slightly above the target instead of below it. It was as though he was testing to see if the result would be symmetrical (see figure 4).

On the third and fourth trials he made what looked like procedural errors as opposed to conceptual

Fig. 3.

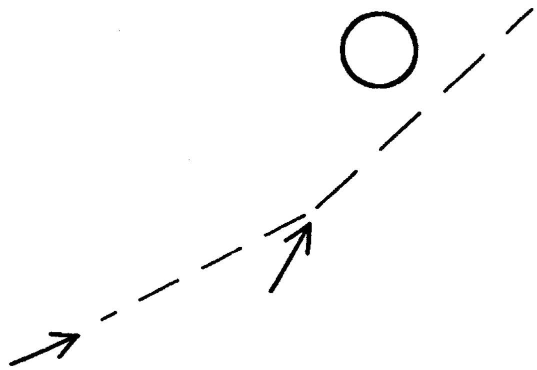
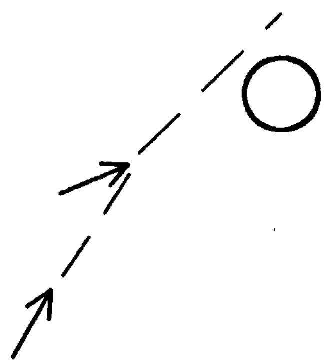


Fig. 4.



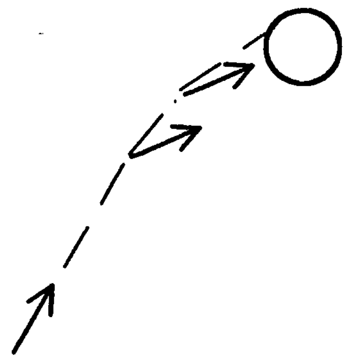
errors.[1]

On the fifth trial, he aimed slightly above the target again and kicked. Then he reaimed, in the same direction as he had done before, but he fired the kick much earlier than previously. When he saw that the dynaturtle was going to miss the target, he kicked again and thereby succeeded in hitting the target. (Illustrated in figure 5)

He described his new strategy by saying that he kicked the second time sooner than he had done on

1. On the third trial, he was executing the same strategy as above and got into a position where he could aim directly at the target. Then he aimed at the target, however, he forgot to actually press the button which gives the dynaturtle a kick. During the fourth trial, he tried the same strategy only he reversed the second sequence of aiming and kicking - he kicked and then aimed instead of aiming and then kicking. This also failed.

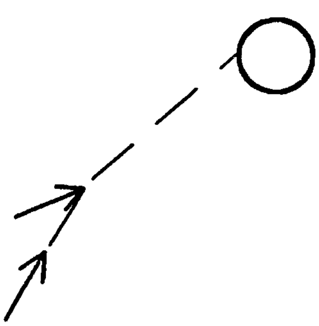
Fig. 5.



prior trials since on prior trials the dynaturtle was late and missed the target. This indicates that he was applying general ideas about timing which he would have acquired from previous experiences in the everyday world. For example, this problem is solved by analogy to the situation, if you arrive at school late, leave earlier next time. This translates into, if you miss the target because you kicked too late, kick earlier next time.

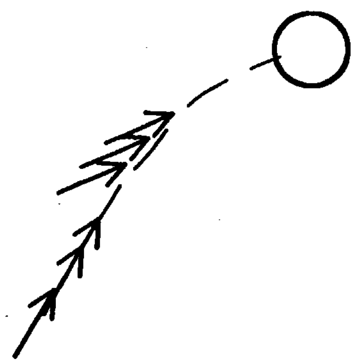
On the following trial, he repeated the same strategy except that he applied the second kick immediately after the first kick. This also succeeded as shown in figure 6.

Fig. 6.



He then continued by experimenting with kicking many times - both after the second and first kicks. The end result was that he would aim slightly above the target, kick several times, then quickly reaim, and continue kicking so that the dynaturtle went in a curved path towards the target and hit with a high speed (see figure 7).

Fig. 7.

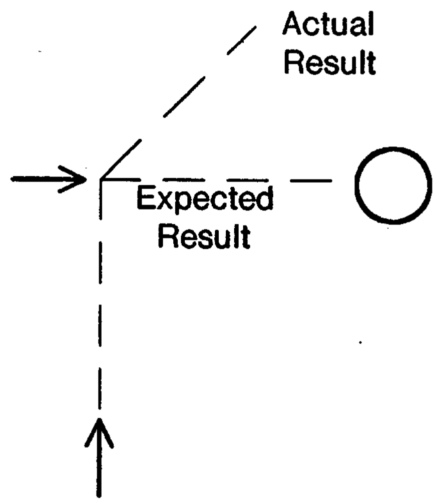


The striking result was the success and systematic approach achieved by this six year old. In fact, he was more able to adapt to this microworld than many of the older physics naive subjects. This may have been because he had fewer preconceived notions as to how forces should affect motion. Alternatively, he might have been more willing to give up his preconceived ideas since he is used to revising his theories of how things behave based on new experiences. Older subjects may be more dogmatic, at least with respect to beliefs concerning how forces affect motion, because they are more sure of their ideas having had many more years of motion related experiences than the six year old.

Although all of the protocols, like that of the six year old, are interesting, it would be too lengthy to present them all here. Thus, in order to enumerate further such interplays between intuitive physics, formal physics, and general problem solving techniques, we will describe some of the most commonly observed strategies used in playing Target and discuss how they could have been developed. This collection of strategies follows closely that discovered by diSessa [1981].

The most obvious strategy to try is to aim where one wants to go and then kick. However, the game is set up so that this is not possible - one cannot aim directly at the target from the starting point. What typically happens is that people give an initial kick. Then, when they get to a position where they can turn and aim directly at the target, they aim at the target and kick again. This strategy is illustrated in figure 8. The naive expectation based on intuitive physics is that the dynaturtle should go in the direction that it is kicked. However, this does not happen within this frictionless Newtonian microworld because the new

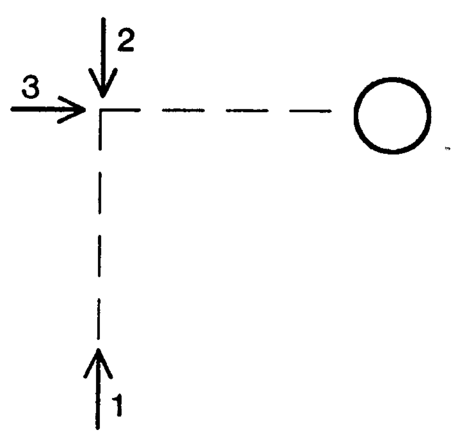
Fig. 8.



component of velocity gets added to the initial velocity and the dynaturtle overshoots the target.

There are several ways to transform this unsuccessful strategy into one that works:
(1) When one gets to a position where one can aim directly at the target, instead of just aiming at the target and kicking, stop first by kicking backwards (i.e. kicking in a direction that is opposite to the current direction of motion). Then aim at the target and kick as shown in figure 9. This strategy succeeds since the belief that *things go in the direction that you kick them* holds true when one is not moving. Hence, if one can stop the dynaturtle, one can now employ this belief to hit the target. This strategy of utilizing a

Fig. 9.

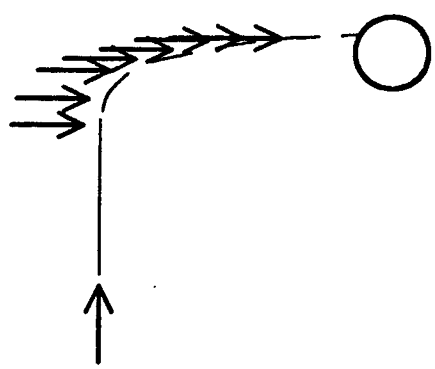


backwards kick thus reduces the situation to a simpler case - the stopped state.

One could claim that knowing how to stop the dynaturtle implies believing that the results of forces add together. However, one could also claim that knowing how to stop appeals to a more primitive notion of cancellation which even people who have never studied physics have from studying scalar arithmetic. In further support of this view is the result of diSessa [in Papert et al. 1979] who reports that most of the elementary school children he observed used "antikick" early on in their attempts to hit the target. Also, as will be reported in a later chapter, high school students could answer the question, "How can you make yourself stop?", more easily than they could other questions concerning how to induce changes in velocity. Thus it has been argued that this strategy could be devised by either a physics naive or sophisticated subject.

(2) Another method is that after kicking and getting to a position where one can aim at the target, instead of kicking just once, kick many times (see figure 10).

Fig. 10.

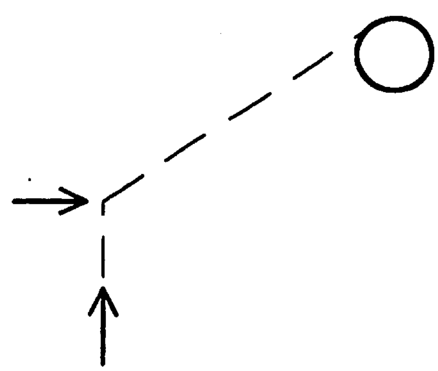


From the point of view of formal physics, this strategy of kicking many times is very similar to the first strategy of stopping and then reaiming. What one is really doing by kicking many times is making the final velocity so large compared to the initial upwards velocity that the initial velocity approximates zero. A physicist might derive both strategies from having the goal of eliminating the initial velocity.

However, this strategy could also be derived using more general problem solving heuristics. While experimenting within this microworld, the naive person would notice that one kick in the horizontal direction produced a turn, not the 90 degree turn expected but a turn nonetheless, and kicking again produces more turning. Hence, he or she might conclude that if one wants to make a 90 degree turn, just keep kicking in that direction until the turn is completed because the more one kicks in that direction, the more the dynaturtle turns in that direction. Thus, this strategy could also be derived by a physics naive person who utilizes observation and feedback combined with the general heuristic, *if one works a little but not enough, try more than one.*

(3) The original strategy can also be modified by reaiming and kicking the dynaturtle before one gets in line with the target (as shown in figure 11)

Fig. 11.



This is the kick early strategy derived by the six year old. It was suggested previously that he derived this strategy by using general knowledge about timing: *if you get there too late, leave earlier next time.* It could also be derived by a form of geometrical reasoning. This could be done by noticing that when one turns 90 degrees and kicks, one ends up going at 45 degrees. If one starts at the target and projects a 45 degree path backwards until it intersects the original upwards path of the dynaturtle, one gets the point at which one could in fact turn 90 degrees, kick, and hit the target (see figure 12).

In contrast, a physicist might derive this strategy in a different way. Immediately after the first kick in the upwards direction, the physicist might ask, "what component of velocity do I have to add to my existing velocity in order to move in the direction of the target?" This process is illustrated in figure 13. The physicist would then deduce that from this particular location, given this particular initial velocity, a kick in the horizontal direction is needed in order to hit the target.

Fig. 12.

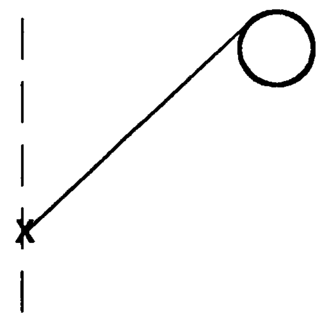
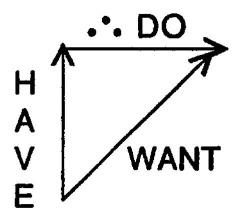
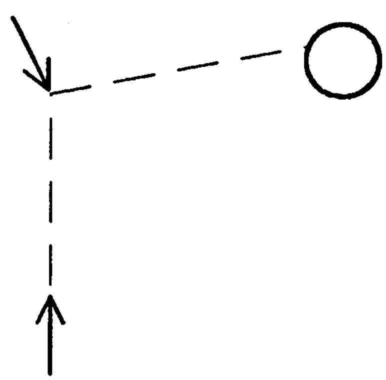


Fig. 13.



(4) Finally, one can modify the original strategy to produce almost a 90 degree turn by kicking nearly backwards (as shown in figure 14). This can be discovered through exploring the phenomenology of this world. The expectation of the physics naive person is that the dynaturtle should always go in the direction that it is kicked. However, through experimenting with kicks, the person discovers that this rule only

Fig. 14.



holds true when one is stopped. In cases where one is already moving, they evolve descriptive rules such as, *if you want to make a turn, you have to reaim the dynaturtle by turning it more than you would think.* From the physicists point of view, they are applying the kick so as to counteract the forward component of velocity.

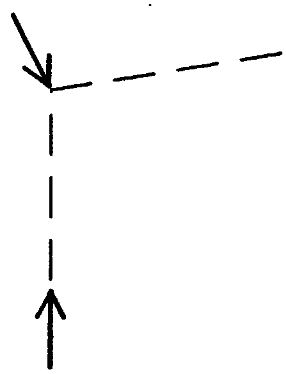
It has thus been argued that these four successful strategies for the game of Target could be derived by someone with no knowledge of formal physics. There are several implications of the result that one can achieve success in this microworld, at least for simple problems, without understanding formal physics. Firstly, it enables a person to get a positive feeling while learning about the behavior of a Newtonian object. One can then motivate the person to evolve a more physics-like explanation of the behavior of this microworld by increasing the complexity of the tasks. Simple rules such as, *you have to turn it more than you would think,* do not work under all conditions. For instance, this rule does not take into account the speed of the object as a factor in determining how much one has to reaim the dynaturtle in order to make a sharp turn. The faster one is going the more backwards one has to reaim. Even if the person does not evolve the physics by themselves, one can motivate the need for the physics as a means of understanding and controlling the behavior of the dynaturtle. A further implication of the result that success can be attained without necessarily knowing the formal physics is that one cannot infer from the person's strategy for a game like Target whether or not the person knows the formal physics underlying the behavior of the dynaturtle.

Another interesting set of results from the pilot study were observations concerning the way people with different levels of formal physics education interacted with the microworld.

The first observation was that the subjects transformed the problem to suit their level. Naive subjects were preoccupied with hitting the target - that was enough of a struggle for them. Subjects with a little physics background first tried hitting the target. After they had succeeded they then became preoccupied with hitting it slowly. This seemed to pose a very challenging task for them. Subjects with a lot of physics background, soon learned to perceive the task in terms of velocity components being added together and were able to hit the target and land slowly. Two of them then spent several hours working on the issue of whether one could hit the target with an arbitrarily low speed. This involved some mathematics and trying out the results with the dynaturtle. Thus, even though given a specific goal to pursue within this Newtonian microworld, subjects were able to transform the task to meet their own interests.

Another important result was the difference between the way physics naive versus advanced physics subjects talked about their strategies for playing the game. For example, with respect to making the path of motion take a sharp turn, naive subjects would make comments such as, "you have to turn it more than you would think". This implies that their original expectation was that one should just have to turn the dynaturtle in the direction that one wants to go, then kick, and the dynaturtle would go in the desired direction. However, if one has already kicked and is moving, one has to turn it more than one expects, about 150 degrees, to make close to a 90 degree turn. (See figure 15).

Fig. 15.



The phrase "more than you would think" represents the adaptation of the subject's original expectation to the Newtonian reality. In contrast, the subjects who had studied a lot of physics, made comments like, "you have to counteract the forward velocity in order to turn". This implies that these subjects are thinking in terms of the addition of velocity components.

Most of the subjects with no physics background seemed to have a lot of difficulty interpreting this microworld. Several thought that turning the dynaturtle should change the direction of motion, like turning the steering wheel on a car. Two had a lot of difficulty figuring out what pressing K on the keyboard did. When they saw motion they assumed a continuous force. Experimenter: "how could you make the dynaturtle stop?" Naive subjects answers: "turn K off" and "isn't there a stop button?" They also all believed that the dynaturtle should go in the direction of the latest kick. They tried kicking, then aiming at the target and kicking again and exhibited dismay. "It isn't right!" None of them ever talked about adding velocities. One expressed a desire not to play anymore and said, "I think if you use this to teach kids, they will find it very confusing."

Subjects who had had several years of college physics, in contrast, did evolve, usually quickly, toward describing the domain in terms of adding velocities. If they had not invented the question of can one hit the target with an arbitrarily small speed, they would probably have lost interest quickly.

Subjects with a little physics, either high school or college, did not find the microworld totally perplexing. On the other hand, they did not adapt to it right away. They seemed to go through interesting struggles between what their naive intuitions told them, *things go in the direction that you kick them*, and what their partially understood physics knowledge told them. These subjects developed strategies that worked, often without the use of formal physics knowledge. Then, in trying to describe why the strategy worked, they sometimes realized that the strategy had a simple interpretation in terms of the formalisms of Newtonian physics. The microworld thus gave the physics explanatory power and thereby helped the student to believe and understand the formalism. It also encouraged the students to work at the interface between their formal physics knowledge and the phenomenology of the everyday world.

Thus the subjects with some physics background, but not a lot, seemed to undergo the most interesting transitions with respect to understanding how velocities add together. For this reason high school students who had studied the relevant physics were chosen as the population to study when analyzing the understanding of Newtonian dynamics and when designing the games.

Another result was that subjects seemed generally unwilling or unable to describe their thinking while playing the game. Their unwillingness to talk about what they were thinking probably occurred because hitting the target required concentration. This was due to the speed at which the dynaturtle moved. Slowing down the dynaturtle by reducing the impact of one kick was tried. This had the effect of making the game less interesting so it was abandoned. Many subjects also seemed unable to describe their strategy after the fact. This may be because they did not have a very good way of describing it to themselves or because their descriptions were in some other form, say visual, and they had trouble translating them into verbal terms. The conjecture that they did not have a good way of describing it seems plausible since subjects with a lot of physics background, who talked in terms of velocity components, had little trouble describing what they had done.

This result, combined with the fact that one cannot infer what a person knows about Newtonian dynamics purely from the strategy they use to hit the target, led to the decision to find some additional means for analyzing students' understanding of Newtonian dynamics. Work was begun on designing a questionnaire where subjects were asked very basic questions concerning how forces affect motion.

In addition to aiding the analysis of students' understanding of Newtonian dynamics, the questionnaire was also needed as a pretest/posttest to assess changes in thinking about force and motion problems as a result of playing the games.

2.2 Analysis of a Preliminary Force and Motion Questionnaire

The questions were all set in the context of a low friction or frictionless environment such as an ice rink, a ball on a table, or outerspace. They all involved the application of impulse forces like a kick or a hit. Usually the applications of the impulse forces were separated in time, as in the Newtonian microworld. Further, all the questions were in one of two forms.

(1) *What would happen if ... ?*

(2) *How could one achieve ... ?*

To illustrate, many of the problems involved controlling the motion of a spaceship propelled by an impulse engine through outerspace. The impulse engine applies a force for a short period of time. It acts like a kick or a shove. Further, the same size force is applied every time the engine is fired. One question was, *suppose we fired the impulse engine once and then turned the engine right around so that it was facing the other way and then fired the impulse engine again - what do you think would happen?* Another question was, *how could you get the spaceship to fly in a square path?* The questionnaire was thus designed to be analogous to situations that could occur in the Newtonian microworld but the questionnaire did not provide the kind of feedback that would allow the "try it and see" approach to work. Many of the subjects in the pilot work were asked trial versions of the questionnaire.

2.2.1 Results.

From answers to the questions and from observing subjects play the game of Target, some areas of difficulty were identified.

The idea behind Newton's first law, that an impulse force would produce constant, eternal motion, was not present in many of the subjects.

Further, many subjects did not correctly predict what the new direction of motion would be when a second or third impulse was applied. They had the belief that motion should always be in the direction of the last impulse. Thus they do not know how the results of impulses combine to affect the direction of motion.

Some subjects thought that if one applied an impulse force to get the object moving and then applied another impulse force in the same direction, the second impulse would not affect the speed. This result indicates a failure to understand the implications of Newton's second law of motion. Also, most subjects gave incorrect answers concerning what would happen to the speed of motion if one applied an impulse in a direction that is different from the current direction of motion. Thus many of the subjects did not believe that the effects of impulse forces combined additively with respect to the speed of motion.

Figuring out how to apply impulses so as to hit the target with a low speed caused even more difficulty. It involves adding velocities so as to achieve what is in the domain of scalar arithmetic a subtractive like result, a slower speed.

In addition to noting these difficulties, it was also observed that correct answers to questions concerning what happens to the direction of motion did not necessarily imply that the person would give correct answers to questions concerning what happens to the speed of motion.

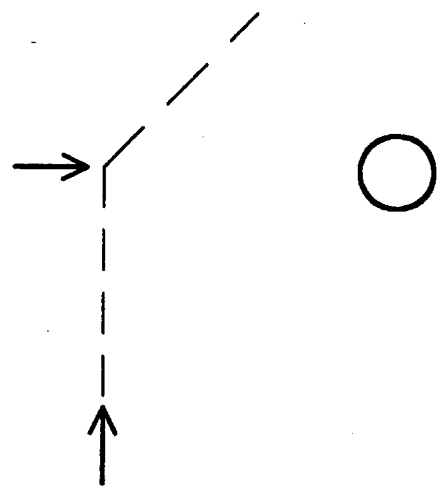
2.2.2 Conclusions.

The fact that knowing what happens to the direction of motion does not imply knowledge of what happens to the speed, suggests that these subjects are not thinking in terms of the vector representation of velocity where speed and direction are linked together: the direction of the vector represents the direction of motion and the length of the vector represents the speed of motion. Computations performed on velocity vectors thereby link together what happens to the direction and speed of motion.

This lack of facility for thinking in terms of vectors is further substantiated by the result that many subjects had particular difficulty figuring out how to apply impulses so as to slow down the motion of the dynaturtle. From the point of view of vectors, the problem involves determining what vector needs to be combined with the current velocity vector in order to achieve a vector with a shorter length.

Another general conclusion from the pilot study was that this microworld does seem to encourage students to integrate their informal physics intuitions with their formal physics knowledge. Even subjects with several years of college physics often approached the game of target with the strategy illustrate in figure 16. Their initial expectation was that the dynaturtle should go in the direction that it is kicked. Upon seeing the result, students with a strong physics background were usually then able to explain the dynaturtle's motion in terms of vector addition.

Fig. 16.



2.3 Summary.

The purposes of the pilot study were:

(1) to determine what population of students would be most helped by a Newtonian computer microworld,

(2) to obtain some data on how people interact with such a microworld,

(3) and to gain some preliminary ideas as to the difficulties students have with understanding Newtonian dynamics.

These goals were achieved by observing subjects with wide variation in their formal physics education play a computer game called Target which was implemented in the Newtonian microworld. Further, many of the subjects were also asked to solve some basic problems concerning the effects of impulse forces applied to objects in a frictionless environment.

Observations indicated that students who had studied the relevant physics but who were not advanced physics students underwent the most interesting transitions.

It was also observed that subjects could utilized their naive intuitions about force and motion combined with general problem solving skills and feedback from the microworld to enable them to evolve strategies which succeeded at hitting the target.

Many of the students gave incorrect answers to questions involving the effects of impulse forces on the speed of motion. They especially had difficulty figuring out how to slow down the speed of motion.

Further, correct answers concerning changes in the direction of motion did not necessarily imply that the subject could correctly answer questions concerning what happens to the speed of motion. This finding led to the conclusion that many of the students were not thinking in terms of vector components of velocity.

These results were utilized in designing the games and the questionnaire.

Chapter III

Design of the Games

This chapter begins with a discussion of the way in which the Newtonian computer microworld represents Newton's laws of motion. Several changes to the original microworld are then proposed. This is followed by a discussion of the design of the games and a description of each of the games is given.

3.1 The Design of the Newtonian microworld.

The formal properties implied by Newton's first two laws of motion are:

(1) that objects in motion stay in motion unless a force is applied to them

(2) and that the results of forces are additive with respect to the motion of objects according to the formula $F = ma$.

The Newtonian computer microworld we have described is a particular way of representing the implications of these two laws. It contains an object, represented by a triangle, in a frictionless environment, represented by a display screen, to which impulse forces can be applied by giving commands via a keyboard. The object responds to the application of impulse forces in accordance with Newton's second law ($F = ma$) by accelerating instantaneously to the appropriate velocity. The resulting motion of the object across the display screen also obeys Newton's laws since it moves with a constant velocity until another force is applied or until it crashes into an obstacle.

At any given point in time the object has a state which includes:

- its position in space,

- its direction of motion,
- and its speed of motion.

In addition, in this particular representation, the dynaturtle has another component of state - the direction in which it is aiming or facing which is distinct from the direction in which it is moving. This direction of aiming exists in order to represent the direction in which an impulse force would be applied if it were applied at that moment in time.

Thus, the directions of forces and the object in motion are represented in the same entity. This might be a source of confusion. An alternative would be to externalize the force by showing a boot kicking the object, for example. This was not done since the decision was made to change the conceptual object from a dynaturtle being kicked to a spaceship being driven by an impulse engine. The change was made because it was thought that the idea of a spaceship in outer space would be a more familiar example of a frictionless environment. The context of outer space, therefore, might enable the student think more easily in terms of no friction.

In order to overcome the potential problem of the direction of aiming being confused with the direction of motion, a circle could be placed around the triangle so that the circle becomes the spaceship and the triangle becomes the impulse engine. (See figure 1).

Fig. 1.

The spaceship can now have one direction of motion while the impulse engine has another direction of aiming with less potential source of confusion. Unfortunately, the particular computer available in the high school where this study was done was unable to display and move both the triangle and the circle at the same time so the circle had to be eliminated.

Several changes were made to the original microworld in the hope of improving it.

Firstly, as has already been mentioned, the conceptual metaphor was changed from a dynaturtle being kicked to a spaceship being propelled by an impulse engine in outer space. This was done primarily because no friction in the context of spaceships in outer space seemed more familiar and, also, partially because this context would be more extendable. The engine could be changed from an impulse to a continuous engine so that constant acceleration could be introduced. Other forces such as gravity could be introduced and the dynamics of orbits could be studied. However, a potential problem with going from a dynaturtle to a spaceship is that probably the students would assume that a spaceship would have a continuously acting engine as opposed to an impulse engine. Even if the students do not assume a continuous engine immediately, they may tend to infer it when they see the engine produce constant motion. As Clement [1979] has documented, constant motion implied constant force to most of his subjects. Hence, when presenting this microworld to people, the fact that the spaceship has an impulse engine as opposed to a continuously acting engine was stressed.

The second change was to have the spaceship leave a trail so that its previous path would be visible. This was done to make the effects of impulses more visible.

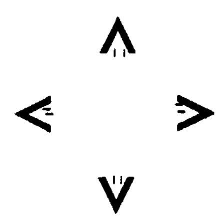
The third change has to do with making it easier to implement impulse forces via the keyboard. In the original version of the microworld the student had three buttons to work with. Two were aiming devices which turned the dynaturtle right (R) or left (L) by thirty degrees and the third button (K) fired the impulse. A difficulty with this arrangement was that subjects typically spent a lot of time deciding whether they wanted to press R or L and then spent even more time finding the button on the keyboard. In addition, subjects often pressed the wrong button and got annoyed and further distracted. In order to eliminate such wastes of time and distraction, the students were given the facility to only reaim to the right instead of to the right and left. This avoided having to make a decision between R or L and it also enabled the student to keep the index finger of his or her right hand on the impulse firing button (K) and the index finger of the left hand on the reaiming button (R). This helped to eliminate time being wasted on searching the keyboard for the appropriate button.

The final change was to have the reaiming button (R) rotate the engine to the right by ninety degrees instead of only thirty degrees. This was done for the first few games in the sequence of games and it was done for two reasons:

(i) Firstly, it was done to make the additive effects of impulse forces more perceptible. In the original microworld, if one kicked and started moving and then reaimed to the right thirty degrees and kicked again, the direction of motion would turn to the right by fifteen degrees since the two component velocities would add together. The naive expectation was that the new direction of motion should be in the direction of the last kick. In other words, one should turn right by thirty degrees not fifteen. Therefore, the difference between the expected change in direction and the actual change in direction was only fifteen degrees. People could thus perceive the change as thirty, consistent with their expectation, instead of the actual fifteen degree turn. Changing the effect of pressing R to ninety instead of thirty has the result of getting a forty-five degree turn instead of an expected ninety degree turn in the above scenario. This forty-five degree difference between expected and actual change in direction is much larger than in the original microworld. Hence, it should be much harder to see this new microworld as being consistent with the belief that things go in the direction of the last impulse.

(ii) Secondly, having R produce a ninety degree change in the heading of the engine means that impulses can only be introduced at headings of 0, 90, 180, or 270 degrees. (See figure 2).

Fig. 2.



This gives the students the capacity to apply only orthogonal impulses in order to alter the motion of the spaceship. Thinking in terms of orthogonal components of velocity is a technique that physicists often find very useful. Possibly, the fact that motion in this microworld is affected by orthogonal impulses only will encourage students to think in those terms.

3.2 Approach used in Designing the Games.

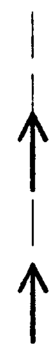
The idea was to focus the students' attention on aspects of Newtonian dynamics that cause them difficulty, one difficulty at a time. An attempt was made to order the games in terms of increasing difficulty. This was done because starting off with hard problems might have frustrated students and caused them to lose interest in the microworld.

There were several sources of ideas relating to what might cause students difficulty when dealing with force and motion problems.

The first was that prior research by diSessa [in Papert et al. 1979] and Clement [1979] has suggested the existence of what might be termed, ignore momentum bugs. The students expect the dynaturtle to always go in the direction that it is kicked, regardless of its current state of motion. Further, when they see constant motion, even in a frictionless environment, they erroneously infer that there must be a constant force producing that motion. They thus possess intuitive beliefs about how forces affect motion which contradict Newton's first and second laws of motion. The behavior of the microworld should help students to realize the falsity of these beliefs. To help ensure this, however, games were included in the sequence which were meant to focus the student's attention on the additivity of forces.

The second source of difficulty might be that the student's previous experiences concerning the combinations and additions of entities have been one dimensional, whereas, velocity addition is a two dimensional[1] form of addition. For instance, scalar arithmetic, e.g. $2+2=4$, is one dimensional as are operations such as combining quantities of liquids. Therefore, students might find situations where impulse forces combine in one dimension (see figure 3)

Fig. 3.

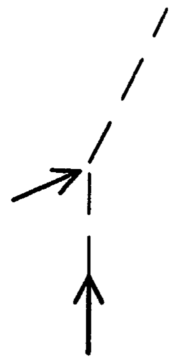


easier to deal with than situations where impulse forces combine in two dimensions. (See figure 4). The one dimensional case maps isomorphically onto the familiar domain of scalar arithmetic. It also has the property that one is only dealing with changes in speed not changes in speed as well as direction. Thus the decision was made to start with games where the impulses are applied in one dimension only.

The pilot study indicated that the majority of subjects observed were not unifying speed and direction into the physicist's concept of velocity. Velocity can be conveniently represented by vectors and thought about in terms of orthogonal components. Instead of doing this, the majority of subjects appeared

1. In fact, vector addition can be done in a space of arbitrary dimensionality. However, all of the problems addressed in this thesis involve only one or two dimensional vector addition.

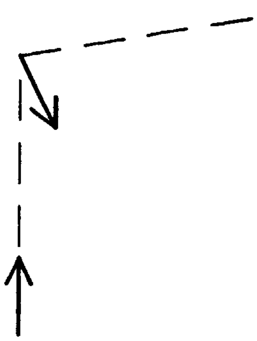
Fig. 4.



to be dealing with changes in speed separately from changes in direction. This could lead to problems since the two are not independent of one another. For example, if one wants to change the direction of motion by a given amount using a fixed sized impulse, the current speed of motion has an effect on the direction of the impulse one must apply. For this reason, a game which focused on such a dependency was included in the sequence.

One final consideration was the fact that most of the subjects in the pilot study found the task of slowing the dynaturtle down very difficult. This led to the conjecture that although adding an impulse in a backwards direction (see figure 5)

Fig. 5.



is formally equivalent, in terms of the vector operation being performed, to adding an impulse in a forwards direction (see figure 6), perhaps conceptually the "subtractive like" case of backwards impulses is

Fig. 6.

more difficult.[1] Based on this conjecture, games were included in the sequence which required the students to apply impulses in backwards directions. Such impulses had to be applied to alter the direction of motion in one of the games and to alter the speed of motion in another of the games.

3.3 The Games.

The purpose of this section is to describe each of the games and to illustrate the strategies which succeed at achieving the goal of the game.

The first few games in the sequence were designed to focus students on the additive effects of forces on motion. It was hypothesized that starting with games where the effects of forces combine in only one dimension would facilitate this realization since the operators and the effects of these operators would map isomorphically onto the familiar domain of scalar arithmetic. A force applied in the same direction as the current direction of motion would increase speed (as in scalar addition). A force applied in the opposite direction would reduce speed (as in scalar subtraction). Starting with one dimension should thus help the students to focus on the additivity of forces with respect to the speed of motion. The next game in the sequence also focused on the additive effects of forces on motion. This time, however, the object was to control the direction of motion in a two dimensional context. It was thereby hoped to focus the students on the additivity of forces with respect to the direction of motion.

1. It is not surprising that these two cases would be perceived as different since the resultants are phenomenologically very different from one another. This is especially true with regard to the effect on the speed of motion. In the first case the speed decreases dramatically. Whereas, in the second case, it increases.

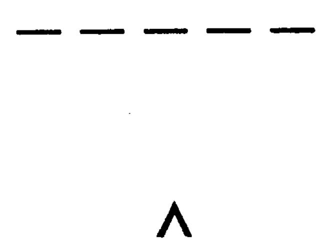
In order to describe the games, we need to define some way of describing the heading of the spaceship: a heading of zero means that the spaceship was facing upwards with respect to the display screen and will be represented pictorially as in figure 7. The remaining headings, 30, 90, 270, etc., are defined as being turned clockwise from zero by the appropriate number of degrees.

The first two games involve the one dimensional case as distinct from the two dimensional case and embody the distinction between combining impulses to increase speed versus combining impulses to decrease speed.

3.3.1 RACE

This game, illustrated in figure 7, involves combining impulses to increase speed in the one dimensional case.

Fig. 7.



The spaceship starts with a heading of zero and an initial speed of zero. The goal is to get the spaceship to cross the line with the fastest possible speed. The speed is printed out when the spaceship crosses the line.

There are two successful strategies for this game. The first simply involves firing as many impulses as one can before the spaceship crosses the line. The other involves going down to the bottom of the screen, stopping, and then firing impulses as many times as possible. This second strategy gives one more time than the first in which to fire impulses and hence allows the spaceship to build up more speed.

3.3.2 DOCK

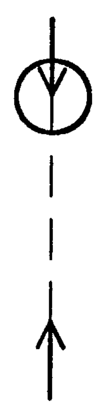
This game, illustrated in figure 8, concerns combining impulses so as to decrease speed in the one dimensional case.

Fig. 8.

The spaceship starts with a heading of zero and an initial speed of zero. The goal is to get the spaceship stopped inside its space port which is represented by a circle.

The successful strategy here involves firing an impulse to get moving and then, when the spaceship is inside the space port, fire an impulse in the opposite direction. This counteracts the first impulse and stops the spaceship as illustrated in figure 9.

Fig. 9.



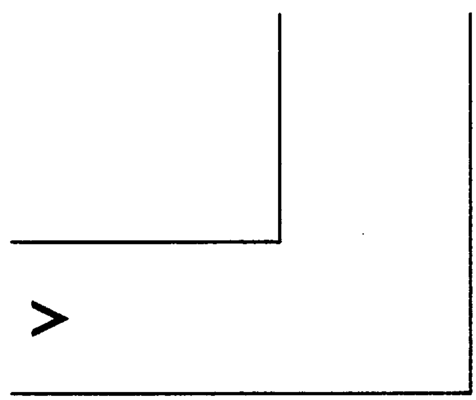
The strategy can be modified by firing several impulses initially and then firing the same number in the opposite direction once the spaceship is inside the space port.

This game is similar to Race in that it involves combining velocities in the one dimensional case. However, the goal here is to combine the velocities so as to stop the spaceship rather than increase its speed.

The following games all involve two dimensional situations where the direction of motion as well as the speed of motion change.

3.3.3 CORNER with only orthogonal impulses available.

Fig. 10.



Here the spaceship's initial heading is 90 degrees and the initial speed is zero. The goal is to get the spaceship around the corner and to the end of the course without crashing into any of the walls. (See figure 10).

This game was presented in order to violate the intuition that things always go in the direction that one kicks them. The naive strategy derived from this expectation would be to fire an impulse to get moving and then, when one gets to the corner, aim upwards (at a zero heading) and fire another impulse - the expectation being that the spaceship would then go upwards in the direction of the last impulse. (See figure 11).

What would in fact happen is that the velocity components produced by the two impulses would add together and the spaceship would go off at 45 degrees and crash into the wall. The expectation is thus dramatically violated in a way that is hard to ignore or perceptually deny. (See figure 12).

There are quite a few successful strategies for this game.

The simplest, in that it reduces the problem to an already handled case, is to stop the spaceship when it reaches the corner and then fire an impulse in the upwards direction. (See figure 13).

Fig. 11.

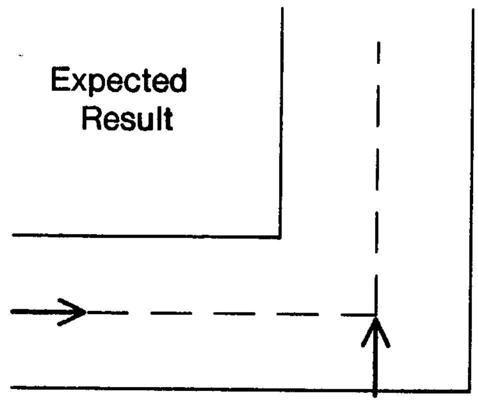
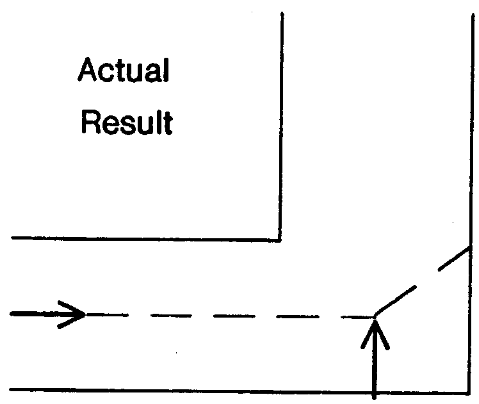


Fig. 12.



A second strategy involves firing an impulse to get moving, then firing an upwards impulse before one reaches the corner - this produces a path at 45 degrees - and then, before the spaceship crashes into the rightmost wall, counteract the horizontal component of velocity by firing an impulse at 270 degrees, i.e. fire away from the wall. (See figure 14).

A third strategy uses a combination of firing early and firing harder. Fire an impulse to get moving, then before the spaceship gets to the corner, fire upwards, and keep firing upwards so that the path curves towards the upwards direction. (See figure 15).

This game thus involves applying impulses so as to control the direction of motion.

Fig. 13.

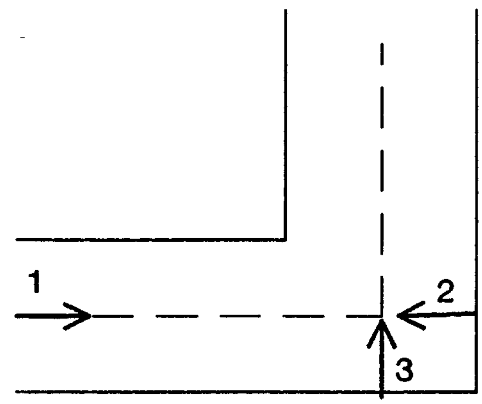
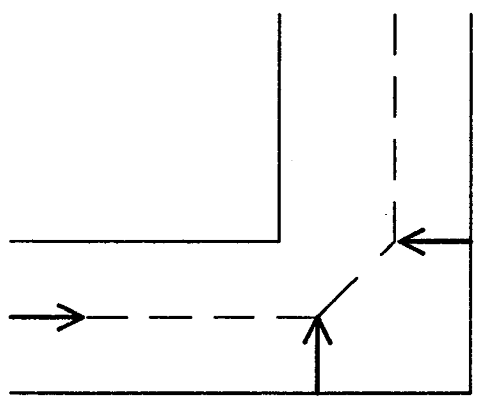


Fig. 14.

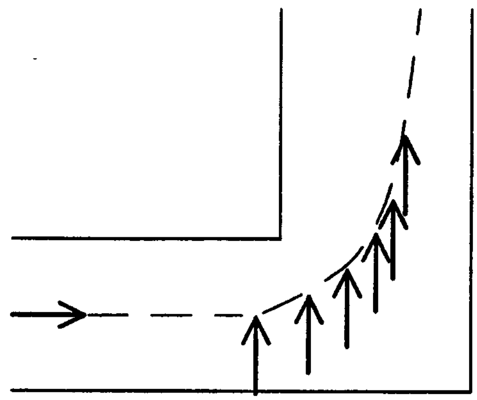


3.3.4 CORNER with an initial speed of 3.

This game is similar to the previous one except that the conditions are modified so as to change the focus of the game. The spaceship again has an initial heading of 90 degrees, however, the initial speed is now 3 - as though the impulse engine of the spaceship had been fired three times before the player gets control.

The strategy the player used to go around the corner in the preceding game has to be modified to take into account how the increased speed alters the effect of an impulse in changing the direction of motion. For instance, the players now have to fire three counter impulses, whereas, before they only had to fire one.

Fig. 15.



This represents a modification of the corner game designed to focus on the interaction between the speed of motion and changes in the direction of motion.

The design of the final corner game was derived from a conjecture formulated during the pilot study: students have difficulty with problems where they have to think to apply an impulse which is almost backwards with respect to the current direction of motion.

In order to be able to apply "almost backwards" impulses, students need to have the facility to apply impulses in various directions, instead of just orthogonal directions. To facilitate this, the effect of pressing the engine rotation button was changed from rotating the engine to the right by ninety degrees to rotating the engine to the right by thirty degrees. To get the students used to this new facility for applying impulses in more directions, an interim game was provided.

3.3.5 CORNER with the ability to apply impulses at 30 degree intervals.

This game was included in the sequence of games to get the person used to the new thirty degree effect of pressing the engine rotation button as opposed to the previous ninety degree effect.

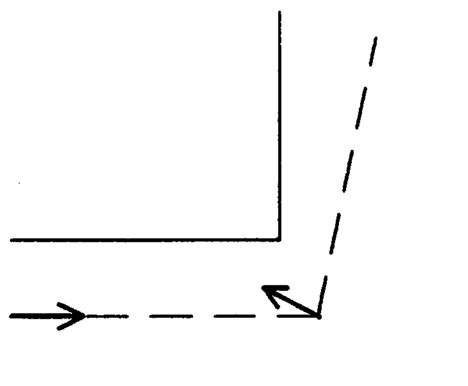
The initial speed was zero and the heading 90 degrees. The goal was the same as for the first corner game, mainly get the spaceship around the corner and to the end of the course without crashing into a wall.

3.3.6 CORNER with only one more impulse allowed.

Here the spaceship's initial orientation is again at 90 degrees but this time the initial speed is 1. The effect of pressing the engine rotation button is to turn the impulse engine to the right by thirty degrees. The goal is to make the spaceship go around the corner by firing only one more impulse.

The successful strategy involves firing an impulse "almost backwards" with respect to the current direction of motion when one gets to the corner. This impulse has the effect of producing almost a right angle turn in the direction of motion. (See figure 16).

Fig. 16.



This game focuses on combining velocities in a "subtractive like" way in order to change the direction of motion. The idea of "subtracting" or reducing the forward momentum to change direction might be foreign since usually reducing has to do with quantities such as speed, size, number, etc. and in no way involves a qualitative effect such as a change of direction.

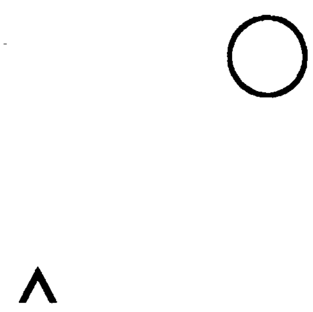
The next four games are similar to the four Corner games except that the context is now hitting a target instead of going around a corner.

3.3.7 TARGET with orthogonal impulses only.

This game is essentially the same game as was used in the pilot study except that the effect of pressing the engine rotation button is to turn the impulse engine to the right by ninety degrees as opposed to thirty degrees and there is no left rotation option. The goal is to hit the target.

This game, illustrated in figure 17, involves combining the effects of impulses so as to control the direction of motion. The focus is on direction as opposed to speed.

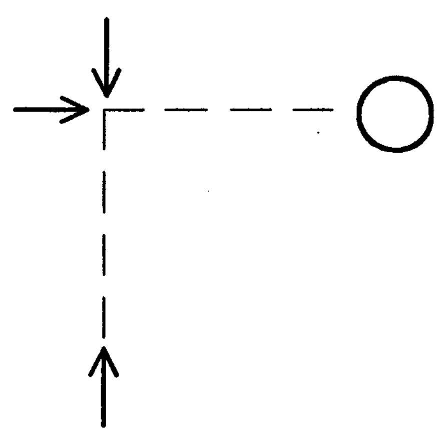
Fig. 17.



The strategies that were appropriate for the first corner game, where one had to guide the spaceship around the corner, can also be used for this game.

(i) reduce to a simpler case (illustrated in figure 18),

Fig. 18.



(ii) fire early and hard (illustrated in figure 19),

and (iii) the delayed anti-impulse (illustrated in figure 20).

There is also another strategy that works for this game that would not work for the Corner game. It involves firing a horizontal and a vertical impulse close together in time. The two velocities add together to produce a path at 45 degrees which will hit the target. (See figure 21).

Fig. 19.

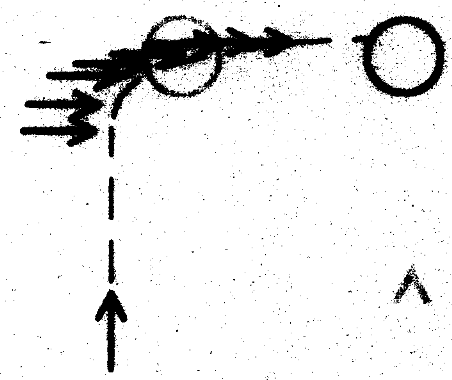


Fig. 20.

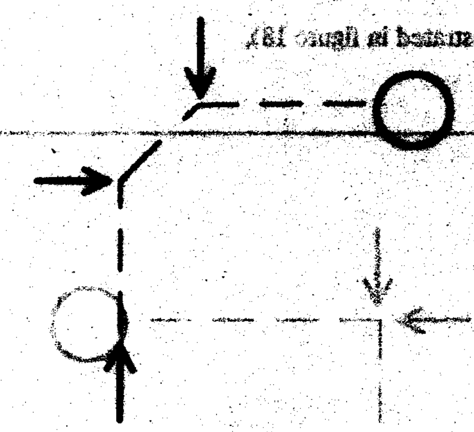
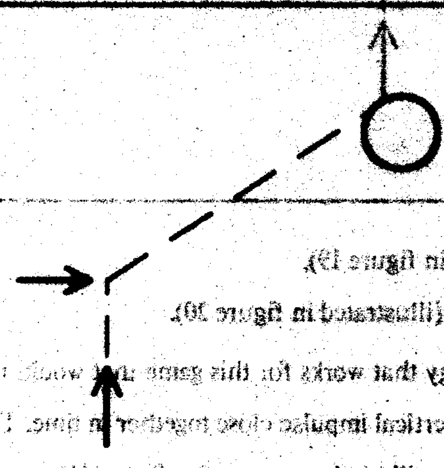


Fig. 21.



3.3.8 TARGET with an initial speed of 3.

This game is analogous to the second Corner game. The initial speed of the spaceship is 3 instead of zero - as though the impulse engine of the spaceship had been fired three times before the player gets control of it.

The strategy the player used to hit the target in the preceding game has to be modified to take into account the increased initial speed. The effect that a single impulse has on changing the direction of motion is lessened due to the large initial speed. This game was thus designed to focus on the interaction between changes in speed and changes in direction of motion.

The final two Target games were designed to require the student to fire impulses in backwards directions with respect to the current direction of motion. The first game concerns controlling the direction of motion, whereas, the last game involves controlling the speed of motion. To enable the firing of backwards impulses, the effect of the engine rotation button was changed from ninety degrees to thirty degrees. An interim game was included in the sequence to get the student used to this new facility for applying impulses.

3.3.9 TARGET with the ability to apply impulses at 30 degree intervals.

This game was included as a transition from the effect of the engine rotation button being right ninety degrees to being right thirty degrees. The goal of this game was to hit the target.

3.3.10 TARGET with only one more impulse allowed.

In this game the initial speed of the spaceship is 1 and the effect of pressing the engine rotation button remains as right thirty degrees. The goal is to make the spaceship hit the target by firing only one more impulse.

Unlike the analogous corner game, there is a range of successful strategies for this game.

One could succeed by aiming right 90 degrees and firing an impulse at the start of the game. (See figure 22).

One can also wait until the spaceship is almost in line horizontally with the target and then fire almost backwards (i.e. right 150 degrees). (See figure 23).

There is also a point in between where one could fire an impulse at right 120 degrees and succeed at hitting the target. (See figure 24).

Fig. 22.

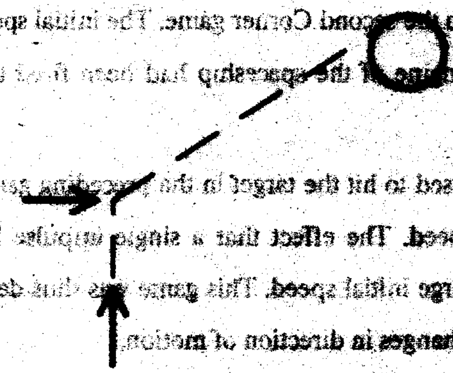


Fig. 23.

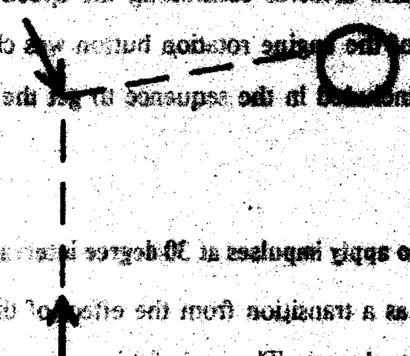
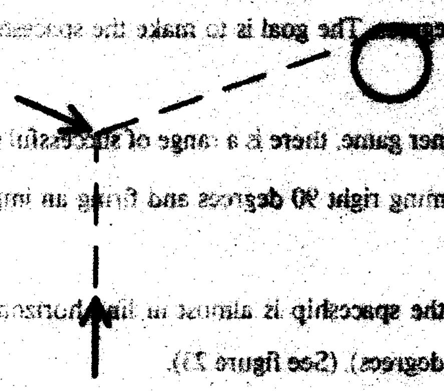


Fig. 24.



This game has thus focused on combining velocities in order to change the direction of motion.

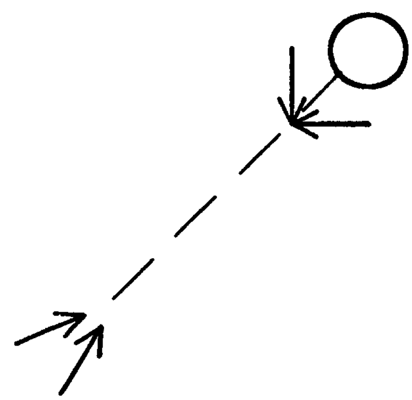
The final game focused on combining velocities in order to reduce the speed of motion.

3.3.11 TARGET land slowly.

Here the effect of pressing the engine rotation button is still thirty degrees and the initial speed is zero. The goal is to hit the target with as low a speed as possible.

There are many strategies that will work for this game including the firing almost backwards strategy that worked for the previous game. All the strategies embody the idea of reducing the speed by canceling some of the forward momentum. As an example consider the strategy illustrated in figure 25.

Fig. 25.



The effects of the first two impulses combine to produce motion in the direction of the target. The second two impulses combine to produce a velocity component in the opposite direction. However, this new component is not as great as the original, thereby, only partially canceling the velocity and thus reducing the speed.

The last two games have thus focused on combining velocities in a "subtractive like" way in order to modify (i) direction and (ii) speed of motion.

3.4 Summary.

The way in which the implications of Newton's laws of motion are represented by the computer microworld was discussed. Several changes to the original microworld were then proposed. These included changing the conceptual metaphor from a dynaturtle being kicked to a spaceship being propelled by an impulse engine in outerspace, having the spaceship leave a trail so that its path would remain visible, and giving the student the facility to fire only orthogonal impulses. A sequence of games and strategies for winning the games were then described. The games attempted to focus the students on the difficulties and distinctions found during the pilot study.

Chapter IV

Design of the Questionnaire

This chapter begins with a general discussion concerning the design of the questionnaire. This is followed by a listing of the questions and includes the reasoning behind the creation of each question.

4.1 Designing the Questionnaire

The questionnaire was designed to include fundamental problems concerning the implications of Newton's laws. Consider an example problem: *suppose we fire the impulse engine of the spaceship twice. Will it go faster or slower or the same speed as when the engine was fired once?*. This type of question differs from typical textbook problems which are usually quantitative in nature, requiring one to compute some value from an appropriate formula. In contrast, to solve the problems in this questionnaire, the students need to invoke their knowledge about what happens, i.e. the qualitative implications of Newton's laws, rather than their knowledge of formulas. If the students understand Newton's second law, then the answer to the above question will be obvious. Failure to correctly answer this type of basic question indicates a serious difficulty with the students' knowledge of Newtonian dynamics.

The problems were all set in the context of a low friction or frictionless environment such as an ice rink, a ball on a table, or outerspace. They all involved the application of impulse forces like a kick or a hit. Usually the application of the impulse forces were separated in time, as in the Newtonian microworld.

All of the questions were phrased in one of two forms.

(1) *What would happen if ...* ?

(2) *How could one achieve ... ?*

The two are fundamentally different kinds of problems. The former could be described as predicting the consequences of an activity, whereas, the latter involves determining the precursors or antecedents to an activity. The games are of the form *how could one achieve*, whereas, textbook problems are more typically of the predictive form, *what would happen if*. Both types of problems were included in the questionnaire in order to acquire data on their relative difficulty. It was hoped that the results would also provide some insights into how these two forms of problem solving differ from one another.

Many of the problems on the questionnaire were designed to be analogous to situations that could occur in the Newtonian microworld, but, the questionnaire did not provide any feedback concerning the correctness of the student's answer.

This parallel between the verbally given problems and the computer games was created for several reasons. Firstly, the questionnaire was used as a tool for exploring students' understanding of the aspects of Newtonian dynamics embodied in this microworld. Secondly, it was used as a means of assessing changes that might occur in the students' understanding of Newtonian dynamics as a result of playing the games. Finally, it was done to see how performance on each game was related to performance on analogous verbal forms of the same problem.

Questions were created to investigate the following:

(i) context effects,

(ii) misconceptions documented by other researchers,

(iii) the distinctions and difficulties found in the pilot work,

- the one dimensional versus the two dimensional case,
- applying impulses in "additive" versus "subtractive" directions,
- controlling direction versus controlling speed,

(iv) and more complex situations.

The following discussion describes further these components of the questionnaire.

When designing the microworld to be used with this sequence of games, the context was changed from a dynaturtle being kicked to a spaceship being propelled by an impulse engine in outer space. This was done in the hope that students could conceive of no friction more easily in such a context. Further, spaceships in outer space is a popular fantasy environment which might help to motivate interest in the microworld. This raises the issue of how the context might affect the students' facility for solving dynamics problems. Examples of contexts include (i) kicking a ball on a smooth surface, (ii) hitting a hockey puck on the ice, and (iii) controlling the motion of a spaceship in outer space with an impulse engine. It could be that the context of a ball being kicked calls up inaccurate beliefs such as the belief that things always go in the direction that you kick them, whereas, the context of outer space might call up abstractions learnt in physics class. If it is found that the context does in fact alter the students' answers, then this factor will have to be taken into account when presenting the computer microworld. In some circumstances one might want to draw out misconceptions and in others one might want to help draw out the students' knowledge of formal physics. This could perhaps be achieved by changing the context. In order to determine whether the context affects the students' answers on force and motion problems, the same question was asked in a variety of different contexts.

Several of the questions were designed to further investigate the misconceptions students have with respect to Newtonian dynamics which have been documented by other researchers. Clement [1979] reports that when subjects are told that a spaceship is drifting in outer space, they infer that there must be a continuous force producing the motion. The belief is thus that a constant force should produce a constant speed. In a frictionless environment such as outer space, constant force yields constant acceleration, not constant speed. A related misconception is that an impulse force should produce motion but that the speed of the object will continually decrease until the object stops. However, in a frictionless environment, an impulse force produces constant velocity. This is Newton's first law of motion. Another misconception reported by diSessa [in Papert et al. 1979] is the belief that the dynaturtle ought to go in the direction that it is kicked. This belief holds true if the dynaturtle is in the stopped state, but, if it is moving, the result of the kick is added to the dynaturtle's current velocity.

Consistent with the design of the games, the questionnaire included problems embodying the distinction between addition in one dimension versus two, as well as problems embodying the difficulties observed during the pilot research. Briefly, these difficulties include:

- combining velocities so as to control the speed of motion and combining velocities to control the direction of motion,
- combining velocities in "subtractive" or opposing directions (↑ + ↘) as distinct from, and possibly harder than, combining velocities in similar directions (↑ + ↗).

Further, the pilot work indicated that many students do not think in terms of vector components of velocity. Thinking in terms of orthogonal components of velocity is often very useful in solving force and motion problems. Many of the games gave the students the facility to apply only orthogonal impulses with the hope that this might encourage students to think in terms of orthogonal components of velocity. For this reason, two questions were included which attempted to determine if the students could solve problems using orthogonal components of velocity.

In addition, some questions were incorporated that were more difficult and that had no direct parallel within any of the games. These included a problem concerning how one would achieve circular motion and a problem involving a continuous force interacting with an impulse force. Both these problems involve continuous operations as opposed to discrete. The first involves a continuous change in the direction of motion and the latter involves a continuously acting force. The Newtonian microworld involved only discrete changes in the direction of motion and discrete forces. These two questions were included to see whether the games had any effect on the student's ability to solve problems that go beyond the focus of this particular microworld.

The original questionnaire included ten problems more than the final version of the questionnaire. The original version was tried out on eleven students from the same population that the final version of the questionnaire was given to. Problems were deleted because it was found to be too much for the students to concentrate on these types of problems for the length of time necessary to complete this version. They were getting tired and bored. Questions were deleted that were considered either redundant, irrelevant, or too easy. They will be discussed along with questions included in the final version of the questionnaire.

The following section contains a listing of the questions and includes the reasoning behind the creation of each question. The correct answers to the questions along with the students' responses will be discussed in a later chapter.

4.2 The Questionnaire

The introduction to the questionnaire attempted to encourage students to reason out loud and to emphasize the distinction between continuous and impulse forces.

I am trying to design some computer games to help people learn physics. I would like to ask you some questions. I am not interested in whether your answers to the questions are right or wrong, instead, I am interested in how you think about these problems and what difficulties you have so that I can design games to help people overcome these difficulties. If you could tell me what you are thinking - sort of reason out loud as much as possible - it would be a big help.

These questions are going to be about moving objects like footballs and spaceships and about things that affect their movement like kicks and engines. Physicists talk about things that affect the motion of objects as forces. You can apply a force continuously like when you push a car to a gas station or you can apply a force for a short time like a punch or a kick. Can you give me an example of a force that acts for a *short time like a kick? Now can you give me an example of a force that acts continuously? Most of these questions will be about forces that act for a short period of time.*

(1) *Now, imagine kicking a ball and then running up to it while it is moving and kicking it again in the same direction. Will kicking it the second time change its speed?*

This question is similar to the following question on the final version of the questionnaire: suppose we fire the impulse engine of the spaceship twice, will it go faster or slower or the same speed as when the engine was fired once?

This problem involves adding velocities in the one dimensional case where there are no changes in direction to complicate the situation. It was thus hoped to be a basic question about the relationship between amount of force and speed.

Ten out of the eleven students answered this question correctly. The one person who got this question wrong also gave the wrong answer to the similar spaceship problem.

Thus, because students apparently had little difficulty with this problem and because the pattern of answers was the same as for the analogous spaceship problem, this question was omitted from the final version of the questionnaire.

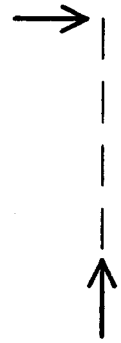
(2) *Suppose we have a kicking machine that always gives the same sized kick. You can turn this machine to face in any direction that you want. If you set the machine up so that it gives a ball a kick forward and then rush ahead and set the machine up so that it gives the ball a sideways kick as it passes by (illustration given - see figure 1),*

(a) Can you show me what the ball will do?
(b) Will the speed of the ball change after the sideways kick?

This question is analogous to a spaceship question on the final version of the questionnaire. It was originally included to see whether asking the question in the context of balls versus spaceships made any difference to people's answers. The pattern of answers[1] was the same as for the spaceship problem so this

1. Ten out of eleven students got part (a) right and six out of eleven got part (b) right.

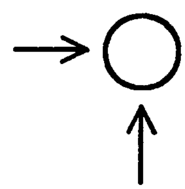
Fig. 1.



question was dropped.

(3) *Suppose we had two kicking machines set up at right angles so that they both kick the ball at the same time (see illustration in figure 2), can you show me which way the ball would go?*

Fig. 2.



This question involves being able to add the results of two simultaneously applied forces. All the prior questions involved forces applied at different points in time. From the point of view of Newton's laws of motion, it makes no difference to the final velocity whether the application of the forces is simultaneous or not. Thus, formally the two situations are equivalent and they can be represented by the same vector diagram. However, the pilot work indicated that many people give the wrong answer concerning what happens to the direction of motion when a second impulse, separated in time from the first, is applied. The wrong answer could be produced by the operation of a belief such as, *balls should go in the direction that you kick them.* Alternatively, the wrong answer could be produced because the person simply does not know how to perform the addition of two forces. In this question, the two impulse forces are applied at the same moment of time. Thus, since the ball cannot go in two directions at once, it is more obviously appropriate to add them - if one knows how. Hence, this question was designed to determine whether the ignore momentum bug was due to simply not being able to add forces, in which case the student would not be able to answer this question, or, whether the ignore momentum bug needs some other explanation.

This question was dropped[1] since very few students completely ignored momentum anyway and everyone got the right answer.

Now, try to imagine a spaceship designed for travelling through outer space. So, there is no friction.

The spaceship is driven by a special impulse engine that gives a sudden burst of force and then shuts itself off. It is not like a car engine that is on all the time, rather, it is more like a kick.

This impulse force is always the same size. That means that whenever you fire the impulse engine, it always turns on for the same amount of time.

When it is fired it gives the spaceship an impulse in the direction that the impulse engine is aimed in. You can rotate the engine so that you can apply an impulse force in any direction that you like.

(4) *What do you think will happen if we fire the impulse engine once? How far will the spaceship go, assuming there are no planets or other spaceships to get in the way?*

This question was included to see if students understood Newton's first law - that an impulse force will produce eternal, constant motion in the absence of other forces.

(5) *What do you think will happen if you keep on firing the impulse engine so that it provides almost a continuous push?*

This question attempted to see whether the students knew that a continuous force would produce constant acceleration. It was dropped because the idea of constant acceleration was considered beyond this

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1. It was, however, asked at the end of the posttest to those students who did consistently ignore momentum and they all got the right answer (except for one person who said it would flip the spaceship over). Thus it does appear that the case of simultaneously applied impulse forces is perceived differently and is easier to understand than the case of time separated impulse forces.

study. This was probably unfortunate since in the major study it became evident that some of the students thought that after the initial impulse, speed remains constant. This question would have determined whether they held that view in this extreme case. As it turned out, five out of twelve people in the preliminary study answered this question incorrectly - three said the spaceship would maintain a constant speed and two said it would be continually speeding up and then slowing down again. This jerky motion answer is consistent with the way the spaceship would behave in a world with friction. The constant speed answer is consistent with the misconception that after the initial impulse, speed remains constant - forces only act to change the direction of motion.

(6) *What happens when you stop firing the engine?*

This question was included to see if students had the erroneous belief documented by Clement [1979] that it takes a continuous force to produce continuous motion. Two out of twelve students said they thought the spaceship would stop once one ceased firing the engine indicating that they did hold this belief. The rest of the students gave the correct answer.

This question was dropped in conjunction with the previous question since it is meaningless without it.

(7) *Suppose we fire the impulse engine of the spaceship twice. Will it go faster or slower or the same speed as when the engine was fired once?*

Does the person realize that applying a second force in the same direction will increase their speed? This question embodies the simple one dimensional case to avoid confounding changes in speed complicated by changes in direction. That is, if one adds velocities in two dimensions, one not only usually changes speed, one also changes the direction of motion. It was, therefore, hoped to be a very basic question about the relationship between amount of force and the resulting speed.

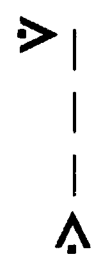
(8) *Suppose the spaceship was sitting in space in a stationary position and you fired the impulse engine once to get the spaceship moving. How could we get the spaceship to stop?* (note: if the students said, "turn off the engine", then the experimenter said, "remember, it is a special impulse engine that gives a sudden burst of force and then shuts itself off", if they said, "turn on the brakes", experimenter said, "there are no brakes, only the impulse engine" and if they said, "hit a planet", experimenter said, "there are no planets or

anything else to hit".)

This question was designed to see if people could figure out how to add forces together so as to stop oneself. It involves the idea of an anti-impulse - applying a force in a direction opposite to motion. Even though it is formally, from a physicists point of view, a vector addition problem, it could intuitively be considered as analogous to a scalar subtraction problem since the object is to reduce speed. Again, it is the simpler one dimensional case with no change in the direction of motion involved.

(9) *Suppose we fired the impulse engine once and then after a little while turned the engine so that it was facing right and then fired the impulse engine again (illustration given - see figure 3),*

Fig. 3.



(a) - which direction do you think that the spaceship would go in? Could you draw a picture of what would happen?
(b) - would it be going faster or slower or the same speed as before the second impulse?

This question was designed to see whether students could add the results of two orthogonal impulses both in terms of predicting what happens to direction and what happens to speed. The reason that speed and direction were separated in this question was that there was evidence from the pilot study that people could answer questions about one but not the other indicating that possibly separate cognitive components are involved or that they have an incorrect theory about the effects of force on motion which yields a right answer to one question and a wrong answer to the other. It was hoped that in discussing their answers to this question with the experimenter that their theories might become evident.

(10) *Suppose we fired the impulse engine once and then turned the engine right around so that it was facing the other way and then fired the impulse engine again - what do you think would happen? (illustration given - see figure 4)*

Fig. 4.

This question is the *what would happen if* form of the question, *how can you make yourself stop.* It was designed to determine whether or not the students understood what an anti-impulse would do. In contrast, the question *how can you make yourself stop* requires that they not only understand what an anti-impulse would do but that they also think of applying one as well.

Further, this question was included because it would allow the bug of ignoring momentum to surface. That is, the student might answer that the spaceship would turn around and go in the opposite direction instead of saying that it would stop.

(11) *Suppose we fired the impulse engine once and then turned the engine right 90 degrees and fired it again and then we turned it right another 90 degrees and fired it again (illustration given - see figure 5).*

Fig. 5.

Which direction do you think the spaceship would end up going in - could you draw in what you think would *happen?*

This question involves adding three velocities together. It could be solved by thinking of the three forces as components, appreciating their commutativity, and seeing that two cancel out. It could also be solved by representing the velocities as vectors and adding all three together in one diagram. Alternately, it could be solved by adding the first two together, getting the result, and adding the third velocity to it. This could be done by either using vectors or doing it qualitatively such as simulating it with one's hands. In both cases one has to pay attention to the fact that the second impulse not only changes the direction but also increases the speed. If one ignores the increase in speed one gets the wrong answer.

(12) *Suppose we faced the impulse engine this way (draw it at 90 degrees) and fired it, then faced it this way (draw it at 0 degrees) and fired it again, and then faced it this way (draw it at 180 degrees) and fired it again. (See figure 6).*

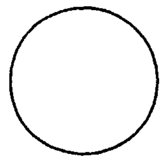
Fig. 6.

Which direction do you think that the spaceship would end up going in - could you draw what you think would happen?

This question is almost identical to the previous question with the exception that the two velocity components which cancel out are right next to each other instead of having a component in between them as in the previous question. This was done to see whether students could think about this question in terms of components when the cancellation was made more obvious.

(13) *Can you draw a picture and explain how you would get your spaceship to fly in a circular path? (See figure 7).*

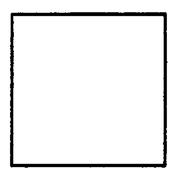
Fig. 7.



This question was included as a more complex problem in adding velocities. It was also included to see whether students spontaneously used vectors in helping themselves solve problems of a form different from what they are used to solving.

(14) *Can you draw a description of how you would get your spaceship to fly in a square path? (See figure 8).*

Fig. 8.

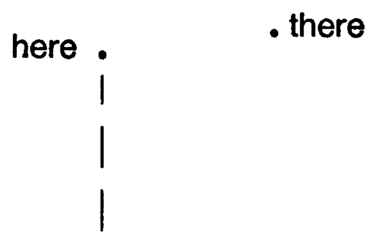


This question was intended to be a problem in determining which sequence of impulses would produce a given path - a square. Given the restriction of a fixed sized impulse, there is only one solution to this problem - stopping at each corner with an anti-impulse and reaiming.

(15) *Suppose you have fired your impulse engine once and got to HERE (illustration given - see figure 9) and you want to get to THERE. In other words, you want to make almost a right angle turn. Suppose that you can fire your impulse engine only one more time. How would you do it? Please draw a picture.*

This question involves determining what velocity needs to be added to an initial velocity in order to produce almost a right angle change in the direction of motion. The reason the task was to produce slightly more than a right angle was that it was meant to be analogous to the Corner game where one had to make almost a right angle turn by firing only one more impulse. This question thus involves applying an impulse

Fig. 9.



that is almost backwards with respect to the current direction of motion in order to change the direction of motion in the specified way.

(16) *Suppose that you were in a part of outer space where there is a strong solar wind blowing (direction drawn). How would that affect your path if you were travelling:*
(a) into the wind?
(b) in the same direction as the wind?
(c) perpendicular to the wind?

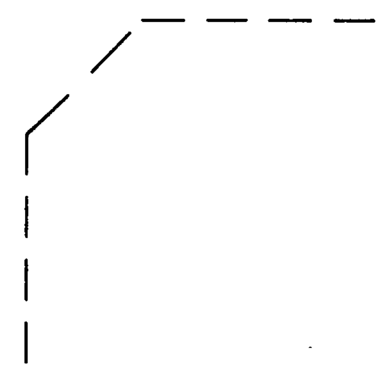
This was included to see whether students could adapt their knowledge of force and motion to a situation involving an impulse force and a continuous force (a wind). For part (c) the students were asked to draw the path of the spaceship as it hit the region of the wind. For parts (a) and (b), if the student said slow you down or speed you up, they were then asked is there any way we could figure out how much.

(17) *You are back in a part of outer space where the is no solar wind. Suppose that you want to dock your spaceship with its mothership which is stationary in space waiting for you. You can dock from any side of the mother ship that you like and you can approach by any path that you like but you want to dock as slowly as possible. How could you do it in order to get yourself going at slower than one impulse worth of speed. Remember, you can only give unit impulses?*

This problem involves determining how to add velocities in a two dimensional setting so as to reduce speed. It was designed to be analogous to the Target game where the goal is to hit the target with as low a speed as possible.

(18) *Suppose your spaceship took this path. (See figure 10).*

Fig. 10.



What would the sequence of impulse engine firings have been?

This is a problem that could be solved very nicely by thinking in terms of orthogonal components. First one gives a vertical component, then one introduces a horizontal component, and finally one needs to cancel out the vertical component. It could also be viewed as a problem of adding velocities so as to change direction where the increase in speed after adding the first two velocities has to be taken into account. This second form of solution motivates the use of vectors in order to be able to represent the change in speed in a way that permits one to determine the direction of the final impulse.

This problem is the *how would you achieve* form of question eleven which involved predicting the path of the spaceship when three orthogonal impulses are applied one after the other.

(19) *Suppose we were in an ice rink playing ice hockey with a puck and we each had a stick to hit the puck with. Suppose you were standing here (illustration given - see figure 11) and I were standing there. You hit the puck forward and then I hit it sideways as it passed me.*
(a) - which way would the puck go? Could you draw in what you think would happen?
(b) - would it change speed after I hit it?

This question is analogous to a previous question where the spaceship's engine fires orthogonal impulses separated in time. It is also analogous to question two of the original version of the questionnaire

Fig. 11.

where a football was kicked twice in directions orthogonal to one another. It involves adding velocities in a two dimensional space where two forces are applied to the same object at different points in time. It was included to see whether asking the same question in different contexts got different responses. For instance, it might have been that spaceships were so foreign to everyday experience that students could answer this question about the motion of hockey pucks but be unable to answer an analogous question about spaceships. On the other hand, the unfamiliarity of spaceships might actually be an advantage when dealing with the implications of a frictionless environment. The students might not have as many preconceived notions concerning the motion of spaceships.

This question was eliminated from the final version of the questionnaire because it was viewed as redundant since the pattern of answers was the same as for the analogous spaceship question. In fact, many students made comments like, "not this question again!". The result that the students realized that this problem is analogous to questions two and nine is very important. It implies that context is not an important factor in determining how the students respond to force and motion problems. Whether it is a ball being kicked, a hockey puck being hit, or a spaceship being propelled by an impulse engine, the students regard the situations as analogous. This suggests that when redesigning the computer games, the context does not need to changed from a spaceship in outerspace. Furthermore, it suggests that whatever the students learn about force and motion in the outerspace context, should transfer to other situations.

(20) *Suppose this time that you hit it very hard and then I hit it sideways much softer than you did. Which way would the puck go? Could you draw a picture of what you think might happen?*

This question was included to determine whether students could add the results of forces of different magnitudes with respect to determining the direction of motion. It involves determining the final direction of motion after two forces of different magnitudes have been applied at different points in time.

Ten out of eleven students answered this question correctly. Thus it was concluded that these students could allow for the effects of differences in magnitude of forces with respect to how they affect direction of motion and the question was dropped from the final version of the questionnaire.

(21) *Suppose there was a river with a strong current flowing at ten miles per hour. What would happen if you tried to swim straight across the river at two miles per hour - can you draw a picture of how it would look from a helicopter looking down on you?*

This problem can be thought about in terms of relative motion rather than in terms of forces. The swimmer is moving relative to the water at a certain velocity and the water is moving relative to the observer with a certain velocity. The motion of the swimmer from the point of view of the observer is represented by the addition of these two velocities. This problem is analogous to the problem of determining the motion of a plane flying through a wind as seen by a person on the ground. It is also has an analogy to the problem where the ball was kicked in two directions at the same time. The velocity of the river and the velocity of the swimmer are also occurring simultaneously.

All of the students were able to correctly solve this problem so it was dropped from the final version of the questionnaire.

4.3 Summary

The questionnaire was created in order to further explore the students' understanding of how impulse forces affect motion and to assess changes that might occur in the students' understanding as a result of playing the games.

The problems were designed to be verbal forms of situations that could occur in the Newtonian computer microworld. They were all phrased in one of two forms:

(1) What do you think would happen if you applied impulse forces in such and such a way?

(2) How could you apply impulse forces so as to make the object attain such and such a speed or such and such a direction?

Questions were created to investigate the following:

(i) context effects,

(ii) misconceptions documented by other researchers,

(iii) the distinctions and difficulties found in the pilot work,

- the one dimensional versus the two dimensional case,
- controlling direction versus controlling speed,
- applying impulses in similar versus opposing directions,

(iv) and more complex situations.

These complex situations included circular motion, the interaction of forces of different magnitudes, and the interaction of forces acting in different ways, for example, a continuous and an impulse force.

The original version of the questionnaire was tried out on eleven students and was found to be too long. Questions that were considered irrelevant, redundant, or too easy were deleted.

Chapter V

The Experimental Study

The major source of evidence for this thesis was a study involving the observation of high school physics students playing the sequence of games implemented in the Newtonian computer microworld. This chapter begins with a description of how this study was conducted. This description is followed by a general overview of the results. A more detailed presentation and discussion of the results is contained in the chapters which follow.

5.1 Procedure

The primary purposes of this study were to gain additional insights into how students reason about the effects of forces on motion and to determine how playing the games affects these reasoning processes.

The effectiveness of the games was assessed by the questionnaire which was given to the students before and after they played the games. A control group was included to provide further data on how people reason about forces and motion and to test the hypothesis that administering the questionnaire twice was, by itself, responsible for any improvement in the student's knowledge state. The control group, consisting of nineteen students, was verbally given the questionnaire on an individual basis on two separate occasions. The experimental group, consisting of twenty-one students, was also given the questionnaire individually on two separate occasions, however, they played the computer games in the intervening period.

This study was conducted at a high school located in an upper middle class suburb in the outer region of the Boston metropolitan area. The majority of students involved in the study were seniors. All of the students who participated did so on a voluntary basis during their free time.

There were two groups of students who participated. The first were from PSSC physics classes and had just completed chapters on vector addition and Newtonian dynamics. The second were students who spent some of their free blocks in the computer room - usually playing dungeons and dragons, doing homework, talking, or working with one of the computer terminals set up in the room. Most of these students had also studied the relevant physics.

Students were selected for the study on the basis of being willing to participate and on having a free block at a time when the computer was available. Students in the PSSC physics classes were recruited through their teacher who collected a schedule of their free blocks and through a letter from the experimenter explaining the purpose of the research and what it would involve. When a PSSC student did not show up, students from the computer room were recruited by the experimenter going up to an unoccupied person, explaining the purpose of the study and what it would involve, and asking them if they would like to volunteer.

Since the students were volunteers, one might expect that they would be an atypical sample. There is some evidence that this was not the case with respect to the students from the physics class. When scores on a class test on a chapter about vectors were compared for students who participated in this study versus those who did not, the difference was not significant at the .05 level ($t=0.38$, $df=53$, $p>.05$). Further, when the same comparison was made for the class test scores for a chapter on Newton's laws, the difference was also not significant at the .05 level ($t=0.44$, $df=46$, $p>.05$). Thus, the students from the physics class who participated in this study did not differ significantly from their classmates in their performance on tests of the relevant physics.

Further, there was no significant difference at the .05 level between the pretest scores of the control group and the pretest scores of the experimental group ($t=0.78$, $df=30$, $p>.05$). Thus there is evidence that the two groups were initially comparable with respect to their understanding of Newtonian dynamics.

The experimental group was individually given the questionnaire during the first week. The sequence of computer games was started if time remained. During the second week, the sequence of computer games was completed and the questionnaire was readministered.

The control group was individually given the questionnaire one week and then given it again during the next week.

The interviews were all tape recorded. In addition, the experimenter kept any diagrams or workings the students made while attempting to work out answers to the problems. The computer was programmed to record the inputs to the games played by the student so that they could be played back for analysis latter.

The role of the experimenter was restricted to asking the questionnaire problems, introducing the games, and recording data. The experimenter avoided intervening in the students' reasoning processes because the purpose of the study was to determine the effect of the games, not the effect of the games in conjunction with a teacher.

The study was conducted in the school's computer room - a large room which also served as a room where students could talk, do homework, play chess, etc.. On occasions, some of the students were interviewed in the math room - a large room across the hall from the computer room where students could do homework, talk, type up papers, etc..

Many efforts[1] were made to ensure a low dropout rate for this study. The no show rate was about one in three. Attempts were made to reschedule people who did not show up. Even so, three people in the experimental group and five people in the control group who started the study did not complete it.

The computer used to run the game programs was the original prototype LOGO stand alone computer system (GTL 3500). It had two CRT displays (one for text and one for graphics), a keyboard, a DEC LSI-11 processor, and a floppy disc unit for storing programs and recording data. It was made available for this study at times when it was not being used for classroom demonstrations.

5.2 Overview of the Results of the Games

The results of the questionnaire will demonstrate that most of the students had only a partial understanding of Newtonian dynamics prior to playing the games. Yet, they were able to succeed at achieving the goals of the games. Thus the hypothesis put forward in the introduction, that knowledge of formal physics is <u>not</u> a necessary prerequisite for success at these games, received support from this study.

Playing the games dramatically improved the students' ability to solve the basic force and motion problems of the questionnaire. Thus it is possible for students with only a partial understanding of Newtonian dynamics to succeed at these games, and, furthermore, to improve their ability to solve basic

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1. A letter was sent to each student explaining the study. Each student was contacted in person to ensure that the time chosen on the basis of their free blocks was indeed convenient for them. A written reminder was sent to the student a few days before each interview. The day before the interview the experimenter reminded the student in person. Then on the day of the interview the student received a written reminder in his/her school mailbox.

force and motion problems as a result of interacting with the games.

The students' descriptions of how they derived their strategies revealed that playing the games evoked many aspects of the students' knowledge. These included:

- general problem solving heuristics,
- ideas from other domains such as scalar arithmetic and geometry,
- partially understood knowledge of the formal physics,
- and intuitions about how forces affect the motion of objects.

The application and debugging of these various components of the students' knowledge combined with feedback from the microworld facilitated the evolution of the students' knowledge of Newtonian dynamics.

The study also revealed the necessity of facilitating the use of general problem solving heuristics, especially for the first few games in the sequence. For example, one of the games, Dock, where general problem solving techniques did not easily work, had some undesirable features. For students who already understood the physics, it was trivial. For those who did not, it was a very difficult and frustrating game.

Another kind of result is that there are features which contribute to making the games easy or hard which have nothing to do with the formal physics underlying the game. For example, games where one has to achieve two goals simultaneously, such as hitting and target and achieving a low speed, can often be more difficult than those where one just has to achieve a single goal.

The following overview of the results with respect to the individual games attempts to further exemplify the role of problem solving heuristics and the many factors which affect the difficulty of a game.

The students had no trouble with the game of Race where one has to cross a finish line with a high speed. Despite the fact that this game was easy, it is suggested that it should remain in the sequence of games since one third of the students, when answering the questionnaire, said that firing a second impulse would not affect the speed of the spaceship. The reason that even these students did not have trouble with this game is that trying the simplest thing, firing another impulse in the same direction, succeeds in increasing the speed of the spaceship.

The game of Dock, where one has to stop the spaceship in a specified location, was harder than the game of Race. The results of the questionnaire fail to support the hypothesis that this result occurred because figuring out how to make the spaceship stop is harder than knowing how to increase the speed. Alternative explanations for this result are that having to stop in a specified location adds difficulty to the game and, furthermore, that just experimenting by firing impulses does not usually succeed in stopping the spaceship. Thus the *just try various impulses and see what happens* problem solving heuristic works more readily in the game of Race than in the game of Dock.

When presented with the game where one has to navigate the spaceship around a corner, fewer students exhibited the "ignore momentum bug" than had been observed with physics naive subjects. Further, those students who may have started to ignore momentum were able to quickly debug their approach. This may have been a result of having studied physics. It may have also have been that the corner edge constraints of this game help simple problem solving techniques to work. For example, the state of being about to run into an edge caused students to fire an impulse away from the edge. This game thus encourages the use of anti-impulses which allowed the students to discover the stop and reaim strategy and the delayed anti-impulse strategy.

Students had no trouble adjusting their strategies to large initial speeds of the spaceship. Again, it is suggested that feedback from the situation combined with simple problem solving heuristics could be partially responsible for the lack of difficulty with this task. For instance, suppose that one is using the strategy of stopping and reaiming the spaceship. If the spaceship has a large initial speed and one tries firing an anti-impulse to stop the spaceship, the result is that one slows down but does not stop. Thus the goal is not achieved but the result is in the right direction. The logical thing to try, given this result, is another anti-impulse. Thus the general heuristic of *if one works a little but not enough, try two* works in this case.

Increasing the facility for applying impulses from only being able to give orthogonal impulses to being able to give impulses at 30 degree intervals of rotation was found to increase the difficulty of games. Several reasons are suggested for this result. Firstly, it is difficult to distinguish an impulse fired at an orientation of 30 degrees from an impulse fired at an orientation of 60 degrees. This may have caused confusion about the effects of impulses if their orientations were difficult to discriminate. Secondly, it is now harder to change the orientation of the spaceship because one has to press the engine rotation button more times to achieve the same amount of turning. Finally, many of the students, when given the facility of being able to fire impulses in more directions, changed their strategy for playing the game. Debugging a new strategy may have been partially responsible for the number of trials taken to succeed at the game.

Giving students the goal of either hitting the target or navigating the corner by firing only one more impulse after an initial impulse had been fired caused added difficulty. It was predicted following the pilot study that this increased difficultly would occur because students might have difficulty with tasks that require the firing of impulses that are almost backwards with respect to the current direction of motion. The results of the games and the questionnaire both support this hypothesis. It is further conjectured that the restriction of only firing one more impulse might also be responsible for the difficulty of this task since if students mistimed their impulse they were not allowed to fire another impulse to recover.

The game where one had to hit the target with a low speed was the most difficult of all of the games. Two reasons are suggested for this result. The first is that the game requires the attainment of two goals - hitting the target and achieving a low speed. It was observed that the most common strategy for reducing speed had a side effect of altering the direction of motion, thereby, often causing students to miss the target. The second hypothesis as to the cause of difficulty with this game is that students have trouble thinking of firing an impulse in a direction that is almost backwards with respect to the current direction of motion in order to reduce the speed of the spaceship. The results of the questionnaire also support this hypothesis.

The fact that the students had so much difficulty with this last game, where one has to hit the target with a low speed, suggests another general result. It concerns the way in which students are able to focus on one aspect of the behavior of the spaceship and ignore others. During the game where the students had to navigate around the corner by firing only one more impulse, they had to fire an impulse almost backwards with respect to their current direction of motion. This had the effect of producing almost a ninety degree turn in the direction of motion and enabled them to navigate the corner. This strategy of firing almost backwards also had the side effect of producing a low speed. Thus the students had achieved a low speed in the context of this game. However, when they were asked to achieve a low speed in the Target game, they did not know how to do this even though they had already done it. Thus, they must have been unaware of this dramatic change in speed which occurred in the Corner game, probably because they were focusing on controlling the direction of motion.

A further implication of the preceding result is that it supports the idea that many of the students were thinking of speed and direction as separate entities, not as a unified vector concept of velocity. This partially explains how they can manage to focus on direction and ignore speed.

A more detailed analysis of the way in which the students interacted with the games is presented in the next chapter.

5.3 Overview of the Results of the Questionnaire

There were several very striking results of this experimental study. The first is that the games appeared to help students to solve force and motion problems. The second is that the questionnaire facilitated the emergence of many new and interesting errors which had not been observed in the work of other researchers. For example, many of the students could not answer correctly questions concerning prediction and control of the speed of motion. Thirdly, there was a very consistent pattern of wrong answers to problems on the questionnaire. This suggests that there are misconceptions concerning Newtonian dynamics which are shared by many of the students. Furthermore, the students' justifications for their answers indicate that there are many diverse sources of such misconceptions - misunderstanding the vector representation, holding beliefs such as *turning uses up energy*, lack of familiarity with two dimensional addition, and so on. In addition, most of the students did not utilize vector diagrams to help themselves solve the problems. Of the students who did use vector diagrams, the majority did so incorrectly.

5.3.1 The most common errors that students made on the pretest:

Many [1] of the students gave incorrect answers to very basic questions about what happens to the speed of the spaceship after a second impulse has been fired. In contrast, nearly all the students could correctly answer similar questions concerning what happens to the direction of motion. This is consistent with the findings of the pilot research which suggested that knowing what happens to direction does not necessarily imply knowledge of what happens to speed.

Further, even students who could answer questions about speed often forgot to take into account changes in speed when answering questions that focused on direction. For example, a common wrong answer to the problem of how could you make the path shown in figure 1 is illustrated on the path. Here, when the students propose the direction of the final impulse, they are ignoring the fact that after the second impulse the speed increased.

Another common error concerned ignoring momentum. When asked how they could make the spaceship fly in a circular path most students gave the answer shown in figure 2. This answer involves completely ignoring momentum - a bug which was common amongst physics naive subjects in the pilot work and in the work of diSessa [in Papert et al. 1979]. However, the bug was not common in the majority of answers given by these students to the other problems in this questionnaire. In particular, it is most strikingly inconsistent with their answer to the problem which immediately follows the circle problem on the questionnaire. When asked how they could make the spaceship fly in a square path, most students gave the answer displayed in figure 3. This answer indicates that the students are taking into account the

1. 1/3 to 1/2 depending on the question.

Fig. 1.

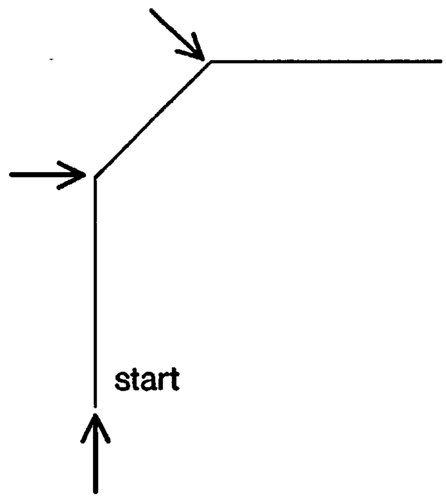


Fig. 2.

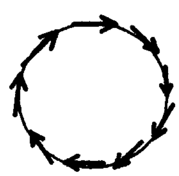
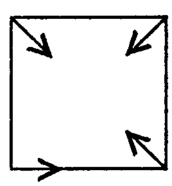


Fig. 3.



additivity of impulses in affecting changes in the direction of motion when considering this problem. Their answer to this question is thus inconsistent with their answer to the circle question where they behaved as though the results of impulses are <u>not</u> additive with respect to the direction of motion. Probably the error in the circle question occurs because students do not know how to apply vector addition to situations where the change in the direction of motion is continuous as opposed to discrete. They then have to revert to using intuitive knowledge of force and motion which yields the error of ignoring momentum.

One further error that most students committed concerned the solar wind problem. Here over three quarters of these students said that the velocity of the wind should be added to the velocity of the spaceship. The students' justifications for their answers to this question indicate that they are making a false analogy between this problem and the problem about a man rowing a boat across a river or an airplane flying through a wind. What makes such an analogy erroneous is the fact that the spaceship is propelled by an impulse engine, not a continuously acting engine, and is only drifting through the wind. Thus when the spaceship is travelling into the wind or against the wind, for example, the force of the wind can completely override the original velocity of the spaceship. Therefore, adding the two velocities is not appropriate in this case.

Potential sources of these errors will be further discussed in the analysis of the individual questions which can be found in chapter seven.

5.3.2 The effects of the games:

A two-way analysis of variance was performed on the data. One factor was whether the subject was in the control group or in the experimental group. The other factor was whether the subject came from the physics class or the computer room. The dependent variable was the difference between the subject's posttest and pretest scores (i.e. d = posttest score - pretest score).

Students who played the games improved their answering of the questionnaire significantly more than those who did not play the games ($F(1,28)=13.046$, $p=0.0006$, one tail). There was no significant difference in the amount of improvement between students from the physics class and students from the computer room ($F(1,28)=0.296$, $p=0.59$). Further, there was no significant interaction effect ($F(1,28)=0.022$, $p=0.88$).

The games were especially helpful to students on the following questions[1] -

Q. If the spaceship is stopped, what is effect of one impulse? (Q.1)
correct answer: impulse force produces constant, eternal motion - application of Newton's first law.

Q. Suppose the impulse engine is fired twice. Will the speed be different from when it was fired once? (Q.2)

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1. These questions are paraphrased for the sake of brevity. It you wish to refer to the actual text of the question, please see chapter seven.

correct answer: faster

Q. How could you make the spaceship go in a circular path? (Q.8)
correct answer: keep firing in towards the center of the circle.

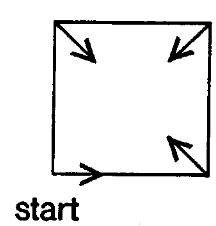
Q. How could you land slowly? (Q.12)
correct answer : fire almost backwards.
This is analogous to the Target game where one has to hit the target with a low speed.

The most surprising result is that the games improved students' answers to the circle question. All of the other improvements have direct parallels in at least one of the games, whereas, this is not true for the circle problem. The issue of circular motion is never addressed. This result is an encouraging indication that the games are teaching something more general about the way impulse forces affect motion. It is also encouraging to note that the games seemed to increase the students belief in Newton's first law.

However, the games did not seem to help students on the following questions:

Q. How could you make a square path? (Q.9)
correct answer : stop at each corner by firing an anti-impulse, then reaim and fire.
common wrong answer: (See figure 4).

Fig. 4.



This answer is incorrect because an impulse fired in at 45 degrees would have to be bigger than the original impulse in order to produce a right angle turn. However, the context here was uniform impulses so the impulse at 45 degrees would not be big enough to produce a 90 degree turn.

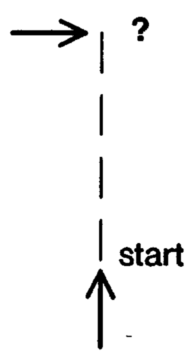
Q. How is motion affected by a solar wind? (Q.11)

One has to worry about the different ways in which the forces are acting to get this question right. The force in the games is always an impulse force and thus the games do no focus on this issue at all so it is not too surprising that they do not help.

There were no declines in performance as a result of playing the games with the exception of the following question:

Q. What is the effect of the second impulse on the speed of motion? (See figure 5).

Fig. 5.



In answer to this question just as many people changed to the wrong answer as changed to the right answer.

These results are discussed in more detail in the chapter seven which contains the analysis of the individual questions.

5.3.3 Solving *what would happen if* vs. *how would you achieve* problems.

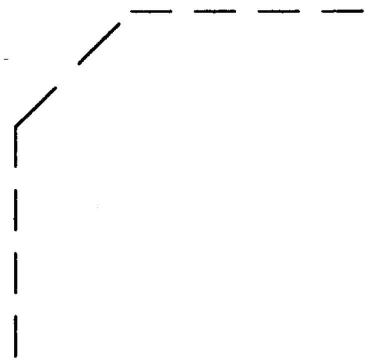
Three problems were presented in both the *what would happen if* and *how would you achieve* forms:

how could you make the spaceship stop?

versus

what happens if you fire two impulses in opposing directions?

Fig. 6.



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how could you produce this path (see figure 6)?

versus

what happens if you fire upwards, then to the right, and then downwards?

how could you slow down?

versus

what happens to your speed if you fire almost backwards?

The first two questions, which involve stopping the spaceship, both had very high success rates, 88% and 83%. For the remaining questions, the *how could you achieve* form was much more difficult than the *what would happen if* form (25% vs. 50% and 25% vs. 73% success rates).

There are undoubtably numerous potential explanations for why it is easier to determine what would happen in a given situation than it is to figure out how to make a specific result happen. A few possible explanations will be discussed here.

Consider the problem: *what happens to our speed if we apply an impulse in a direction that is almost opposite to our current direction of motion.* Compare this to the analogous problem: *how can you apply an impulse in order to slow yourself down.* The prior form of the problem may be more suggestive of vector diagrams than the latter, especially since the prior form was presented with the diagram shown in figure 7. Even if the students decide to utilize vector drawings for the latter form of the problem, they then need to

Fig. 7.

know that the resulting vector must be shorter than the vector representing the original velocity. With the *what happens if* form of the problem, the original velocity vector and the impulse vector are the same length so the students do not have to pay attention to lengths of vectors until they get to the resultant. Hence the *what happens if* form of the problem makes it more likely that the student will use vector diagrams and involves a vector diagram that is easier to construct.

Further, suppose the student does not utilize vector addition and subtraction but rather searches through possible results and answers. The *what would happen if* form of the problem only has three possible answers - the speed will be (1) slower, (2) faster, or (3) unchanged. Thus the student has to consider the reasonableness of only three possibilities. For the *how would you achieve* form of the question, the student has to consider the space, or at least some subset of the space, of all possible combinations of impulses. This is a much larger task.

Another explanation for why *how would you achieve* problems are more difficult might be that the students have catalogued experiences with impulses according to the operations as opposed to the effects of those operations. For example, they have indexed it by the operation of "applying an impulse in a direction that is almost opposite to the current direction of motion" as opposed to by the effect of "reducing the speed of motion". The result would be that it is harder for them to determine the operation that would cause a given effect than it is for them to determine the effect of a given operation.

One final explanation is that the diagram given with the *what would happen if* form of the question matches some reference example attached to the principle of cancellation in the students' memory. Thus the answer is given by a pattern recognition and retrieval process. Whereas, the *how would you achieve* form of the question involves realizing that it is appropriate to apply the principle of cancellation. It involves recognizing what one needs which may be a more difficult process. In other words, ways of accessing information vary. Therefore, it is possible to know something but not to know that it is relevant to the current situation.

5.4 Summary

The major source of evidence for this thesis was the study described in this chapter. It involved the observation of high school physics students playing the sequence of games implemented in the Newtonian computer microworld. The primary purposes were to gain additional insights into how students reason about the effects of forces on motion and to determine how playing the games affects these reasoning processes.

The effectiveness of the games was assessed by the questionnaire which was given to the students before and after they played the games. A control group was included to provide further data on how people reason about forces and motion and to test the hypothesis that administering the questionnaire twice was, by itself, responsible for any improvement in the student's knowledge state.

There were two groups of students who participated. The first were from PSSC physics classes and had just completed a chapter on Newtonian dynamics. The second were students who spent some of their free blocks in the computer room. Nearly all of these students had also studied the relevant physics.

It was found that the students were able to succeed at the games even though the questionnaire results indicated that they did not understand the formal physics prior to playing the games. Their descriptions of how they derived their strategies indicate that they were using a combination of:

- general problem solving heuristics,
- ideas from other domains such as scalar arithmetic and geometry,
- partially understood knowledge of the formal physics,
- intuitions about how forces affect the motion of objects,
- and feedback from the microworld.

The results also support the hypotheses made following the pilot study, that:

- controlling motion in one dimension is easier than controlling motion in two dimensions (since one dimension maps onto the familiar domain of scalar arithmetic),
- solving problems that require the application of impulses which are "backwards" with respect to the current direction of motion are difficult,
- knowing what happens to the direction of motion does not imply knowledge of what happens to the speed of motion.

The relative difficulty levels of games were found to be complicated by other factors. For instance, it was conjectured that games where one has to achieve two goals simultaneously, such as hitting a target and achieving a low speed, are inherently more difficult than games where one has to achieve only one goal.

The results of the questionnaire yielded several striking results. The first is that the games appeared to help students to solve force and motion problems. The second is that the questionnaire facilitated the emergence of many new and interesting errors which had not been observed in the work of other researchers. For example, many of the students could not answer correctly questions concerning prediction and control of the speed of motion. Thirdly, there was a very consistent pattern of wrong answers to problems on the questionnaire. This suggests that there are misconceptions concerning Newtonian dynamics which are shared by many of the students. Furthermore, the students' justifications for their answers indicate that there are many diverse sources of such misconceptions - misunderstanding the vector representation, holding beliefs such as *turning uses up energy*, lack of familiarity with two dimensional addition, and so on. In addition, most of the students did not utilize vector diagrams to help themselves solve the problems. Of the students who did use vector diagrams, the majority did so incorrectly.

Chapter VI

Results of the Games

This chapter commences with a description of how difficult the games were relative to one another. This is followed by a brief discussion of the major faults of the games that were uncovered while the students were working with them. Then the results of the individual games are described with respect to difficulty level, strategies used, and the merits and deficits of each particular game are discussed.

6.1 Relative Difficulty of the Games

The following is a listing of the games in the order in which they were presented to the students.

Race - cross the finish line with a high speed.

Dock - stop the spaceship inside its spaceport.

Corner - with the facility for applying impulses separated by 90 degrees.

Corner - with a large initial velocity.

Corner - with the facility for applying impulses separated by 30 degrees.

Corner - with only one more impulse permitted.

Target - with the facility for applying impulses separated by 90 degrees.

Target - with a large initial velocity.

Target - with the facility for applying impulses separated by 30 degrees.

Target - with only one more impulse permitted.

Target - with the goal of landing slowly.

If all of the games were of equivalent difficulty, the expectation would be that the later games in the sequence would be easier due to a practice effect. However, the games were not designed to be equivalent. For example, the first two games in the sequence, Race and Dock, were meant to be the easiest. They both involve controlling motion in one dimension only. It was hypothesized that dealing with one dimension should be easier than dealing with two dimensions because forces applied in one dimension affect motion in accordance with the familiar properties of scalar arithmetic.

The sequence of games was designed to focus the students on difficulties observed during the pilot study, one difficulty at a time:

- realizing that the affects of forces are additive with regard to changes in the speed of motion and the direction of motion (games which focus on overcoming difficulties with this: Race (speed), Dock (speed), navigate the Corner (direction), and hit the Target (direction)),
- taking into account changes in speed when attempting to control the direction of motion (games which focus on overcoming difficulty with this: Corner and Target with a large initial velocity),
- controlling the speed or direction of motion by an impulse which is applied almost backwards with respect to the current direction of motion (Games which focus on overcoming difficulty with this: Corner and Target with only one more impulse allowed (direction) and Target with the goal of landing slowly (speed)).

The following is a listing of the games in terms of the average number of trials that it took for the students to succeed. They are presented in order of increasing averages.

<u>Game description:</u> <u>Average number of trials until success</u>

Race - cross the finish line with a high speed: 1.00

Target - with a large initial velocity: 1.06

Corner - with a large initial velocity: 1.24

Target - with impulses separated by 90 degrees: 1.37

Corner - with impulses separated by 90 degrees: 1.50

Dock - stop spaceship inside spaceport: 1.67

Target - with impulses separated by 30 degrees: 1.67

Corner - with impulses separated by 30 degrees: 1.71

Corner - with only one more impulse permitted: 2.29

Target - with only one more impulse permitted: 2.87

Target - with the goal of landing slowly: 3.65

Note that this listing is not just the reverse order of the previous listing. This suggests that the games are not of equivalent difficulty. In fact, the game of Race is the number one game on both of these lists which implies that it is a very easy game. It is the first game that the students were presented with and yet they have no difficulty with it even though they have had no prior experience within this computer microworld. The result that the game of Race is so easy is consistent with the hypothesis that games which involve controlling motion in one dimension only should be easier than games which involve controlling motion in two dimensions.

However, comparing the relative difficult levels of the games should be done with some hesitation. There are features of the games that make them easier or harder than one another that have nothing to do with understanding physics. For example, consider the other one dimensional game, Dock, where one has to stop the spaceship in a specified location. This game followed the game of Race in the sequence of games. Thus the students had had more experience with the microworld when they played the game of Dock. Yet, the students took more trials, on the average, to succeed at this game than they did for the game of Race. This result implies that figuring out how to stop the spaceship is more difficult than figuring out how to increase its speed. However, the results of the questionnaire do not support this inference. Thus there must be some other reason why the students took more trials, on the average, to succeed at Dock. One possibility is that in the game of Dock there is the additional factor of having to stop the spaceship at a specific location. The student has to not only achieve the stopped state but also has to do it at a specific location. This second factor adds difficulty to the game that has nothing to do with understanding how to make the spaceship stop.

Given such reservations, the relative difficulty of the games will now be described. The results of the questionnaire will provide much more direct evidence as to the sources of conceptual difficulty that contribute to making these games easy or hard.

This discussion of the relative difficulty of the games is based on the average number of trials that it took for students to succeed at each game. It is a descriptive analysis only, no inferences are intended beyond the population of students who participated in this study. The results are used to provide further evidence as to sources of the students' difficulty in understanding Newtonian dynamics, to suggest improvements that could be made to the games, and to assist in determining a suitable ordering of the games.

The most difficult games were those where to achieve the desired goal, students had to fire an impulse in a direction almost opposite to the direction of motion. These included the games where the students had to go around a corner or hit a target by firing only one more impulse after the initial impulse and the game where they had to hit the target with a low speed. The hardest game was the one where they had to hit the target with a low speed. These results add support to the hypothesis that students have difficulty with problems which require for their solution the application of an impulse force in a direction which is almost opposite to the current direction of motion.

The students had less difficulty with the games where they had to navigate the corner or hit the target, even though these games were presented earlier in the sequence and the students were thus less experienced at dealing with the microworld. Both of these games were meant to focus the students on the falsity of the intuition that *objects should always move in the direction of the most recent impulse.* The fact that the students had little difficulty with these games may suggest that the students were not applying this intuition or that the games were structured so as to help them succeed even though they may have initially applied this misconception.

Similarly, the students had little difficulty adapting their strategy for navigating the corner or hitting the target to a situation where the spaceship had a large initial speed. This seems to indicate, contrary to the results of the pilot study, that the students are able to take into account the speed of motion when attempting to control the direction of motion. However, the results of the questionnaire further support the hypothesis that students have difficulty taking into account the speed of motion when focusing on the direction of motion. The relative lack of difficulty that students had while playing the games could be explained by the use of feedback from the situation combined with simple problem solving heuristics. For instance, suppose that one is using the strategy of stopping and reaiming the spaceship. If the spaceship has a large initial speed and one tries firing an anti-impulse to stop the spaceship, the result is that the spaceship slows down but does not stop. Thus the goal is not achieved but the result is in the right direction. The logical thing to try, given this result, is another anti-impulse. Thus the general heuristic of *if one works a little but not enough, try two* works in this case.

The games where the facility for rotating the spaceship's engine was changed from 90 degree rotations to 30 degree rotations, caused many students some added difficulty. This may have been due to the fact that it was now harder to make the spaceship's engine turn. One had to press the button more times to get the same amount of turning. It may also be due to the fact that this added flexibility in the directions in which impulses could be fired, caused many of the students to change their strategies. Debugging the new strategy could be responsible for many of the students not succeeding immediately on a game they had previously mastered.

6.2 Problematical Features of the Games

Having the engine rotation button set so that it rotated the engine to the right by 30 degrees was found to be undesirable. It is very difficult to distinguish an orientation of 30 degrees from an orientation of 60 degrees without any references. This is especially true when one is moving and under time pressure [1] as is the case in these computer games. If the students had trouble distinguishing these orientations, they may have perceived them as being the same. This would have the effect of making the spaceship's behavior appear unpredictable. What are perceived as being the same impulses have different effects. Thus it was concluded that changing the engine rotation button from 90 to 45 degrees would have been a better choice than from 90 to 30 degrees.

The game of Dock, where the students had to make the spaceship stop at a given position, was found to be problematical. For most students, the game was trivial. They knew how to make the spaceship stop. Those students who did not know were not likely to discover the answer by experimentation. Firing impulses with 90 degree variations in their directions gets the spaceship into states from which firing more impulses separated by 90 degrees does not often accidentally cause it to stop.

Many of the students complicated their strategies for playing the games by firing several impulses at the start of each game. They did so in order to get up to what they considered a reasonable speed. However, it meant that they had to account for that number of impulses when they wanted to alter the state of the spaceship. Although this is an important thing to learn, it was usually not the feature of the microworld that students were supposed to be focusing on. It was therefore concluded that the effect on speed of one impulse should have been increased so that the students would not have fired several impulses in order to get up to what they considered to be a desirable speed.

6.3 The Results of the Individual Games.

The following is the introductory statement[2] given by the experimenter to introduce the students to the games.

Do you remember the questions that I asked you about the spaceship? Well, all of these games are going to involve a spaceship like the one that we were talking about. Remember, it had a special impulse

1. If, when the spaceship is moving, one does not fire an impulse at a certain point in time, the spaceship will alter its position and it will be too late for that particular impulse to have the desired effect.
2. This statement, and all of the statements which introduced the games, were repeated and elaborated until the experimenter judged that the student understood what was being said.

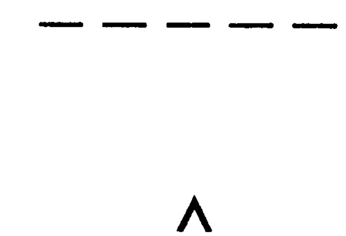
engine that gave a thrust like a kick whenever it was fired.

This is the spaceship (experimenter points to the triangle on the display screen) and the rest of the screen is outerspace, so there is no friction. You can control the spaceship's impulse engine by using two buttons on the keyboard. This one, K (experimenter points to button labeled K on the keyboard), fires the impulse engine which gives the spaceship a thrust like a kick in whatever direction it is aiming. This button, R (experimenter points to button labeled R on the keyboard), changes the direction of the spaceship's impulse engine. It does not affect the motion of the spaceship, it just turns the engine (experimenter demonstrates by pressing R a few times). See, pressing R reaims the impulse engine by turning it to the right by 90 degrees. If you press K now, it will fire an impulse in that direction (experimenter points in the direction in which spaceship is pointing). If the spaceship runs into the edge of the screen, it will crash and the game will be over.

6.3.1 The game of Race.

This game, illustrated in figure 1, involves adding velocities to increase speed in the one dimensional case.

Fig. 1.



The spaceship starts with a heading of zero and an initial speed of zero. The goal is to get the spaceship to cross the line with the fastest possible speed. The speed is printed out when the spaceship crosses the line.

Experimenter: *In this game, all you have to do is to get the spaceship across that line (experimenter points to the finish line) with as high a speed as you can make it have when it crosses the line. The speed will be printed out here (experimenter points to the bottom of the screen) when you cross the line.*

Results.

All of the students succeeded at this game on the first trial and all employed the strategy of firing several impulses in the upwards direction to increase their speed.

Discussion.

None of the students utilized the more sophisticated strategy of going down to the bottom of the screen first in order to be able to fire more impulses before they crossed the finish line. Possibly this strategy was not seen because students did not find the game challenging enough to want to try to do better nor were they asked to try again by the experimenter. Those students who had difficulty with this game made comments such as, "you cannot make it go faster", or, "I do not know how to make it go faster". However, when they were told to just try, they succeeded. This is because trying the simplest thing, firing another impulse in the same direction, works for this game.

It could be argued that since all of the students succeeded on the first trial of this game, it is unnecessary and should be eliminated from the sequence of games. However, some of the students were uncertain about what the effect of firing another impulse would be. This is further supported by the questionnaire result where one third of the students said that a second impulse would not alter the speed of the spaceship. This game would focus them on the fact that, in the one dimensional case, the effects of impulse forces on the speed of motion obey the familiar laws of scalar addition.

6.3.2 The game of Dock.

This game, illustrated in figure 2, concerns combining velocities so as to decrease speed in the linear case.

The spaceship starts with a heading of zero and an initial speed of zero. The goal is to get the

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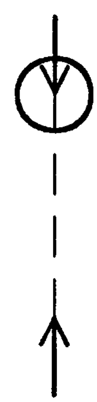
Fig. 2.

spaceship stopped inside its space port. The spaceport is represented by a circle.

Experimenter: *Now, in this game you have to get the spaceship inside its spaceport (experimenter points to the circle) and you also have to make it stop right in the middle of the spaceport. So, you have to figure out a way to make it stop. Remember, it is K to fire an impulse and R to make the impulse engine face in a different direction.*

The successful strategy here involves firing an impulse to get moving and then, when the spaceship is inside the space port, fire an impulse in the opposite direction. This counteracts the first impulse and stops the spaceship. (See figure 3).

Fig. 3.



The strategy can be modified by firing several impulses initially and then firing the same number in the opposite direction once the spaceship is inside the space port.

This game is similar to Race in that it involves combining velocities in the linear case. However, the goal here is to combine the velocities so as to stop oneself rather than increase the speed.

Results.

On the average, students took one and two thirds trials to succeed at this task. Students thus found this game more difficult than the previous game where the goal was to increase the speed of the spaceship as opposed to stopping it at a specified location.

Discussion.

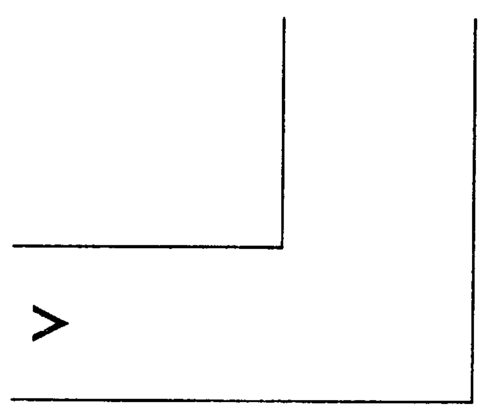
Many students found this game trivial. They knew how to make the spaceship stop. Those who did not, often made comments such as, "I cannot make it stop", or "where is the stop button?". Telling them to just try and see what happens did not seem to help. Firing orthogonal impulses got them into complex states and firing more orthogonal impulses usually did not result in accidentally undoing all the previous impulses and thereby stopping the spaceship. Thus experimenting usually only frustrated the student. In fact, one student got so annoyed that the decision was made to move on to another game even though she had not succeeded at this one.

This game has the undesirable feature of requiring the students to understand the solution before they can succeed at the task. General problem solving heuristics, such as, *experiment to see what happens*, do not achieve useful results. Possibly having the engine rotation button set at 180 degrees instead of 90 degrees would remedy this situation. Just experimenting, under such conditions, would be very likely to achieve the goal of stopping the spaceship. This game could then help students to discover the additive affects of forces on the speed of motion.

6.3.3 The game of Corner.

Here the spaceship's initial heading is 90 degrees and the initial speed is zero. The goal is to get the spaceship around the corner and to the end of the course without crashing into any of the edges. (See figure 4). The effect of pressing the engine rotation button is to turn the engine to the right by 90 degrees, the same as in the previous game.

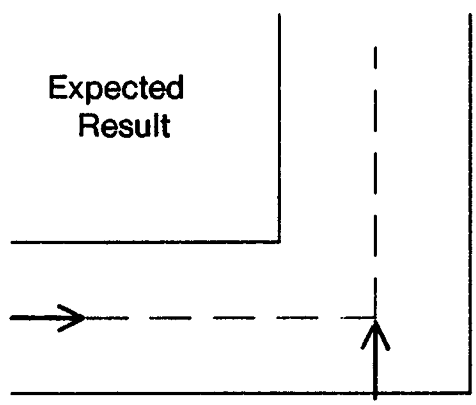
Fig. 4.



Experimenter: *In this game you have to make the spaceship go around this track. You do not have to stop it when you get to the end and you can go in any path that you like, just stay within the boundaries. If you hit one of the edges, you will crash.*

This game was presented in order to violate the expectation that things go in the direction that one pushes them. The naive strategy derived from this expectation would be to fire an impulse to get moving and then, when one gets to the corner, aim upwards and fire another impulse - the expectation being that one would then go upwards in the direction of the last impulse. (See figure 5).

Fig. 5.



What would in fact happen is that the velocity components produced by the two impulses would add together and one would go off at 45 degrees and crash into the wall. The expectation is thus dramatically violated in a way that is hard to ignore or perceptually deny. (See figure 6).

Results.

On the average students took one and a half trials to succeed at this game. Almost two thirds of the students succeeded on their first trial.

Strategies.

When first confronted with this task, one third of the students tried firing many times in the upwards direction when they reached the corner. This strategy was usually not successful. (See figure 7).

Fig. 6.

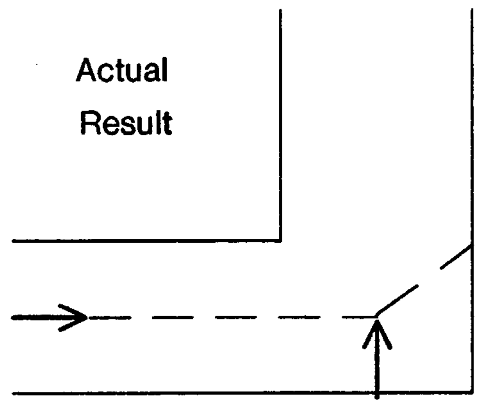
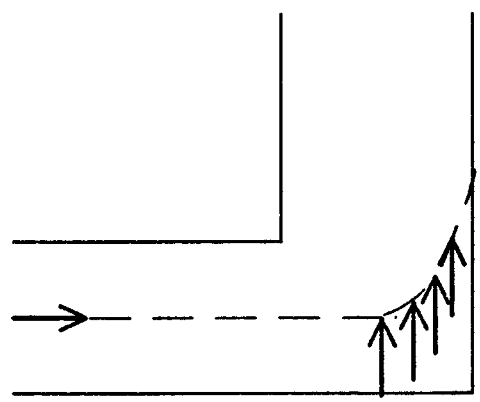


Fig. 7.



Another third of the students, stopped when they reached the corner and then fired an impulse in the upwards direction. (See figure 8).

Four of the nineteen students fired an upwards impulse and then fired an impulse to cancel the horizontal motion. This was called the delayed anti-impulse strategy. (See figure 9).

Two of the students tried just firing one impulse in the upwards direction when they reached the corner. This strategy does not succeed. (See figure 10).

Upon subsequent trials of this game, all but two of the students discovered the delayed anti-impulse strategy. Two thirds used the stop and reaim strategy and two students tried to debug the strategy of firing upwards many times by starting to fire upwards impulses before they reached the corner.

Fig. 8.

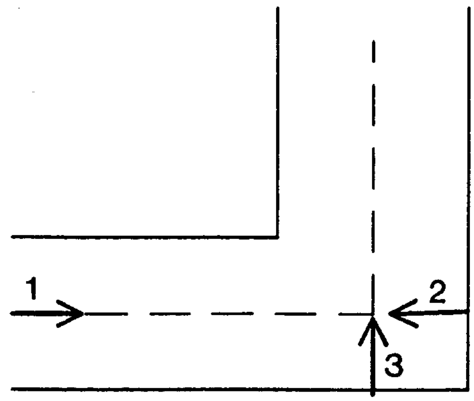
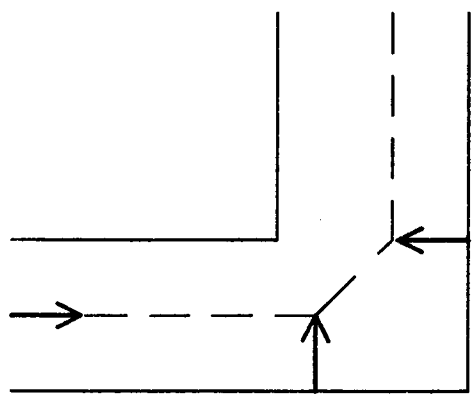


Fig. 9.



This approach is illustrated in figure 11.

Discussion.

Determining whether students exhibited the bug of ignoring momentum from their initial strategy for this game was, in some instances, difficult. Two out of the nineteen students clearly did exhibit the bug since they used the strategy shown in figure 10 which ignores momentum.

Six of the students clearly did not ignore momentum since they employed the strategy of stopping and reaiming when they reached the corner. (See figure 8).

The behavior of the remaining eleven students was more questionable. Seven tried firing upwards

Fig. 10.

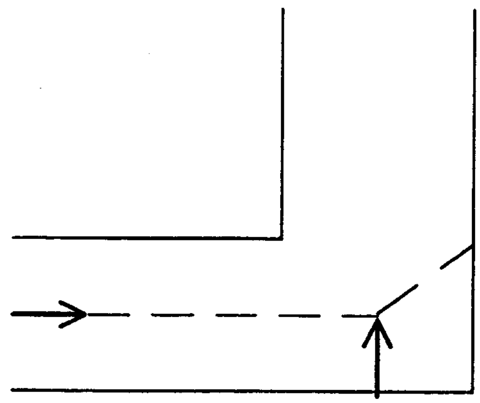
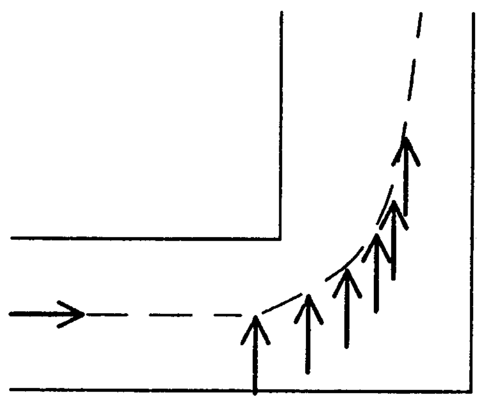


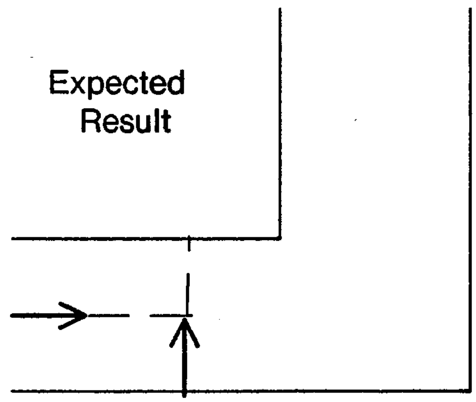
Fig. 11.



many times when they reached the corner. This strategy is shown in figure 7. Four used the delayed anti-impulse strategy illustrated in figure 9.

The problem with interpreting these last two strategies is that they could be a reaction to feedback from the situation rather than indicating a belief in additivity. When the students fire more impulses, as in the first strategy, they could just be reacting to the fact the the first impulse fired did not quite work so they introduce another impulse in the same direction. With respect to the second strategy, the students could be firing an impulse away from the edge that they are about to hit as opposed to thinking in terms of canceling horizontal momentum. Four of the students using these strategies fired their first upwards impulse before they reached the corner. This indicates that they were not ignoring momentum otherwise the expected result of such an impulse would be to crash into the corner as illustrated in figure 12.

Fig. 12.



So, half of the students clearly did not ignore momentum. Two clearly did. The rest are questionable. However, the students usually did not exhibit surprise at the behavior of the spaceship and were able to quickly debug their strategies. Also, most students did not exhibit the bug of totally ignoring momentum [1] when solving the problems in the questionnaire. These additional results suggest that when it comes to predicting the direction of motion of the spaceship, the majority of these students have the foundations for not ignoring momentum. Many probably started this game by ignoring it but, when given the appropriate feedback, they are able to debug their strategies quickly.

It is thus conjectured that as many half of the students could have begun this game by ignoring momentum. The proportion of students ignoring momentum in this games was, therefore, much lower than amongst people who have not studied the relevant physics who almost all ignore momentum. Despite the lower proportion, it still seemed to be a worthwhile game since it helped those who did ignore momentum to link their intuitions with their knowledge of formal physics. Comments such as, "of course, I should have expected that to happen", were common reactions to the change in the spaceship's direction of motion when a second impulse was applied. Further, students said they liked this game better than either Race or Dock. In fact, one of the most frequent comments made by students, when asked how could this sequence of games be improved, was that more maze like games should be included.

The result that almost all of the students discovered the delayed anti-impulse strategy whilst playing this game was interesting. DiSessa [1981] notes it as a potential strategy for the game where one has

1. The exception to this was the question concerning how to make the spaceship move in a circular path. The most common answer to this question completely ignored momentum.

to hit a target but comments that he never observed anyone using it. One reason it was never observed could be that one would have to be thinking in terms of components of velocity in order derive this strategy. In contrast, this game of going around a corner seems to have helped students discover this strategy. Possibly this is because they were firing an impulse away from the wall that they were about to crash into rather than because it inspired them to think in terms of velocity components. However, having derived this strategy might be useful in helping the students begin to think about components of velocity. This could be encouraged by simply asking the student to explain why the delayed anti-impulse strategy works. Thus this game could be useful not only for illustrating that one cannot ignore momentum but also for encouraging thinking in terms of velocity components.

Note. Several students commented that the button which allows one to reaim the spaceship's engine should have had the effect of turning the spaceship left 90 degrees instead of right 90 degrees for this game. This is especially true for making the application of the delayed anti-impulse strategy easier. If the reaiming button turned the spaceship left instead of right, one would only have to press it twice when implementing this strategy instead of six times. Alternatively, the direction of the corner could have been changed from a left turn to a right turn. This would have made the facility for reaiming the spaceship's engine to the right by 90 degrees efficient.

6.3.4 The game of Corner with an initial speed of 3.

This game is similar to the previous one except that the conditions are modified so as to change the focus of the game. The spaceship again has an initial heading of 90 degrees, however, the initial speed is now 3 - as though the impulse engine of the spaceship had been fired three times before the player gets control.

The strategy the player used to go around the corner in the preceding game has to be modified to take into account how the increased speed alters the effect of an impulse in changing the direction of motion. For instance, the players now have to fire three counter impulses, whereas, before they only had to fire one.

This game thus represents a modification of the original Corner game in order to focus on the effect that changes in speed have on attempts to change direction of motion.

Experimenter: *I am going to change the game this time so that the spaceship will already be moving when you start. It is as though the impulse engine had been fired three times before you get control of it. So, the* *spaceship will be moving with three impulses worth of speed when you start.*

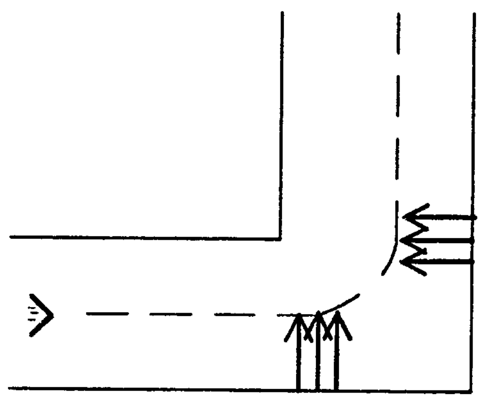
Results.

All but one of the students succeeded at this task on the first trial.

Strategies.

Almost two thirds of the students chose to modify the delayed anti-impulse strategy for this task. They did so by firing three impulses whenever they would have fired one impulse previously. This approach is illustrated in figure 13.

Fig. 13.



Four of the students modified the stop and reaim strategy as shown in figure 14.

Three chose to fire two anti-impulses immediately so as to reduce the task to being the same as in the previous game. They then employed the delayed anti-impulse strategy as shown in figure 15.

Discussion.

The fact that students had so little difficulty with this task may reflect the use of common sense reasoning combined with feedback rather than an a priori understanding of how their strategy needed to be modified to take into account the initial velocity of the spaceship. To illustrate, suppose that the student is using the stop and reaim strategy. When the student fires the first anti-impulse, the result is that the spaceship merely slows down instead of stopping. This result partially fulfills the goal that the student

Fig. 14.

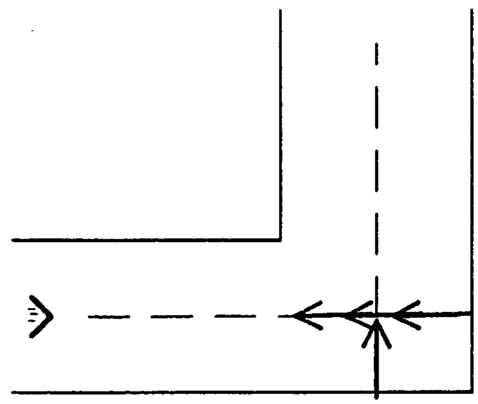
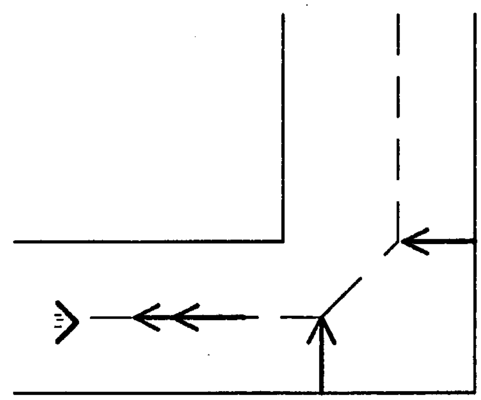


Fig. 15.



was trying to attain, so, the student fires another anti-impulse hoping that it will complete the task. With this kind of reasoning combined with feedback, the student can successfully modify his or her strategy to succeed at this game.

6.3.5 The game of Corner with 30 degrees separations in impulse directions.

This game was included in the sequence of games to get the person used to being able to apply impulses at 30 degree separations as opposed to only being able to apply impulses at 90 degree separations as was the case in the previous games.

The initial speed was zero and the heading 90 degrees. The goal was the same as for the first

Corner game, that is, to get the spaceship around the corner and to the end of the course without crashing into an edge.

Experimenter: *Now I am going to change the effect of pressing R from 90 degrees to 30 degrees. So, it will turn the spaceship's impulse engine by a smaller amount whenever you press it. The spaceship will be stopped this time, as it was before.*

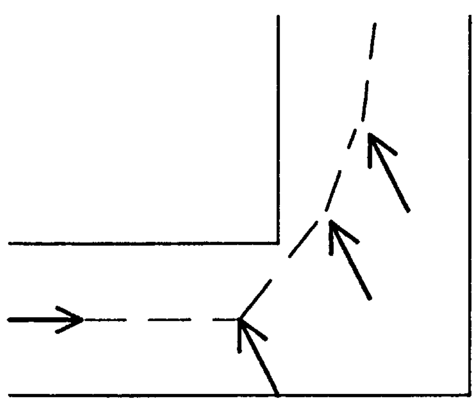
Results.

It took students 1.7 trials, on average, to succeed at this game. More than half of the students succeeded on their first trial.

Strategies.

The most frequently employed strategy was to fire partially backwards several times at minus thirty degrees. (See figure 16). This strategy was used by almost half of the students.

Fig. 16.



One third used the stop and reaim strategy that they had used previously. (See figure 17).

Three utilized the delayed anti-impulse strategy illustrated in figure 18.

Two chose to fire an upwards impulse early and hard. (See figure 19).

One person fired a second impulse almost backwards at minus sixty degrees as shown in figure 20.

Fig. 17.

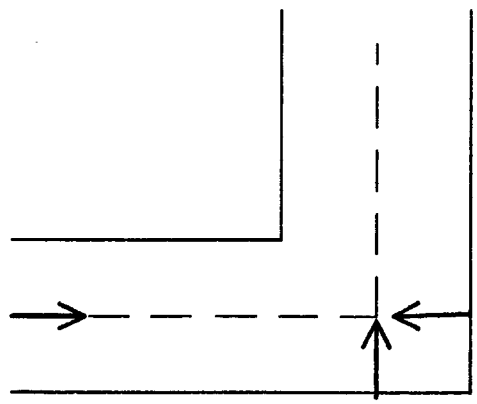
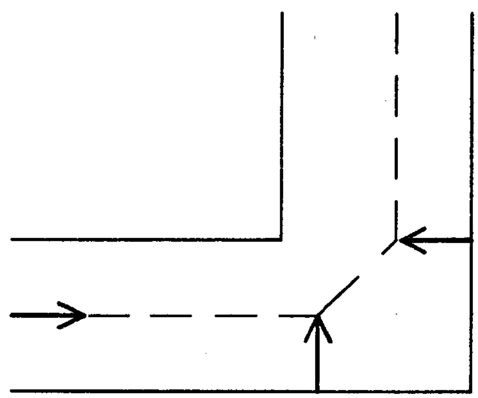


Fig. 18.



Discussion.

The result that students did not immediately master this game, even though it has the same goal as the first Corner game which they have already mastered, probably reflects two factors:

(1) Half of the students, in response to the ability to apply impulses at thirty degree intervals, changed their strategy. They were thus involved in the process of debugging a new strategy which would account for their lack of success on the first trial.

(2) With the ability to fire impulses at thirty degrees instead of ninety, it takes three presses of the engine rotation button to achieve a rotation that used to take only one press. The problem with this occurs when one wants to make large rotations. For example, when one wants to fire an anti-impulse, it takes more time than it used to. The added time factor by itself would make it harder to implement strategies and harder to

Fig. 19.

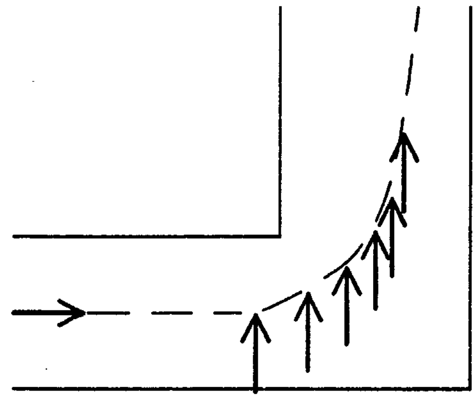
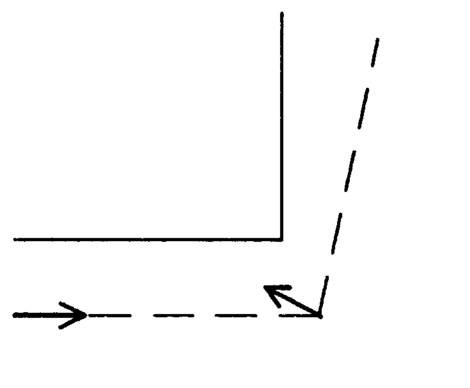


Fig. 20.



recover in time from errors. In addition, in order to achieve the result in time, one starts pressing the rotation button faster. Pressing the button faster increases the error rate for rotating the engine more than one had intended. This means that one then has to go around again until one gets into the desired position. If this cannot be done fast enough one crashes into a wall and loses the game.

Thus, making it possible to apply impulses at thirty degree intervals instead of ninety, increases the number of strategies available to the subject but it also has the side effect of increasing the likelihood of performance (as distinct from conceptual) errors. The reason one should pay attention to this class of errors when designing educational games is that such errors tend to frustrate the students and to cause them to focus on factors that have nothing to do with understanding Newtonian dynamics.

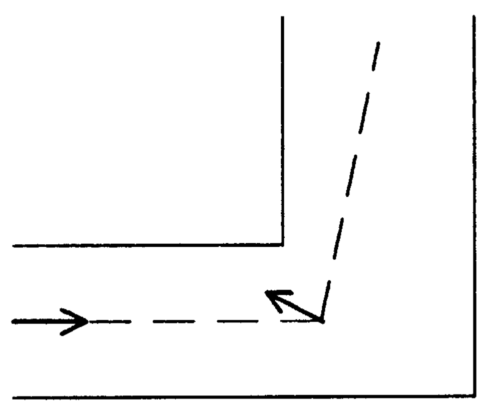
6.3.6 The game of Corner with only one more impulse.

Here the spaceship's initial orientation is again at 90 degrees but this time the initial speed is 1. The effect of pressing the rotation button is to turn the spaceship right 30 degrees. The goal is to make the spaceship go around the corner by firing only one more impulse.

Experimenter: *Now this time, to make things harder, the spaceship will be moving with one impulse's worth of speed when you start. You have to go around the track by firing only one more impulse. The impulse can be in whatever direction that you want and fired whenever you want, but, you can only fire once.*

The successful strategy involves firing an impulse "almost backwards" (i.e. at a heading of minus 60 degrees) when one gets to the corner. (See figure 21). This impulse has the effect of producing almost a right angle turn in the direction of motion.

Fig. 21.



Results.

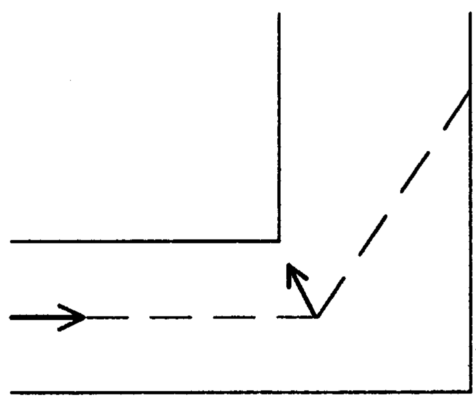
On the average, students took 2.3 trials to succeed at this task. One third of the students succeeded on their first trial. One student did not succeed after many tries. When computing the average, it was assumed that he would have succeeded on the next trial.

Strategies.

Almost half of the students utilized the potentially successful strategy of firing an impulse almost backwards at minus 60 degrees on the first trial. The only possible error that occurs with this strategy is to fire the backwards impulse too soon or too late. Several students made this error and had to work on the timing of their impulse.

The remaining half of the students employed the strategy of firing backwards at minus 30 degrees when they reached the corner. (See figure 22). This strategy does not produce enough change in the direction of motion to navigate the corner.

Fig. 22.

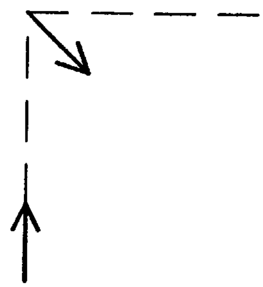


These students then typically fired backwards at minus 60 degrees on the second trial. This usually succeeded in navigating the corner. A few students had to debug the timing of the backwards impulse.

Discussion.

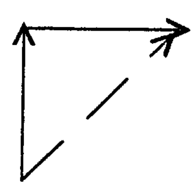
The students had more difficulty with this game than with any of the previously encountered games. This result supports the hypothesis that people have difficulty with problems whose solution requires an impulse to be fired in a direction that is almost opposite to the current direction of motion. However, the results of the questionnaire, which will be described in the following chapter, suggest that this hypothesis needs some modification. When it comes to controlling the direction of motion, the students do think to apply backwards impulses. Their answers, however, are usually incorrect because they have a rather specific bug in their reasoning. To elaborate, when asked a question analogous to this game, *how would you produce almost a 90 degree turn in the direction of motion*, nearly all of the students said that an impulse applied backwards at 45 degrees with respect to the current direction of motion would produce

Fig. 23.



such a turn. (See figure 23). This answer is incorrect because an impulse applied backwards at 45 degrees would need to be larger than the initial impulse in order to produce such a turn. The context here was fixed sized impulses and, therefore, an impulse applied backwards at 45 degrees would not produce anywhere near a 90 degree turn. This incorrect answer reflects two general patterns of wrong answers which occurred on the questionnaire. The first is that the students exhibited a preference for impulses applied at 45 degrees. This may have been because many of their textbook examples contain 90 and 45 degree angles in the example vector diagrams (see figure 24).

Fig. 24.



Thus the students may have erroneously concluded that there is something preferable about a 45 degree answer. Secondly, the students consistently ignored anything related to the lengths of vectors. In this problem, for example, they are ignoring the fact that an impulse applied backwards at 45 degrees would have to be bigger than the original impulse. Thus it is not that the students do not think to apply backwards impulses in order to control the direction of motion. Rather, they have several misconceptions that affect which backwards impulse they choose.

This belief that an impulse applied at minus 45 degrees should produce a 90 degree turn in the direction of motion probably accounts for why over half of the students thought that an impulse applied at minus 30 degrees would be sufficient to go around the corner. These students only resorted to minus 60 degrees on the second trial, after minus 30 had failed.

Another feature of this game that could contribute to its relatively high difficulty level is the restriction of firing only one more impulse. If the impulse is mistimed, the students cannot compensate for it in anyway since they are not allowed to fire another impulse.

Note: When these games were replayed for analysis, it was observed that it was difficult to distinguish between an impulse at minus 30 versus an impulse at minus 60. Often one had to place a vertical referent on the screen in order to determine the direction of the impulse. It would be even more difficult for students to make this distinction since they are under time pressure. Also, when the games were replayed for analysis, arrows were drawn to indicate the direction of each impulse thus making it easier to determine the impulse's direction. The students did not have this advantage. However, only one student vacillated between firing impulses at minus 30 and minus 60 degrees during this last Corner game. This suggests that she might not have been able to distinguish an impulse at 30 degrees from one at 60 degrees. The rest of the students either fired backwards at minus 60 degrees on the first trial or else chose minus 30 on the first trial and then minus 60 on the second trial. Possibly students were able to make the discrimination when playing this particular game because the corner provided somewhat of a referent and because they could count the number of times they pressed the engine rotation button. The counting technique would be feasible here because the game involves firing only one impulse and hence involves making only one engine rotation which is always made from the same starting orientation.

The following four Target games have the same foci as the preceding four Corner games. The context now is hitting a target instead of going around a corner.

6.3.7 The game of Target.

The goal of this game is to hit the target. The game thus involves combining the effects of impulses so as to control the direction of motion. The effect of pressing the engine rotation button is to turn the engine to the right by 90 degrees, as it was in the first two Corner games.

Experimenter: *This next game is a little different. The idea is just to hit this target (the experimenter points to the circle). The effect of pressing R is back at 90 degrees again, like this (experimenter demonstrates by pressing R a few times).*

Results.

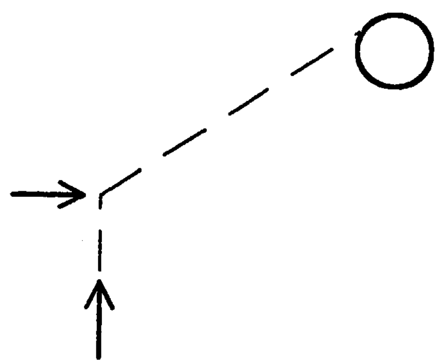
The average number of trials taken to win this game was 1.37. Over three quarters of the students succeeded on the first trial.

Strategies.

The strategies will be described in order of frequency of use.

The strategy shown in figure 25 was used by one third of the students on their first trial of this game. After successive trials, almost two thirds of the students had discovered this strategy.

Fig. 25.



The delayed anti-impulse strategy, which was the most frequently used strategy in the first Corner game, was the next most common strategy here. (See figure 26). It was utilized by almost one third of the students on their first trial at this game. Its frequency of use did not increase appreciably on successive trials.

Stop and reaim was employed by one third of the students at some point while playing this game. (See figure 27).

Fire early and hard was also a strategy that was used by more than one quarter of the students. (See figure 28).

One student had a unusual strategy which created an S curve. (See figure 29).

It was also noted that many students, when they unexpectedly missed the target, reverted to the strategy of aim in the direction that one wants to go and fire hard. (See figure 30).

Fig. 26.

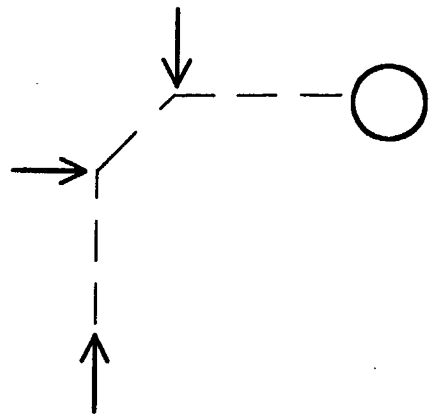
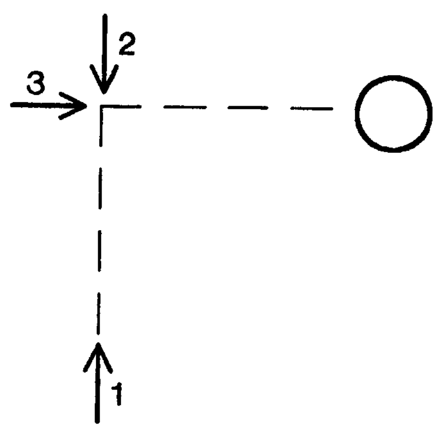


Fig. 27.



Discussion.

All of the strategies that worked for the Corner game also worked for this game. In addition, other strategies were possible for the Target game. The combination of two orthogonal impulses at the start of the game and the S curve produced by one student are examples of alternate strategies for this game. Two thirds of the students did discover at least one of these strategies, although the majority took more than one trial to do so. Also, the fire early and hard strategy works for this game, whereas, it did not usually work in the Corner game. More than one quarter of the students utilized it. Thus, although there was a lot of transfer of strategies from the Corner game to this game, two thirds of the students did discover new strategies for this game.

Fig. 28.

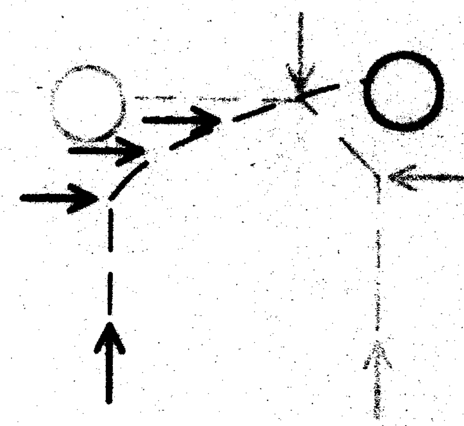


Fig. 29.

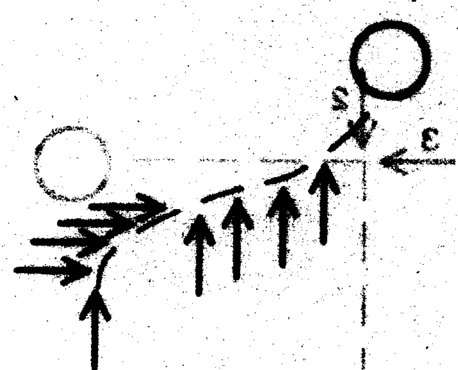
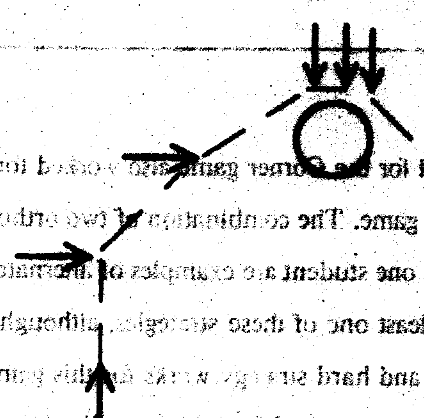


Fig. 30.



The students did not have much difficulty with this game but nearly all of them said that they liked it. It has the feature of permitting more strategies to succeed than the Corner game and, in that sense, it is probably an easier game. Whether Target should precede Corner or vice versa is debatable. More strategies succeed at Target. On the other hand, the Corner game was designed to help students realize that one cannot ignore momentum. This is likely to be an important feature in helping students succeed with these games. However, initial success within this microworld is probably the most important factor. This would be especially true for more naive subjects such as some of the people observed during the pilot work. Most of them found this microworld to be extremely counterintuitive. To avoid frustrating such subjects, the Target game should probably have preceded the Corner game since more strategies succeed with the Target game.

6.3.8 The game of Target with an initial speed of 3.

This game is analogous to the second Corner game. The initial speed of the spaceship is 3 instead of zero - as though the impulse engine of the spaceship had been fired three times before the player gets control of it.

Experimenter: *I am going to change the game this time so that the spaceship will already be moving when you start. It is as though the impulse engine had been fired three times before you get control of it. So, the spaceship will be moving with three impulses worth of speed when you start.*

The strategy that the player used to hit the target in the preceding game has to be modified to take into account the increased initial speed. The change in the direction of motion, after a single impulse has been fired, is lessened. This game was thus designed to focus on the interaction between the speed of motion and the application of impulses in order to alter the direction of motion.

Results.

The average number of trials that it took for students to succeed at this game was 1.06. All but one of the students succeeded on the first trial.

Strategies.

Five of the students modified the stop and reaim strategy shown in figure 31.

Five modified the fire early and hard strategy shown in figure 32.

Fig. 31.

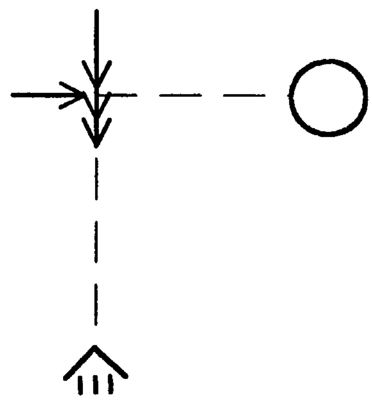
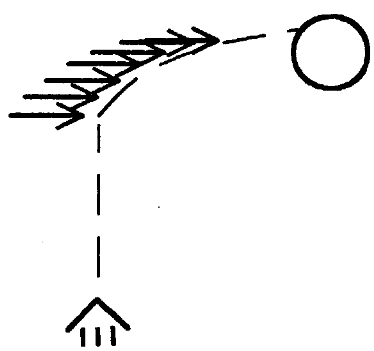


Fig. 32.



Three utilized the delayed anti-impulse strategy shown in figure 33.

Two used the two initial orthogonal impulses strategy shown in figure 34.

Finally, the one student who had produced the S curve strategy for the Target game, modified it for this game as shown in figure 35.

Discussion.

The students thus had no difficulty with this game. This may be partially due to the fact that they had been given a similar task in the context of going around a corner. Including this game in the sequence, as well as the analogous Corner game, seemed unnecessary. Probably a version of this game with the facility to apply other than orthogonal impulses would have been more useful. When only orthogonal

Fig. 33.

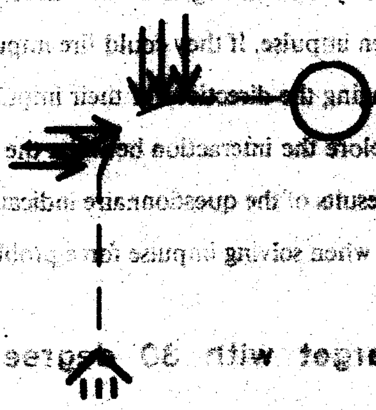


Fig. 34.

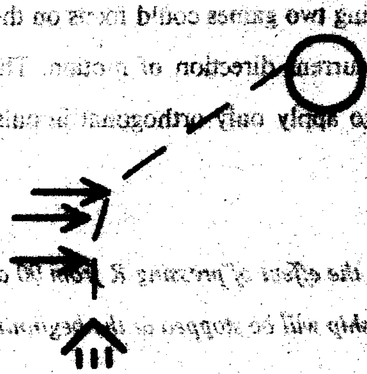
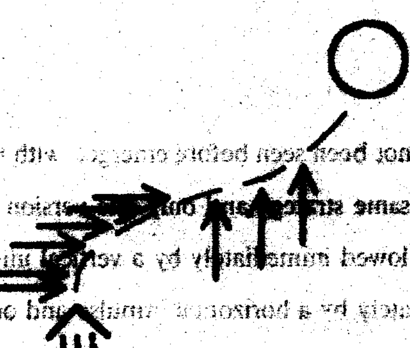


Fig. 35.



impulses are available, students modify their strategies to account for the increased initial speed by altering the number of times they fire a given impulse. If they could fire impulses at 30 degree intervals, they could also modify their strategies by changing the directions of their impulses, not just the quantity. This would give them the ability to further explore the interaction between the speed of motion and the effect of an impulse. This is needed since the results of the questionnaire indicated that the majority of students failed to take into account this interaction when solving impulse force problems.

6.3.9 The game of Target with 30 degree separations in impulse directions.

This game was included as a transition game for altering the effect of the engine rotation button. Instead of turning the engine right 90 degrees as in the previous game, it now turned the engine right 30 degrees. This was done so that the following two games could focus on the application of an impulse in a direction that is almost opposite to the current direction of motion. The application of such impulses would not be possible with the facility to apply only orthogonal impulses. The goal of this game was simply to hit the target.

Experimenter: *Now, I am going to change the effect of pressing R from 90 degrees back to 30 degrees again. The goal is still to hit the target. The spaceship will be stopped at the beginning of each game.*

Results.

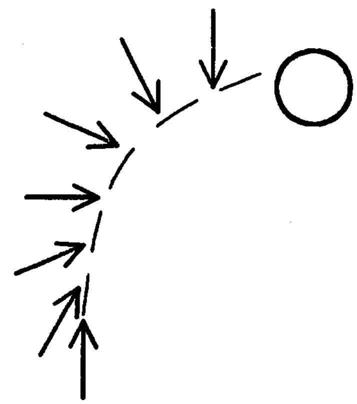
On the average, the students took 1.71 trials to succeed at this game. Two thirds of them succeeded on the first trial.

Strategies.

Several strategies that had not been seen before emerged with this game. Symmetrically equivalent strategies were considered as the same strategy and only one version of it will be diagrammed here. For example, a horizontal impulse followed immediately by a vertical impulse was considered the same as a vertical impulse followed immediately by a horizontal impulse and only the latter is diagrammed below. The strategies that the students used will be presented in order of decreasing frequency.

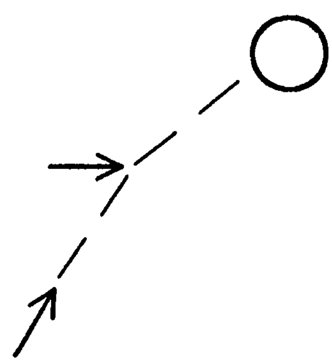
The strategy illustrated in figure 36 produced a curved path by repeatedly turning the engine right 30 degrees and firing an impulse.

Fig. 36.



The next strategy shown in figure 37 is also a previously unseen strategy.

Fig. 37.



The strategy shown in figure 38 is similar to the preceding one.

The stop and reaim strategy, illustrated in figure 39, was also observed.

The strategy shown in figure 40 is the combination of two orthogonal impulses that was observed previously.

The next strategy, shown in figure 41, is a modification of the preceding strategy which takes into account the ability to fire impulses at 30 degree separations.

The strategy of aiming and firing hard in the direction that one wants to go was used especially when

Fig. 38.

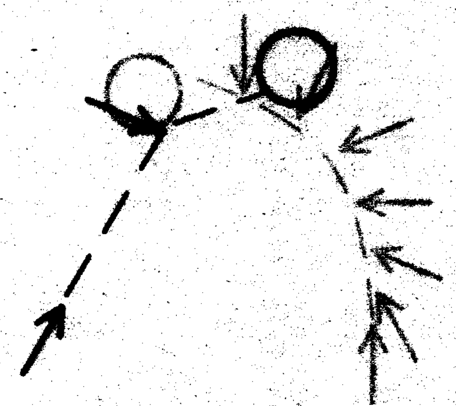


Fig. 39.

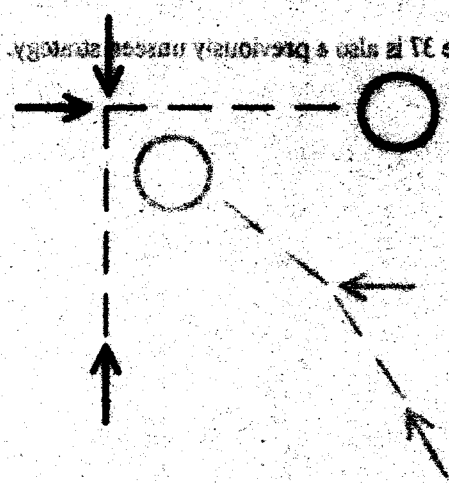


Fig. 40.

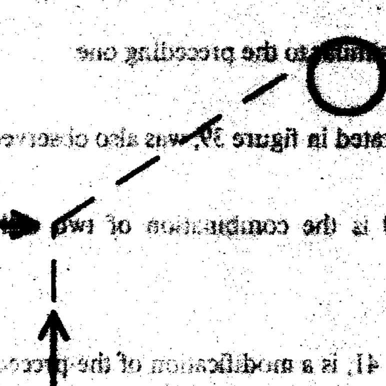
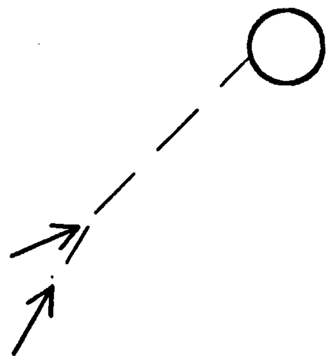
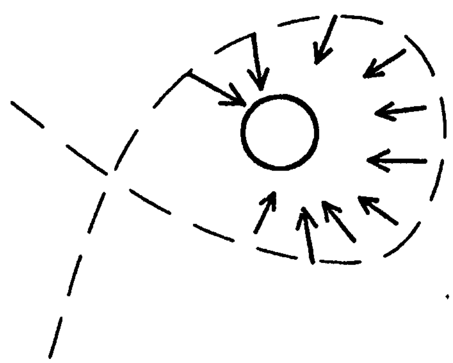


Fig. 41.



students were about to miss the target. This strategy is displayed in figure 42.

Fig. 42.



The delayed anti-impulse strategy, shown in figure 43, was also observed.

The one subject who produce an S curve strategy during the previous Target games, utilized the same strategy again here as shown in figure 44.

Discussion.

When confronted with this game, the majority of students used strategies that they had not used before. This reflects the facility to apply impulses at 30 degree intervals and the fact that the game of hitting the target permits many strategies to succeed. There are not the constraints introduced by navigating a maze which exist in the Corner games. This game thus encouraged students to explore the effects of more varied combinations of impulses than were tried in the Corner game. However, imposing

Fig. 43.

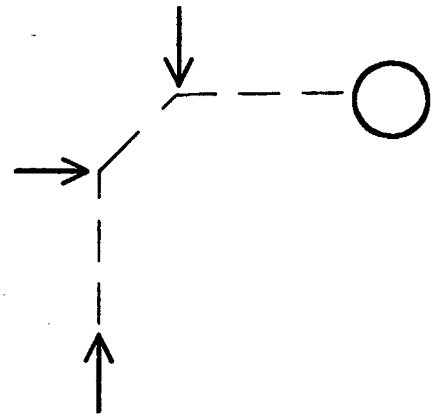
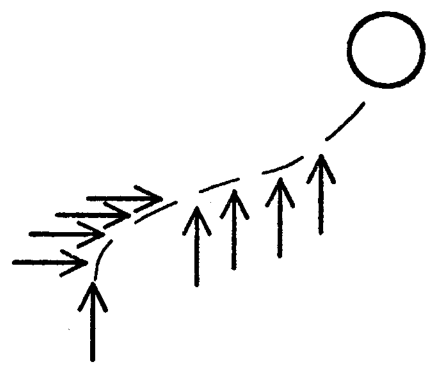


Fig. 44.



constraints on what the students can do plays a valuable role in helping them to discover the implications of Newton's laws. For example, the Corner game caused students to:

- discover anti-impulses in that they used the stop and reaim strategy to avoid hitting the edge,
- discover the delayed anti-impulse strategy which it was conjectured could be used to help students think in terms of orthogonal components of velocity,
- and to focus on the fact that one cannot ignore momentum.

One of the main tasks of the designer is to determine what kinds of goals and constraints are useful in helping the students to discover the phenomenology of the microworld.

6.3.10 The game of Target with only one more impulse.

In this game the initial speed of the spaceship is 1. The effect of pressing the engine rotation button is still to turn the spaceship right 30 degrees. The goal is to make the spaceship hit the target by firing only one more impulse.

Experimenter: *Now this time the spaceship will be moving with one impulses worth of speed when you start. You have to hit the target by firing only one more impulse. The impulse can be in whatever direction that you want and fired whenever you want, but, you can only fire once.*

Results.

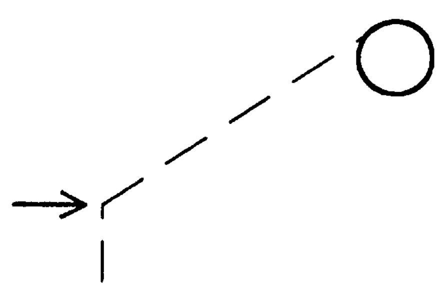
The students took 2.87 trials, on the average, to succeed. One quarter of the students succeeded on their first trial. One student had 8 trials and never succeeded.

Strategies.

Unlike the analogous Corner game where only one strategy works, there are three successful strategies for this game. All occurred with about equal frequency.

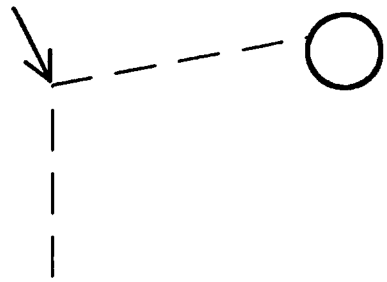
The first strategy involves firing a horizontal impulse at the start of the game. (See figure 45). Unless this impulse is fired as soon as the game begins, this strategy will fail. Two of the students who tried this strategy did not make it work.

Fig. 45.



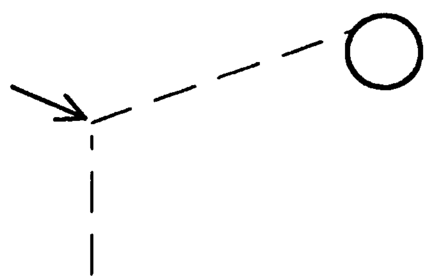
The next strategy involves firing an impulse almost backwards with respect to the direction of motion. (See figure 46). It is the same strategy that worked for the analogous Corner game.

Fig. 46.



The last strategy involves firing an impulse backwards with respect to the direction of motion but not as far backwards as in the previous strategy. (See figure 47). This impulse, therefore, needs to occur sooner than the impulse in the previous strategy.

Fig. 47.



Discussion.

The students found this game more difficult than any of the other games they had encountered so far. It was even more difficult than the analogous Corner game which all of the students had already mastered. This result is somewhat surprising since more strategies work for this game than in the Corner game. Thus there are more ways to succeed at this task. This seems to be an example of how the lack of constraints can hurt rather than help. The goal in the analogous Corner game is more narrowly defined. One has to produce as close to a right angle change in the direction of motion as one can when one gets to the corner. In contrast, several different angular changes in the direction of motion intersect the target provided they occur at the right position. Thus the task in the Target game is more ill defined. This, combined with the fact that the strategy of firing a horizontal impulse immediately at the start of the game works in theory but is difficult to implement, may partially account for the difficulty that the students experienced with this game.

6.3.11 The game of Target with the goal of landing slowly.

The goal of this game is to hit the target with as low a speed as possible. An impact speed of less than one was considered a success. The initial speed of the spaceship is zero and the effect of pressing the engine rotation button is still to turn the spaceship right 30 degrees.

Experimenter: *This time, try to hit the target with as low a speed as you can. See if you can hit with a speed of less than one.*

Results.

On average, it took students 3.65 trials to succeed at this task. One student achieved the goal on his first trial at this game. Two students did not succeed after many tries. When calculating the average, it was assumed that they would have succeeded on the next trial. Three of the students hit the target with a low speed after having just missed the target and having fired many impulses in rapid succession in order to save themselves. Their success at achieving a low velocity was judged to be accidental. However, their success was still incorporated when computing the average.

Strategies.

There are many strategies that will work for this game. All of the strategies embody the idea of reducing the speed by canceling some of the forward momentum.

One strategy that was frequently used was the fire almost backwards strategy which was one of the strategies that worked in the previous game. (See figure 48). It is also the only strategy that worked for the Corner game where one could fire only one more impulse after the initial impulse.

Another strategy that was often used is the combination of a horizontal and a vertical impulse at the start of the game followed by an almost backwards impulse to reduce speed when the spaceship is close to the target. (See figure 49).

One student employed a strategy which had been observed frequently during the pilot work with people who had a lot of physics background. It was entitled balanced backfiring. (See figure 50).

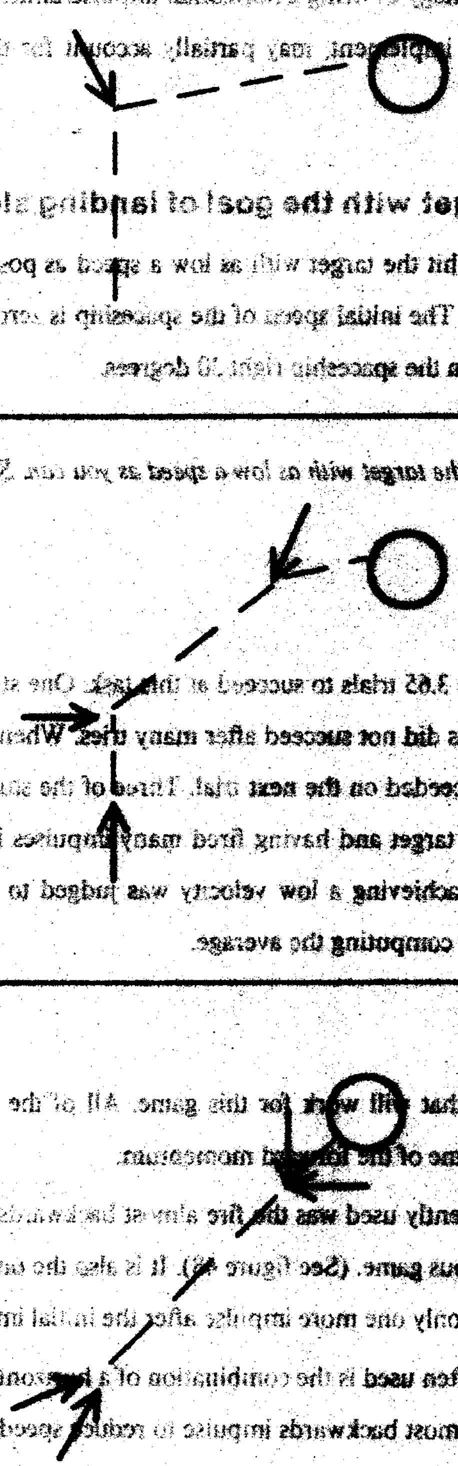


Fig. 50.

Discussion.

The average number of trials to success for this game, 3.65, is probably lower than is appropriate. Two students never did succeed at this game. In computing the average it was assumed that they would have succeeded on their next trial. This is most likely an erroneous assumption. Further, the success of several of the students was judged to be accidental since it occurred after they had fired several impulses in rapid succession to avoid missing the target. This should not really have been considered a success. In addition, many of the students could not repeat their success on subsequent trials indicating their success was also accidental. Thus the average for this game is probably inappropriately low.

Even though the average number of trials to success is lower than it should be for this game, it is still higher than for any of the other games. Further, it was the last game in the sequence so the students were very experienced at applying impulses by the time they played this game. This experience should have made the game easier. The game is, therefore, even more difficult than is suggested by the average number of trials to success. The students found this to be a very challenging game and most spent more time on this game than on any of the prior games.

The result that this was the most difficult game is a little hard to explain. When the students played the Corner game, where they had to navigate the corner by firing only one more impulse after the initial impulse, they all used the strategy[1] of firing an impulse almost backwards with respect to the direction of motion. A side effect of this strategy is to achieve a very low speed. Thus all of the students have achieved a low speed prior to encountering the Target game where they had to hit the target with a low speed. The fact that the majority of students have difficulty in achieving a low speed, even though they have done it in the past, demonstrates how people ignore features of the situation that are not relevant to what they are focusing on. In the Corner game the students were focusing on the direction of motion and probably did not even notice the low speed that they had attained. This is demonstrated even more dramatically by the fact that five students used this same fire almost backwards strategy when playing the Target game where one could fire only one more impulse. This game immediately precedes the game where one has to hit the target with a low speed. Three out of these five students still had difficulty with achieving a low speed. Thus although they had achieved a very low speed in the game immediately prior to this one, they could not reproduce the result in this context. Probably this is because they did not even pay attention to the low speed since they were focusing on the direction of motion at the time.

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1. The reason that they all used this approach is because it is the only one that works.

One possible reason why this task was so difficult may be that succeeding at this game requires the attainment of two goals - hitting the target and achieving a low speed. These students have already demonstrated that they can hit the target. However, when an impulse is applied so as to reduce the speed it also changes the direction of motion unless balanced backfiring is utilized. Most of the students did not use balanced backfiring so that their attempts to reduce speed also changed the direction of motion. This sometimes caused them to miss the target and thereby contributed to the difficulty level of this game. However, this alone does not account for the difficulty that students had with this game. Most of them had trouble achieving a low speed indicated by comments such as, "I don't think that it is possible to get a speed of less than one". This conclusion is corroborated by the result that three quarters of the students could not correctly answer a problem on the questionnaire concerning the reduction of speed.

The fact that this was the most difficult game supports the hypothesis that people have trouble with problems that involve the application of backwards impulses for their solution. In particular, they have difficulty in thinking to apply almost backwards impulses in order to control the speed of motion. The results of the questionnaire add further support to this hypothesis. In addition, there is some evidence to suggest that students do <u>not</u> have similar difficulty in thinking to apply impulses in almost the same direction as the current direction of motion in order to increase the speed of motion. To elaborate, at the conclusion of the study, several of the students were asked to hit the target and to navigate the corner by achieving as high a speed as they possibly could. In contrast to the game where they had to hit the target with a low speed, none of these students had any difficulty attaining a high speed. The conclusion is that what the students have major problems with is using impulses fired almost backwards with respect to the direction of motion to reduce the speed of the spaceship. This conclusion is further supported by the results of the questionnaire where students readily suggest partially backwards impulses to control the direction of motion (although, they do it incorrectly[1]), but, cannot suggest how to reduce the speed of the spaceship without stopping it.

6.4 Summary

Observing students interacting with the games revealed that many factors influence the difficulty of a particular game. These factors include:

(1) Whether or not the students can employ general problem solving heuristics to achieve useful results.

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1. They have the misconception that an impulse applied backwards at 45 degrees will produce a 90 degree turn. This is not true in the context of fixed sized impulses.

(2) How the constraints of the game operate to help the students to discover the implications on Newton's laws. For instance, the constraint of not hitting the edges in the Corner game was useful in helping the students to realize that the effects of impulse forces are additive.

(3) How the goals of the game act to focus the students' attention. For example, many of the students did not appear to notice changes in the speed of motion when they were focusing on controlling the direction of motion.

(4) How the students' intuitive beliefs and knowledge of formal physics influence the way they interpret the behavior of the spaceship.

One of the main tasks of the designer is to take into account such factors when trying to create situations which will help students to discover the phenomenology of the microworld.

Chapter VII

Results of the Questionnaire

The students' responses to the questionnaire yielded many interesting results. These findings were outlined in chapter five and will be briefly summarized again here. The results of each question will then be presented and discussed. This is followed by a description of the students' use of vector diagrams. Finally, the chapter closes with the presentation of some additional evidence obtained by asking students attending a graduate psychology seminar to answer a force and motion problem.

7.1 Overview of the Results

Students who played the games improved their overall answering of the questionnaire significantly more ($p=.0006$) than those who did not play the games. This result suggests that the games were helpful in enabling the students to solve very basic problems concerning the effects of impulse forces on the motion of objects. For instance, the question, *if the spaceship is stopped, what is effect of one impulse*, is a very basic question pertaining to Newton's first law of motion. Yet, many of the students could not correctly answer this question. Playing the games improved the students' ability to answer such questions.

The questionnaire also facilitated the emergence of many new and interesting errors which had not been observed in the work of other researchers. For example, many of the students could not correctly answer questions concerning prediction and control of the speed of motion. Further, even students who could answer questions about speed, often forgot to take into account the influence of changes in speed when answering questions which focused on determining the direction of motion.

There was a very consistent pattern of wrong answers to problems on the questionnaire. This suggests that there are misconceptions concerning Newtonian dynamics which are shared by many of the students.

Further, the students' justifications for their answers indicate that the sources of these misconceptions are many and diverse. Sources of misconception include:

- misinterpreting the vector representation,
- holding erroneous beliefs such as, *turning uses up energy*,
- and interference from knowledge of scalar addition and subtraction.

Finally, most of the students did not utilize vector diagrams to help themselves solve the problems. Of the students who did use vector diagrams, the majority did so incorrectly. This result adds further support to the conjecture that the majority of students are not thinking in terms of a vector concept of velocity, where speed and direction of motion are unified into a concept of velocity which can be represented by a vector.

These results will be discussed in more detail in the following analysis of the individual questions.

7.2 Results and Discussion

When reporting the proportion of students who could correctly answer each question on the pretest. students who did not complete the study by doing the posttest were also included. This provided a larger and therefore more representative sample (n=40 vs n=32) from which to compute this measure of the difficulty that students had with the individual problems.

In order to provide a way of assessing how helpful the games were to students who could not solve a given problem on the pretest, a Fisher exact probability test was performed on the data for each question. The proportion of students who changed to the right answer on the posttest, having answered the question incorrectly on the pretest, was compared for the experimental versus the control group.

(1) *What do you think will happen if we fire the impulse engine once? How far will the spaceship go, assuming there are no planets or other spaceships to get in the way?*

Correct answer: an impulse force will produce constant, eternal motion in the absence of other forces. This is an application of Newton's first law.

Most common wrong answer: the spaceship will go for a ways and then stop.

Results:

pre-test , post test	x ✓	✓ x	✓ ✓	x x
Expt.	6 /18	0 /18	12/18	0 /18
Cont.	0 /14	0 /14	13/14	1 /14

Of the 40 people[1] who did this questionnaire at least once, 80% had the correct answer on the pretest. All of the people who got the wrong answer originally and who played the games changed to the right answer after playing the games. However, only one person in the control group could not answer this question correctly on the pretest. There is thus a ceiling effect[2] for the control group which interferes with the determination of whether the games really helped or not. No one went from the right answer on the pretest to the wrong answer on the posttest.

A Fisher exact test performed on this data yields a probability of 0.143. This is the probability that the null hypothesis is true, i.e. the probability that playing the games does <u>not</u> help students who answered this question incorrectly on the pretest, to answer it correctly on the posttest. The two by two contingency table from which this probability is computed can be derived by taking columns one and four of the table of results. Henceforth, when presenting the result of a Fisher exact test, the following format will be used ($p=0.143$, Nexpt.= 6/6, Ncont.= 0/1). Nexpt. gives the number of students in the experimental group who changed to the correct answer on the posttest as compared to the total number of students in the experimental group who answered the question incorrectly on the pretest. Similarly, Ncont. gives the number of students in the control group who changed to the correct answer on the posttest as compared to the total number of students in the control group who answered the question incorrectly on the pretest.

Discussion: Even though they had just studied Newton's laws in class, one out of three people in the experimental group answered this question incorrectly indicating a failure to understand Newton's first law of motion. This lack of understanding derives from living in a world with friction where objects do not

1. This includes dropouts.
2. ceiling effect: very few people got the wrong answer.

move forever after a force has been applied to them. There is some evidence that the games remedy this situation. This is probably because the Newtonian computer microworld simulates the fact that without friction an object in motion does not stop unless a force is applied.

(2) *Suppose we fire the impulse engine of the spaceship twice. Will it go faster or slower or the same speed as when the engine was fired once?*

Correct answer: it will go faster.

Most common wrong answer: firing the impulse engine twice as opposed to once will make no difference to the speed.

Results:

pre-test , post test	x ✓	✓ x	✓✓	x x
Expt.	3/18	0/18	15/18	0/18
Cont.	1/14	0/14	8/14	5/14

Two thirds of the people who were asked this question got it right the first time they were asked it. All of the people in the experimental group who got the question wrong on the pretest got it right on the posttest as compared to only one person in the control group went from wrong to right ($p=0.048$, Nexpt. = 3/3, Ncont. = 1/6).

Discussion: One out of three people answered this question incorrectly on the pretest. The situation is in fact even worse than that since some people specified faster but not two times faster. This was counted as "correct" because students were not asked how much faster so this spontaneous qualification was ignored for consistency's sake.

The error rate for this question is rather alarming since it suggests a basic misconception about the effects of forces on motion. Instead of believing that the more impulses, the more speed, one out of three believe that after the initial impulse, the speed remains constant, even if one applies a new force in the same direction. There is evidence that the games help the students to overcome this misconception.

One justification given by some of the students who gave the answer that the speed remains the same was, "the second impulse is the same size as the first, so it will produce the same speed". It is true that the second impulse will produce the same size component of velocity as the first. What they are failing to realize is the fact that the two add together.

These students may not realize the additivity because they believe that speed is a property associated with an impulse and not an additive quantity. For instance, if one has two objects both going at five miles per hour and one ties them together, they still go five miles per hour not ten. On the other hand, they may be answering this question on the basis of their everyday experience which tells them that the same size force will produce the same speed. This holds true in the everyday world since objects are usually stopped when one applies a force to make them move. This belief thus has an implicit assumption that the object is stopped when one applies the force. However, people may not have noticed this fact since the only state they ever deal with is stopped. They thus apply this idea, regardless of the state of the object, because they have not learnt that state is relevant.

The belief that the second impulse will make the spaceship go faster, but not two times faster, is somewhat mysterious. Such a belief implies that the effects of the impulses are somewhat additive - the students think that something gets lost somewhere. This idea may relate to the experience from the everyday world where one kicks a ball and then kicks it again while it is still moving. The result of the second kick is to make the ball go faster but not two times faster because friction has slowed the ball down in the mean time.

(3) *Suppose the spaceship was sitting in space in a stationary position and you fired the impulse engine once to get the spaceship moving. How could we get the spaceship to stop?* (note: if the students said, "turn off the engine", then the experimenter said, "remember, it is a special impulse engine that gives a sudden burst of force and then shuts itself off", if they said, "turn on the brakes", experimenter said, "there are no brakes, only the impulse engine" and if they said, "hit a planet", experimenter said, "there are no planets or anything else to hit".)

Correct answer: fire an impulse in the direction opposite to motion.

Most common wrong answer: it is impossible to make the spaceship stop.

Results:

pre-test , post-test	x ✓	✓ x	✓ ✓	x x
Expt.	3/18	0/18	15/18	0/18
Cont.	0/14	1/14	11/14	2/14

Most people, 88%, got this question right. All of the people in the experimental group who had the wrong answer on the pretest changed to the right answer on the posttest. Whereas, none of the people in the control group did (p=0.100, Nexpt.=3/3, Ncont.=0/2).

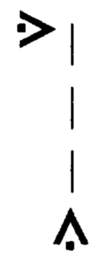
Discussion: The correct answer has several origins. One is derived from the mathematical notion of cancellation. Equal and opposite things cancel - three combined with negative three is zero. Another is the everyday experience of blocking. If one places one's hand in front of a rolling ball, the hand provides a force in the direction opposite to the motion of the ball and the ball stops.

Many students answered this question correctly who could not correctly answer other questions concerning speed. This is probably because this question appeals to the familiar notion of cancellation and because this question involves both speed and direction. People who believe that directions are additive, but not speeds, would still get this problem right.

The origin of the answer that it is impossible to make yourself stop comes from the belief that motion should be in the direction of the last impulse. This idea is derived, in a similar fashion to the misconception revealed in the previous question, from living in a world with friction. The belief that motion should be in the direction of the last impulse, like the belief that the same size impulse always produces the same resulting speed of motion, has the implicit assumption that the object is in the stopped state when one applies the impulse. This is the normal case in a friction filled environment and is responsible for these erroneous beliefs. The answer that it is impossible to make yourself stop comes about as follows: if one believed that the motion should always be in the direction of the last impulse, then one could not apply an impulse so as to make oneself stop, because, every impulse, no matter what the direction, would result in motion.

(4) *Suppose we fired the impulse engine once and then, after a little while, turned the engine so that it was facing right and then fired the impulse engine again (illustration given - see figure 1),*

Fig. 1.



(a) - which direction do you think that the spaceship would go in? Could you draw a picture of what would happen?

(b) - would it be going faster or slower or the same speed as before the second impulse?

Correct answer for part (a): ↗

Most common wrong answer for part (a): →

Results for part (a):

pre-test , post test	X ✓	✓ X	✓ ✓	X X
Expt.	0/18	0/18	18/18	0/18
Cont.	0/14	0/14	12/14	2/14

In response to part (a) of this question concerning how the direction of motion would change, most people, 93%, got the correct answer. Also, no one changed their answer from the pretest to the posttest. Since no one in the experimental group got the wrong answer on the pretest, there was no room for improvement on this question after playing the games (p=1.000, Nexpt.=0/0, Ncont.=0/2). Thus a ceiling effect prevents determining whether the games would help people who could not answer this question.

Discussion: Only three people in this study exhibited the bug of ignoring momentum in this context. That is, three people said that the new direction of motion would be in the direction of the last impulse. This is in contrast to the results of diSessa [in Papert et al. 1979] and of the pilot study where this was a very frequent bug. This suggests that studying formal physics helps slay this bug - at least in this situation. Possibly this question refers to a reference example that the students have learnt from their physics course, because, when more complex questions about changes in direction were asked, such as the circle question, the ignore-momentum bug reappears in most of the students.

Correct answer for part (b): speed would be faster after the second impulse.

Most common wrong answer: no change in speed.

Rarer wrong answer: the speed would be slower after the second impulse.

Results for part (b):

pre-test , post test	x ✓	✓ x	✓ ✓	x x
Expt.	3/18	4/18	6/18	5/18
Cont.	0/14	1/14	5/14	8/14

With respect to the question concerning speed, half of the students gave the wrong answer. Even students giving the correct answer often did so with hesitation and uncertainty indicated by comments like "I guess maybe faster, but, I'm not sure, it might even be slower, but, I'll say faster". One third of the people in the experimental group changed their answer from the pretest to the posttest - half changed to the right answer ($p=0.100$, Nexpt.= 3/8, Ncont.= 0/8) and half changed to the wrong answer. Only one person in the control group changed their answer and that was to the wrong answer.

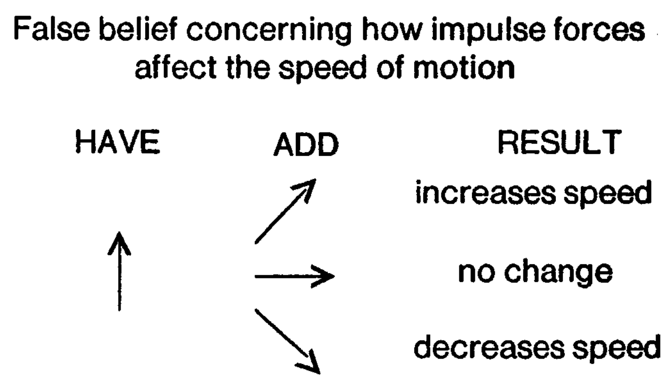
Discussion: The games seem to hinder as much as help for this question - just as many people change to the wrong answer after playing the games as changed to the right answer. The games thus increased the students' uncertainty about their beliefs concerning speed as reflected by the high proportion of people in the experimental group who changed their answers.

People who gave the wrong answer of "slower" to this question gave reasons like, "it goes slower because energy gets used up in turning, so you slow down". This idea may have its origins in experiences with driving cars and riding bicycles. When one goes to change the direction in which one is driving a car or riding a bicycle, one slows down in order to prevent skidding. Thus turning gets associated with slowing down. This association may be partially responsible for some students believing that any change in the direction of motion slows one down. Another misleading feature of cars and bicycles is that whenever one wants to change the direction of motion, one has to turn the steering wheel or handle bars. It thus takes effort and energy to make the vehicle turn. This association may be responsible for the idea that turning uses up energy. This, however, represents a misconception about the relationship between force and energy since turning the spaceship has to do with force not energy.

The majority of people who answered this question incorrectly said that the speed would remain unchanged after the orthogonal impulse.

One interesting origin of this wrong answer which, unlike wrong answers for the prior questions, has nothing to do with living in a world with friction, is the following. Some students said that 90 degrees should be the zero point in terms of adding velocities. They believe that an impulse applied at less than 90 degrees speeds one up, an impulse applied at greater than 90 degrees slows one down, but, an impulse applied at 90 degrees has no effect on the speed of motion. This is illustrated in figure 2.

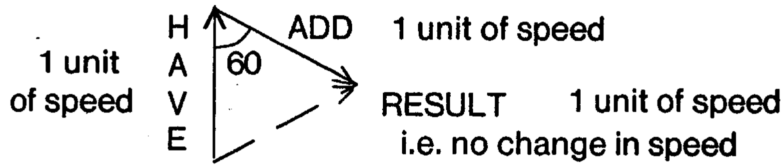
Fig. 2.



In fact, 120 degrees is the zero point, not 90. (See figure 3). This is somewhat counterintuitive in that it violates symmetry and is counter to the zero point in scalar arithmetic. However, this misconception is not likely to be the only source of the high proportion of wrong answers because many of the students had difficulties with all of the problems in this questionnaire where speed was a factor.

Another source of the answer that the speed remains constant derives again from living in a frictional environment. Students could be reasoning, as they did on question 2, that since the second

Fig. 3.



impulse is the same size as the first, the same speed will result. This belief holds true in the everyday world because objects are usually stopped when one applies a force to make them move. When the object is moving, as is the case here, the resulting motion after the impulse is dependent on the initial velocity of the object. Hence the same size impulse will not always result in the same speed.

(5) *Suppose we fired the impulse engine once and then turned the engine right around so that it was facing the other way and then fired the impulse engine again - what do you think would happen? (illustration given - see figure 4)*

Fig. 4.

Correct answer: the spaceship would stop.

Most common wrong answer: the spaceship would turn around and go the other way.

Results:

pre-test , post test	x ✓	✓ x	✓ ✓	x x
Expt.	4/18	0/18	14/18	0/18
Cont.	0/14	1/14	11/14	2/14

Most of the students, 83%, got this question right. All of the people in the experimental group who got this question wrong on the pretest changed to the right answer after playing the games. Whereas, in the control group no one changed from the wrong answer to the right one and one person changed from the right answer to the wrong answer ($p=0.067$, Nexpt. = 4/4, Ncont. = 0/2).

Discussion: The bug of ignoring momentum does surface here for almost one fifth of the students - in more cases than for the previous question involving orthogonal impulses. This supports the hypothesis that the orthogonal impulse situation is a reference example from physics class. What is somewhat surprising here is that two people who gave the correct answer to the *how do you make yourself stop* question, i.e. apply an anti-impulse, then answered this question incorrectly. This indicates that either they did not understand the question properly or else that their knowledge is very context dependent. They apply different and inconsistent ideas to different forms of the same question.

(6) *Suppose we fired the impulse engine once and then turned the engine right 90 degrees and fired it again and then we turned it right another 90 degrees and fired it again (illustration given - see figure 5).*

Fig. 5.

Which direction do you think the spaceship would end up going in - could you draw in what you think would *happen?*

Correct answer: →

Most common wrong answer: ↘

Rarer wrong answer: go in the direction of final impulse.

Results:

pre-test , post test	x ✓	✓ x	✓✓	x x
Expt.	4/18	0/18	8/18	6/18
Cont.	1/14	0/14	9/14	4/14

About one half of the students got this question right. Of those who got this question wrong on the pretest, almost half of the people in the experimental group changed to the right answer after playing the games and one person in the control group did (p=0.434, Nexpt.= 4/10, Ncont.= 1/5). No one changed from a right answer to a wrong answer.

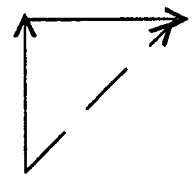
Discussion: Most of the students did not think of this question in terms of components and therefore did not perceive the cancellation. Instead, they added the first two velocities and then added the third velocity to the resultant. Some of them did this using vectors but most simulated it with their hands or in their head.[1] The common wrong answer is caused by two factors. Firstly, the students are ignoring the increase in speed caused by the second impulse. Secondly, they have a preference for choosing 45 degree angles as answers (supported by wrong 45 degree answers to questions 6,7,9,10,11c,&13).

This preference for 45 degrees might have several origins. One might come from physics class where examples of vector diagrams often involve vectors drawn at 45 and 90 degrees as in figure 6.

Another source of the 45 degree preference might come from geometry where students also get lots of examples of lines drawn at 45 and 90 degrees. These angles might thus become part of a set of primitive or reference angles. The result may be that one needs a special excuse not to use one of these reference

1. This is a conjecture - although some students did say, "I imagined it in my head".

Fig. 6.



angles in an answer since they are "simpler" or more familiar than "weird" or unusual angles.

The rarer wrong answer of *it goes in the direction of the final impulse* is the ignore-momentum bug which occurred consistently in a few of the students.

(7) *Suppose we faced the impulse engine this way (draw it at 90 degrees) and fired it, then faced it this way (draw it at 0 degrees) and fired it again, and then faced it this way (draw it at 180 degrees) and fired it again. (illustration given - see figure 7).*

Fig. 7.

Which direction do you think that the spaceship would end up going in - could you draw what you think would happen?

Correct answer: →

Most common wrong answer: ↘

Rarer wrong answer: go in the direction of final impulse.

Results:

pre-test , post test	x ✓	✓ x	✓✓	x x
Expt.	3/18	0/18	9/18	6/18
Cont.	1/14	0/14	9/14	4/14

This question is almost identical to the previous question with the exception that the two velocity components which cancel out are right next to each other instead of having a component in between them as in the previous question. This was done to see whether students could think about this question in terms of components when the cancellation was made more obvious. However, the pattern of answers to this question was virtually identical to the previous one ($p=0.545$, Nexpt.= 3/9, Ncont.= 1/5). Only half of the students got this question right on the pretest.

Discussion: This result was rather surprising since the question immediately prior to these two was what happens if you fire forwards and then backwards. Thus one would think that the context had been set for the students to be likely to see the cancellation. This, however, was not the case. The reason a lot of the students do not see the cancellation, even though it is made "obvious", is that they have a stepwise approach to solving this problem - take the first two velocities and add them together and then add the third velocity to the resultant - and, therefore, they never consider the result of adding the last two velocities together. The games thus do not appear to help students see this question in terms of components of velocity.

(8) *Can you draw a picture and explain how you would get your spaceship to fly in a circular path?*

Correct answer: keep firing impulses in towards the center of the circle.

Most common wrong answer: keep turning and firing in the direction that you want to go in.

Results:

pre-test , post test	x ✓	✓ x	✓✓	x x
Expt.	6/18	0/18	5/18	7/18
Cont.	0/14	0/14	2/14	12/14

Only 20 % of the students got this question right on the pretest. About one half of the thirteen students in the experimental group who got the wrong answer switched to the right answer on the posttest - no one in the control group did ($p=0.01$, Nexpt.= 6/13, Ncont.= 0/12). No one switched from a right answer on the pretest to a wrong answer on the posttest.

Discussion: This question produced a resurgence of the ignore-momentum bug which is a common bug amongst physics naive people. The origin of this bug, discussed previously, derives from living in a world with friction.

This was a difficult problem for the majority of students as indicated by long response times and hesitation in their answers. Very few used vectors to help themselves analyze this problem. Even those who gave the correct solution often could not explain their answers in terms of the vector representation or in any other terms.

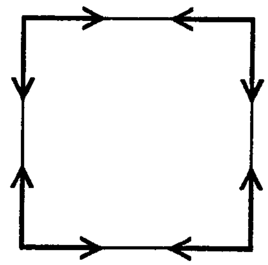
Possibly the reason most students do not apply vectors to this problem is that they think vectors are only applicable in situations where one has discrete changes in the direction of motion. They do not think to approximate the circle by a polygon and hence either do not know how or do not think to apply vectors in this case of continuously changing direction of motion.

This lapse to the "ignore momentum" way of thinking may indicate a return to beliefs about force and motion which the students held prior to their exposure to formal physics - the transfer occurring when confronted with a difficult problem. This result supports the hypothesis that people can hold multiple ideas relevant to forces and their effects upon motion, different pieces of knowledge being utilized in different contexts. Furthermore, it supports the conjecture that these beliefs are not necessarily consistent with one another.

(9) *Can you draw a description of how you would get your spaceship to fly in a square path?*

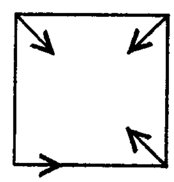
Correct answer: stop at each corner, reaim, and fire. (See figure 8)

Fig. 8.



Most common incorrect answer: when you reach a corner, fire in towards the center of the square. (See figure 9).

Fig. 9.



Results:

pre-test , post test	x ✓	✓ x	✓ ✓	x x
Expt.	0/18	1/18	5/18	12/18
Cont.	0/14	2/14	5/14	7/14

Less than half the students, 38%, got this problem right on the pretest. The games did not produce any improvement in the proportion of correct answers ($p=1.0$, Nexpt.= 0/12, Ncont. 0/7).

Discussion: The most common answer involves firing in towards the middle of the square at 45 degrees from your current path. This answer would represent a possible correct solution if the size of the impulse force could be arbitrary. However, the size of the impulse is fixed for this problem set. Thus an impulse aimed in towards the center of the square would not be big enough to produce a right angle turn in the direction of motion.

There are several potential explanations for this wrong answer. The first is that the students are simply forgetting the restriction of the fixed sized impulse. The second is that they are failing to worry about the size of the impulse as a factor. That is, they know how to worry about the size of the impulse but they are forgetting to do it. The third is that they do not know how to take into account the size of the impulse. For example, they may not know that the length of the vector maps onto size of the impulse.

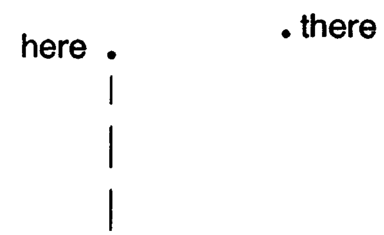
It is hard to explain why the games do not produce improvement on this problem since one of the games involves trying to negotiate a right angle turn by firing only one more impulse after an initial impulse has been given. In attempting to play this game, most people try the strategy described above for the square problem. They get to the corner and try firing backwards at 45 degrees. They cannot quite do this in the game context because they are only given the capacity to make 30 degree turns. What they typically try is to aim backwards at 30 degrees. This usually fails so then they try aiming backwards at 60 degrees. This succeeds in negotiating the corner, although, even 60 degrees backwards does not produce a right angle turn (but it is close enough to go around the corner). Thus the students have direct evidence from playing this game that even a 60 degree backwardly aimed impulse does not produce a right angle turn so why they should continue to believe that a 45 degree backwardly aimed impulse will produce a right angle turn hard to explain. They must either be forgetting the fixed size impulse limitation or else not noticing the results of their successful strategy for the corner game.

The result is also puzzling because this is not a situation where the correct solution of stopping and reaiming at each corner is beyond their grasp. On the contrary, the right answer involves using anti-impulses to stop oneself and, as was found in previous problems from this questionnaire, the majority of students know how to do this. In addition, many students used anti-impulses in playing some of the games. In fact, one game, DOCK, is specifically designed to focus on this idea. Possibly the students think that the right answer or best answer must involve using only one impulse in order to turn.

(10) *Suppose you have fired your impulse engine once and got to HERE (illustration given - see figure 10) and you want to get to THERE. In other words, you want to make almost a right angle turn. Suppose that you can fire your impulse engine only one more time. How would you do it? Please draw a picture.*

Correct answer: fire almost backwards.

Fig. 10.



Most common wrong answer: fire backwards at 45 degrees.

Results:

pre-test , post test	x ✓	✓ x	✓✓	x x
Expt.	4/18	0/18	1/18	13/18
Cont.	0/14	0/14	1/14	13/14

This question involves determining what velocity one needs to add to an initial velocity in order to produce almost a right angle change in the direction of motion. Almost none, 5%, of the students got the correct answer to this problem on the pretest. Four students in the experimental group changed to the correct answer on the posttest - no one in the control group did ($p=0.087$, Nexpt. = 4/17, Ncont. = 0/13).

Discussion: It is difficult, as with the previous question, to explain the fact that only a few people changed to the right answer to this question after playing the games. This question is analogous to one of the games where the person has to produce almost a right angle turn (in order to go around a corner) by firing only one more impulse after an initial impulse has been given. The students eventually succeed in doing this by firing backwards at 60 degrees (only 30 degree turns were possible).

Possibly giving students the capacity to make only 45 degree turns would be a more direct and inescapable contradiction to the commonly held belief that after an initial impulse, firing backwards at 45 degrees will produce a right angle change in the direction of motion.

The common wrong answer to this question again seems to indicate that students have a special preference for impulses given at 45 degrees. This conjecture is also supported by the most common wrong answers to questions six, seven, and nine.

(11) *Suppose that you were in a part of outer space where there is a strong solar wind blowing (direction drawn). How would that affect your path if you were travelling:*

(a) into the wind?

(b) in the same direction as the wind?

(c) perpendicular to the wind?

This question was included to see whether students could adapt their knowledge of force and motion to a situation involving an impulse force and a force acting in a more complex way. For part (c) the students were asked to draw the path of the spaceship as it hit the region of the wind. For parts (a) and (b), if the student said slow you down or speed you up, they were then asked is there any way we could figure out how much.

Correct answer for (a) and (b): you would end up going with the same velocity as the wind.

Most common wrong answer for (a) and (b): combine the velocity of the spaceship with the velocity of the wind.

Results:

pre-test , post test	x ✓	✓ x	✓✓	x x
Expt.	4/18	0/18	1/18	13/18
Cont.	2/14	0/14	4/14	8/14

pre-test , post test	x ✓	✓ x	✓✓	x x
Expt.	2/18	1/18	1/18	14/18
Cont.	0/14	0/14	4/14	10/14

The pattern of answers to parts (a) and (b) was very similar. Few people, 20%, got the right answer. A few people in the experimental group changed to the right answer on the posttest - fewer people in the control group did (part(a): p=0.613, Nexpt.= 4/17, Ncont.= 2/10) (part(b): p=0.369, Nexpt.= 2/16, Ncont.= 0/10). One person in the experimental group changed from a right to wrong answer on question (b) - no one else changed to the wrong answer on either question (a) or (b).

Correct answer for part (c): there could be some debate as to the correct answer. One could argue that the forward component of the spaceship's velocity would not be affected by a wind blowing perpendicular to the spaceship's direction of motion. The result of the wind would thus be to continuously increase the spaceship's component of velocity in the direction of the wind until it reaches the same velocity as the wind. The spaceship would still maintain its forward velocity, therefore, the path of the spaceship through the wind would curve until the spaceship's velocity reached the combined velocities of the perpendicular wind and the original forward velocity of the spaceship. However, in reality, a wind involves moving particles which create frictional forces. These forces would gradually eliminate the spaceship's forward motion and the resulting final velocity would be equivalent to that of the wind. The spaceship's path through the wind would thus curve until it reached the same velocity as the wind. For the purposes of this study, both of these answers were considered correct.

Most common wrong answer: the path through the wind would be a straight line, often drawn at 45 degrees. This straight line path implies that the velocity of the wind immediately adds to the velocity of the spaceship.

Results:

pre-test , post test	x ✓	✓ x	✓✓	x x
Expt.	0/18	0/18	1/18	17/18
Cont.	0/14	1/14	1/14	12/14

Few students, 13%, got the right answer for part (c). No one changed from a wrong answer to a right answer (p=1.0, Nexpt.= 0/17, Ncont.= 0/12). One person in the control group went from a right answer to a wrong answer.

Discussion: There are several possible explanations for why students add velocities here when it is not appropriate to do so.

One explanation is that a mental set has been created. In all of the ten questions preceding this one, it is appropriate to add the component velocities that are created by impulse forces. Hence, the students may assume, without thinking about it, that this is also appropriate here. Further, none of the prior questions involve frictional forces and, suddenly, by introducing a wind, friction becomes a factor. The claim is that the students have been tricked into a wrong answer and that ordinarily they could answer this question. This claim could be tested by asking this question by itself or by asking this question first. If the students then get the correct answer, one can conclude that their wrong answers here were caused by the context.

Another explanation is that the students think they are dealing with two similar forces and that it is, therefore, appropriate to add the resulting velocities. There could be a misconception that the wind acts instantaneously like an impulse force. For instance, they might think that forces like winds and streams are analogous to stepping onto a moving train when in fact the effects happen more gradually. On the other hand, they may be exhibiting the bug noted by Clement [1979]: the spaceship's constant motion may cause them to slip into believing that the spaceship is being driven by a continuous force. Then, if they perceive the wind as a continuous force, it is appropriate to add the resulting velocities.

One final explanation is that the students may not differentiate an impulse force from a continuous force or from a more complex force like a wind. Thus they would treat all these forces as the same and add the resulting velocities together. This is an unlikely explanation since in a preliminary version of this questionnaire, students from the same population were asked to give examples of an impulse force and a continuously acting force and they were all able to do so.

(12) *You are back in a part of outer space where the is no solar wind. Suppose that you want to dock your spaceship with its mothership which is stationary in space waiting for you. You can dock from any side of the mother ship that you like and you can approach by any path that you like but you want to dock as slowly as possible. How could you do it in order to get yourself going at slower than one impulse worth of speed. Remember, you can only give unit impulses?*

Correct answer: fire almost backwards in order to slow yourself down.

Most common wrong answer: it is impossible to get a speed of less than one but greater than 0.

Less common wrong answer: think it can be done but cannot figure out how.

Even less common wrong answer: Take a zig - zag path in towards the mothership, the turning will slow you down.

Results:

pre-test , post test	x ✓	✓ x	✓✓	x x
Expt.	9/18	0/18	4/18	5/18
Cont.	2/14	1/14	3/14	8/14

One quarter of the students solved this problem on the pretest. Nine of the students in the experimental group changed to the right answer on the posttest - two of the students in the control group also did ($p=0.041$, Nexpt.= 9/14, Ncont.= 2/10). One student in the control group changed from a right answer on the pretest to a wrong answer on the posttest - no one in the experimental group did.

Discussion: Some students answered fairly quickly that they thought it was impossible to attain a speed of less than one but greater than zero.

This is a predictable answer for students who consistently exhibit the bug of ignoring momentum. For them, an impulse always results in motion in the direction of the impulse. Further, since all fixed sized impulses result in the same speed of motion, it is impossible to attain a speed of less than one.

For students who believe that the results of forces combine additively to change the direction of motion but not the speed of motion, this is also a likely answer. They think that, with the exception of anti-impulses, all equal sized impulses result in the same speed of motion. Thus speeds of less than one but greater than zero are not possible.

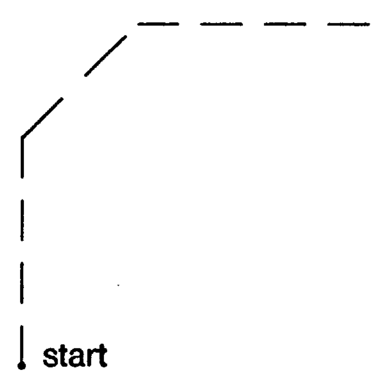
Most students took a long time to answer and many could not solve this problem. Some tried to draw vectors but many did not seem to know that they should be looking for a resultant vector with a shorter length. This is consistent with the general pattern of results that many students cannot answer questions which involve speed as a factor.

The belief expressed by some students that *turning will slow you down* is a misconception that probably derives from the experience of living in a world complicated by friction. In the everyday world one uses friction in order to be able to turn. Furthermore, one usually slows down to make a turn to avoid skidding, i.e. exceed the limits of friction's ability to help one turn. Hence, one associates turning and slowing down.

There is evidence that the games helped a lot of students with this problem. This is not surprising since this problem is a verbal form of one of the games where the object was to hit the target as slowly as possible. Hence, the students have solved this problem before in a context where they received feedback as to whether or not their answer was right. What is surprising, however, is that the game did not cure everybody. This may indicate that although they can succeed at the game they did not understand what they were doing well enough to transfer it to this verbal form of the same problem. This may be reflected in the fact that students usually found this game difficult and typically spent a lot of time on it. Often they eventually succeeded by a trial and error process but possibly did not fully understand why their answer worked. If they do not understand the answer, i.e. describe it to themselves in terms of partially canceling speed, this may account for why they were not be able to reproduce it at a later time.

(13) *Suppose your spaceship took this path: (See figure 11).*

Fig. 11.



What would the sequence of impulse engine firings have been?

Right answer: (See figure 12).

Most common wrong answer: (See figure 13).

Fig. 12.

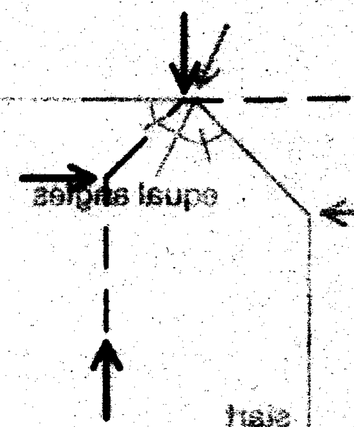
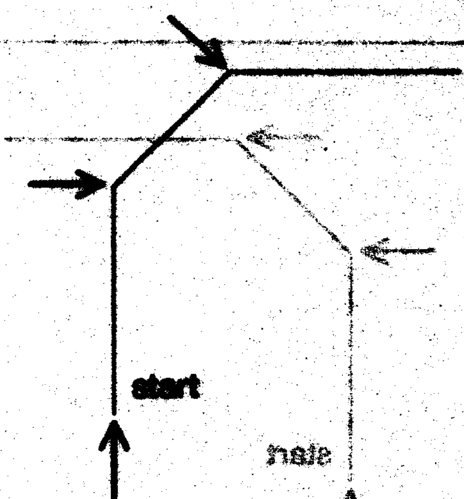


Fig. 13.



Less common wrong answer: (See figure 14).

Even less common wrong answer: (See figure 16).

Least common wrong answer: (See figure 16).

Fig. 14.

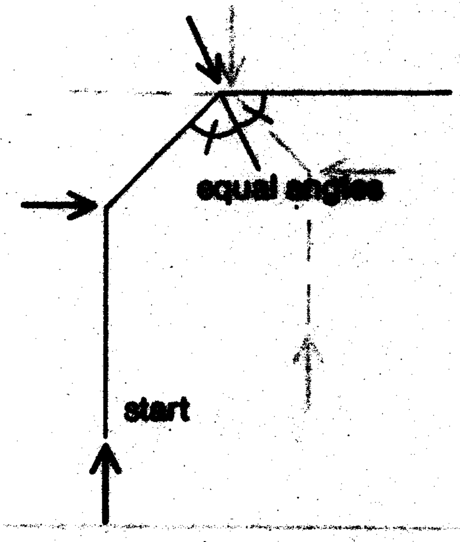
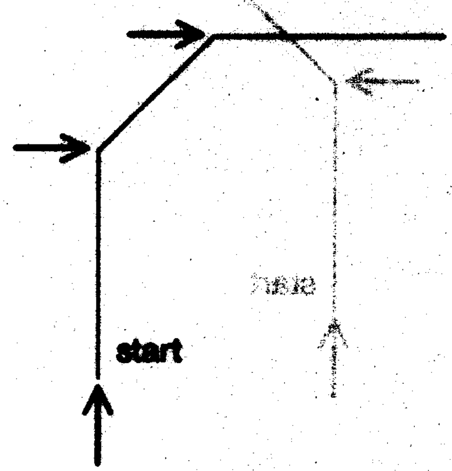


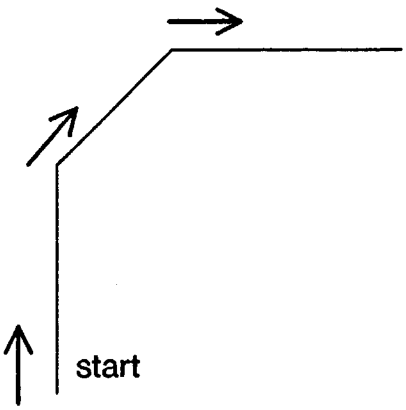
Fig. 15.



Results:

pre-test , post test	x ✓	✓ x	✓ ✓	x x
Expt.	8/18	1/18	2/18	7/18
Cont.	3/14	1/14	3/14	7/14

Fig. 16.



One quarter of the students solve this problem correctly on the pretest. A higher proportion of people giving the wrong answer on the pretest switched to the right answer on the posttest in the experimental group, 60%, than in the control group, 20% (p=0.231, Nexpt. 8/15, Ncont.= 3/10). One person in the experimental group and one person in the control group switched from a right answer on the pretest to a wrong answer on the posttest.

Discussion: Nearly all of the students gave correct answers for the first two impulses. This part of the problem involves producing a 45 degree turn in the direction of motion. This maps onto the previously encountered problem of what happens if you apply a vertical impulse followed by a horizontal impulse. Here we have the *how can you produce the path* as opposed to the *what happens if* form of the same problem. It was conjectured that the high proportion of right answers to the latter question could be due to its being like example problems from the physics course. The application of such a standard reference example could also account for the high proportion of correct answers to this segment of this problem.

The exceptions to the success of students with respect to the direction of the first two impulses were students who gave the answer illustrated in figure 17. These students ignored momentum completely by exhibiting a belief that the motion should be in the direction of the most recent impulse.

The next rarest wrong answer was the one shown in figure 18. Here the first two impulses are correct but the third impulse is in the wrong direction. This may be due to a having used a reference example to produce the directions of the first two impulses and then either, (1) utilizing the belief that motion should go in direction of the last impulse, i.e. ignore momentum, to produce the direction for the third impulse, or (2) using the result from the reference example that one impulse in the horizontal

Fig. 17.

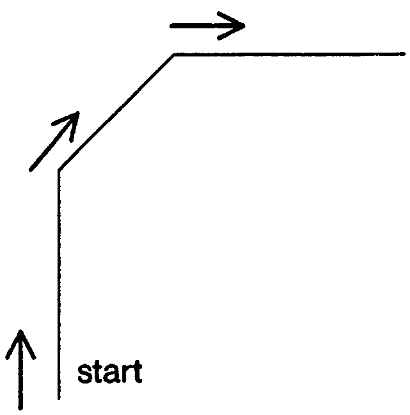
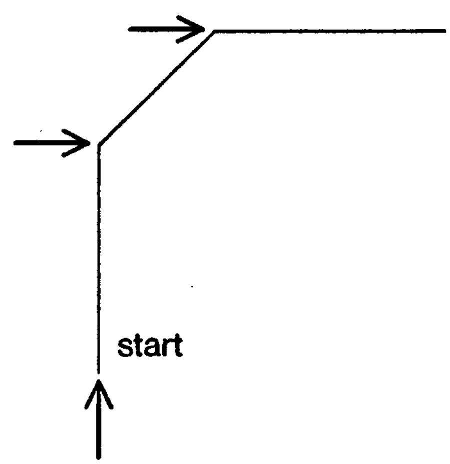


Fig. 18.



direction produced a 45 degree turn in the direction of motion to infer that an additional horizontal impulse should produce another 45 degree turn. Both conjectures represent state independent theories about the effects of forces on motion.

The next most common wrong answer is illustrated in figure 19. This answer is again wrong with respect to the final impulse. Here, the final impulse is "in between" the current direction of motion and the desired direction of motion. This answer may reflect a misapplication of an "in the middle" or "in between" description of how impulse forces affect motion. To elaborate, if the student describes the effect of ↑ combined with → as in the middle (see figure 20) then he or she may misapply the descriptor "in the middle" to the situation where one has ↑ and want → according to the representation shown in figure 21. These diagrams represent only direction and thus reflect the fact that the students are not paying attention

Fig. 19.

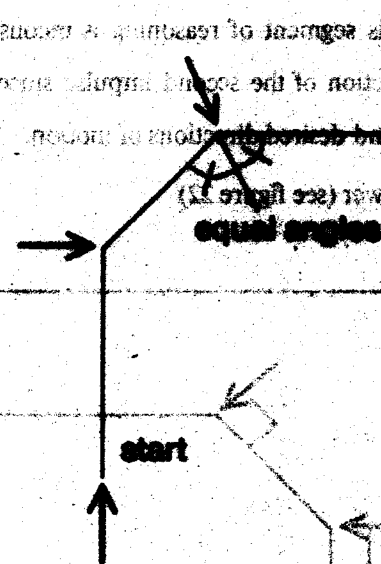


Fig. 20.

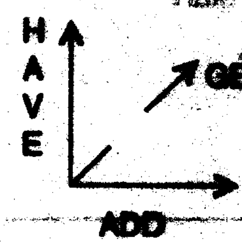
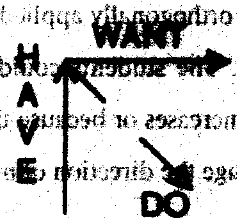


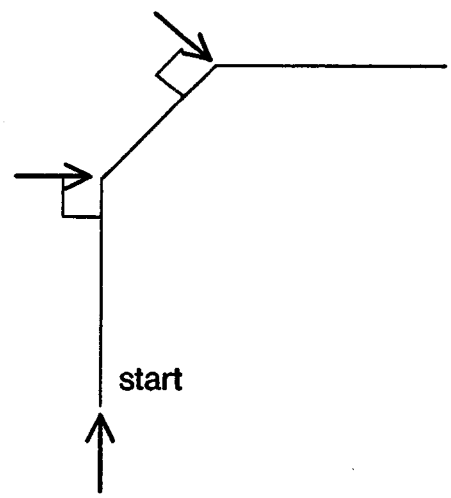
Fig. 21.



to changes in speed.[1] Note that this segment of reasoning is inconsistent with whatever reasoning the students used to determine the direction of the second impulse since the second impulse is not "in the middle" with respect to the current and desired directions of motion.

The most common wrong answer (see figure 22)

Fig. 22.



involves thinking that the application of the third impulse is analogous to the application of the second impulse - both impulses are orthogonal with respect to the current direction of motion. In order to perceive these two situations as analogous one has to ignore the increase in speed after the second impulse. This increase in speed would result in the orthogonally applied third impulse producing less turning than the orthogonally applied second impulse. The students could be ignoring the increase in speed either because they do not know that the speed increases or because they forget to worry about speed as a factor affecting the capacity of an impulse to change the direction of motion.

This problem could be solved very easily by thinking in terms of orthogonal components. First one gives a vertical component, then one introduces a horizontal component, and finally, since one only wants to be left with horizontal motion, one cancels out the vertical component. However, very few students

1. This way of representing vector addition and subtraction combined with descriptors such as "in the middle" will be discussed in more detail in the following chapter.

were able to see this problem in terms of components and thereby see that a cancellation was needed. Instead, they approached the problem in terms of the resulting velocities, not the components. Thus they did not see that the diagonal velocity had a vertical and horizontal component of motion. This is similar to the approach applied by students to question six which is the *what would happen if we had* - (see figure 23)

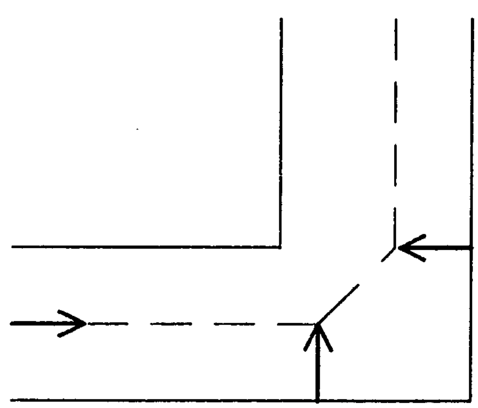
Fig. 23.

---

form of this problem. The conclusion is that the students do not think in terms of commutative, orthogonal components when solving this type of problem.

The result that the games improved ($p=0.2$) the students' performance on this question is not surprising since this path is analogous to a path produced by an often employed strategy for the game where one had to guide the spaceship around a corner. (See figure 24). Thus many of the students have had experience producing such a path in the context of playing the games.

Fig. 24.



7.3 Additional Questions Asked of Some of the Students

After some of the results had been studied and it was found that one half of the students could not answer a question about the change in speed caused by firing an orthogonal impulse, the hypothesis was proposed that the students were not able to answer correctly because they thought that firing at 90 degrees might be the zero point with respect to changing velocity. To test this, two questions were asked of the eleven subjects who had not yet completed the study. The questions were given following the posttest so as to not interfere with the pattern of answers of the rest of the questionnaire.

The questions were (1) what would happen to your speed if you fired a vertical impulse and then turned the engine 45 degrees and fired another impulse (fire almost forwards again) and (2) what would happen to your speed if you fired a vertical impulse and then turned the engine 135 degrees and fired another impulse (fire almost backwards). Four out of eleven people answered the first question correctly and eight out of eleven answered the second question correctly.

The high proportion of people answering the first question incorrectly supports the notion that there is a deep misconception about the effects of impulse forces on motion indicated here. Possibly the reason there is a higher proportion of right answers to the second question is that it is close to the question about applying an anti-impulse which most students got right whereas the first question is close to the question about firing two impulses in the same direction where there were a lot of wrong answers. Further the answer of slower to the second question is consistent with the view that turning slows you down - a theory believed by some of the subjects. Thus it was possible to get the right answer to the second question by the wrong means. It should also be noted that although most of the students correctly answered question two here, most could not answer the question of how could you slow yourself down. Thus they can tell you what would happen but they do not spontaneously think of it when they need it in solving a problem.

7.4 The Students' use of Vector Diagrams.

These observations are derived from the students' pencil and paper workings made while solving the problems. It is difficult to make precise observations for the following reasons:

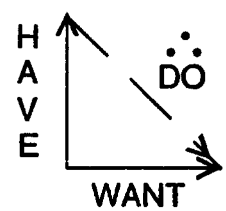
- the students' diagrams are messy and were often made very quickly making it difficult to see what the student was doing,
- the experimenter did not interrupt the student and ask for clarification unless the final answer was unclear,
- by just looking at the diagram it is difficult to tell the order in which the student drew the

components. Listening to the tape recording usually does not disambiguate the order. Video tapes would have been helpful here.

Results:

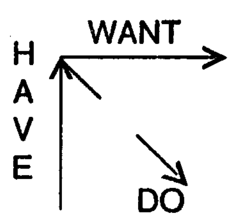
Only one third of the students used vector diagrams to help themselves solve any of the problems. Of those who utilized vector diagrams, three quarters used them only once or twice. Only two of the students gave any indication of worrying about the length of the vectors that they drew. Further, no one was seen to subtract vectors properly. That is, no one was observed subtracting as in figure 25.

Fig. 25.



Instead, they subtracted as in figure 26.

Fig. 26.



An alternate method for vector subtraction used by some students was to add vectors that they thought were potential candidates to see if they produced an appropriate result.

Discussion:

There are several possible explanations as to why the students did not use vector diagrams to help themselves solve the problems. The first is that possibly they think that vectors are not relevant to these problems. This is unlikely since when the students were asked what aspects of their physics course these problems related to, more than half said vectors. Perhaps they do not think that it is appropriate to draw vector diagrams unless the problems are expressed quantitatively. Also, they may have felt that vector diagrams were unnecessary. They could solve the problems without them. Alternately, they may not have known how to use vectors to help themselves solve these problems. This last conjecture is supported by the fact that the majority of students who used vectors, used them to attain incorrect answers.

This chapter closes with the presentation of some additional evidence obtained by asking students attending a graduate psychology seminar to solve a force and motion problem, similar to the final problem on the questionnaire.

7.5 Results of Question Given to Students in Graduate Psych Class

In order to get some additional data relevant to people's ideas concerning how impulse forces affect motion, eight students attending a graduate seminar in psychology were asked the following question.

Try to imagine a spaceship designed for travelling through outer space. So, there is no friction.

The spaceship is driven by a special impulse engine that gives a sudden burst of force and then shuts itself off. It is not like a car engine that is on all the time, rather, it is more like a kick.

This impulse force is always the same size. That means that whenever you fire the impulse engine, it always turns on for the same amount of time.

When the engine is fired, it gives the spaceship an impulse in the direction that the impulse engine is aimed in. You can rotate the engine so that you can apply an impulse force in any direction that you like.

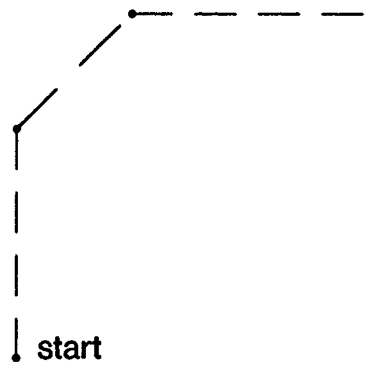
Which way did the spaceship's pilot fire his impulse engine at each of the points indicated in order to make the following path? (See figure 27). Why did he have to fire in those directions?

What do you think happens to the speed of the spaceship as it follows the above path and why?

This question is identical to the final problem on the questionnaire given to the high school students except that here the students are also asked about what happens to the speed of the spaceship and they are also asked to give reasons for their answers.

The question about speed was introduced to help determine the relationship between students'

Fig. 27.



beliefs about what happens to direction versus what happens to speed.

They were asked why in order to prompt them to explain the reasoning behind their answers. The high school students had been asked to reason out loud while solving problems, but often they just gave answers without giving any justification for their answer. It was hoped that this technique of asking why might be a better way of encouraging students to articulate their reasoning process.

The physics background of the students attending the psychology seminar was varied. One had not taken any physics courses. Two had taken several MIT undergraduate physics courses and the rest of the students had taken physics in high school.

Results with respect to the directions of the impulses:

Two students gave the correct answer indicated in figure 28. Both of these students had taken physics in high school but not college.

Four students gave the incorrect answer displayed in figure 29 which ignores the increase in speed that occurs after the second impulse and thereby treats the third impulse as analogous to the second. Both the second and third impulses are thus applied at right angles to the current direction of motion.

One student gave the answer shown in figure 30 which tries to treat the third impulse as analogous to the second by indicating that both are given at ninety degrees to the current direction of motion by drawing the symbol for ninety degrees. In reality, the student drew the third impulse in a direction that was "in the middle" with respect to the current direction of motion and the desired direction of motion and hence not at ninety degrees.

Another student oscillated between the correct answer and the most common wrong answer. He

Fig. 28.

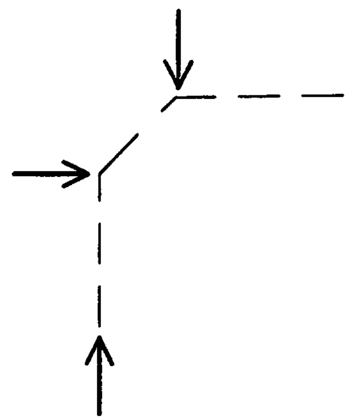
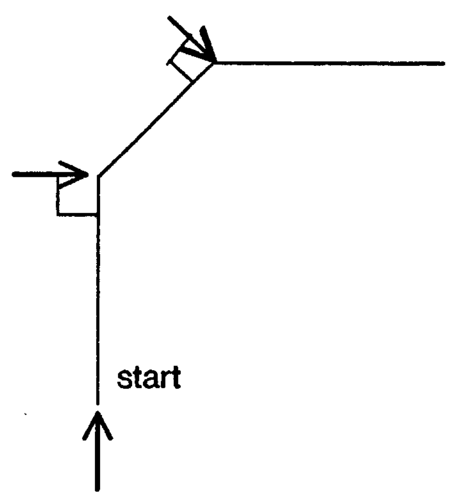


Fig. 29.

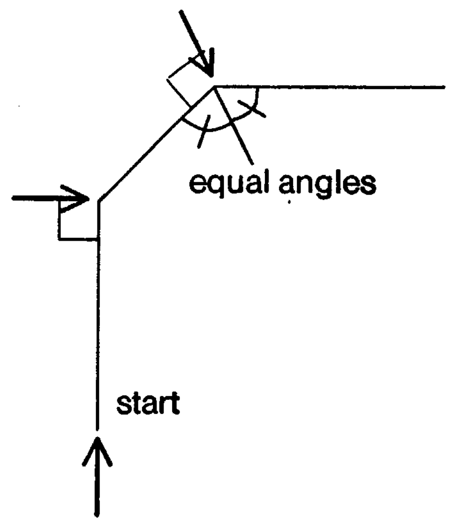


expressed uncertainty as to whether one has to take into account the increase in speed after the second impulse when trying to determine the direction of the final impulse or, whether, speed is an independent issue and that the direction of the third impulse should, therefore, be analogous to the second impulse.

Results with respect to what happens to the spaceship's speed:

The correct answer is that the speed would increase after the second impulse and then resume its original speed after the third impulse. One student indicated a correct answer by saying that the new velocity would be the vector sum of the current velocity and the impulse velocity. This student had taken

Fig. 30.



two MIT undergraduate physics courses.

Five out of the eight students said that the speed would increase after each impulse. One added the qualification that the increase would be a factor of one and a half each time instead of two because you lose some speed in order to change direction. Another added the qualification that if you fire backwards, it will stop you, so, if the angle of the impulse is greater than something (he was not not sure what angle), maybe the impulse will slow you down. This piece of reasoning is an important step towards realizing the relationship between the direction of the impulse and the change in the speed of motion.

One student said that the speed would be slower after each impulse because the force gets used up in moving the spaceship off its original course.

Another students said that she had no idea what happens to the speed but guessed that it would remain constant.

Discussion: The most common error with respect to applying impulses so as to change direction concerned ignoring the increase in speed caused by the second impulse when determining the direction of the third impulse. This factor cannot be ignored because the change in the direction of motion produced by an impulse is dependent on the spaceship's speed of motion. The faster the spaceship is traveling, the less effect a given impulse will have on altering the direction of motion. Thus if one wants to change the direction of motion by a given amount, but the spaceship is now travelling faster, one has to alter the direction in which one applies the impulse to get the same effect.

The most common error with respect to predicting the speed of the spaceship concerned ignoring the effect that the direction of the impulse would have on the resulting speed.

Both these errors express beliefs that directions of impulses are additive and that speeds produced by impulses are additive. What they ignore is the interaction between speed and direction. The resulting velocity after a fixed size impulse is dependent both on the initial velocity of the spaceship and on the direction of the impulse. These subjects are treating speed and direction as additive but as independent of one another.

Two of the subjects also expressed the erroneous belief that turning uses up some of the force. This idea was present in some of the high school students.

This pattern of errors was very similar to those of the high school students. What these results show more clearly, because of the inclusion of the question about what happens to speed, is that the common error of ignoring the increase in speed after the second impulse, when determining the direction of the third impulse, may not be due to simply forgetting to pay attention to speed. but, rather, to failing to realize or forgetting that there is an interaction between changes in speed and changes in direction. Many of these subjects treated the two as independent factors - speeds added regardless of direction and directions added regardless of current speed. Ignoring the interaction between speed and direction by treating them as independent factors is a convenient simplification of the problem, but it does not match a Newtonian reality. These results suggest that the graduate students, like the high school students, were not unifying the speed and direction of motion into a vector concept of velocity. Computing with vectors ties the speed and direction of motion together and, thereby, helps to avoid the bug of treating them as independent.

7.6 Summary

This chapter has described and analyzed the students' answers to the questionnaire problems. The results raised some interesting questions. For instance, why do students have incorrect ideas concerning what happens to the speed of motion when impulse forces are applied. In particular, why do they have so much difficulty figuring out how to apply an impulse force in order to reduce the speed motion and why do they fail to worry about the lengths of vectors when drawing vector diagrams? Further, why do students fail to pay attention to the speed of motion when solving problems which involve predicting and controlling the direction of motion?

The next chapter will present an analysis of the origins of the difficulties displayed by the students when solving these basic force and motion problems.

Chapter VIII

Why is Learning Newtonian Dynamics so Difficult?

This chapter commences by summarizing some of the errors which students made when solving the problems on the questionnaire. The origins of such errors are then outlined. The remainder of the chapter is devoted to a more in depth discussion of the many and diverse sources of difficulty encountered when learning to solve Newtonian dynamics problems.

8.1 An Overview of the Origins of Difficulty in Learning Newtonian Dynamics.

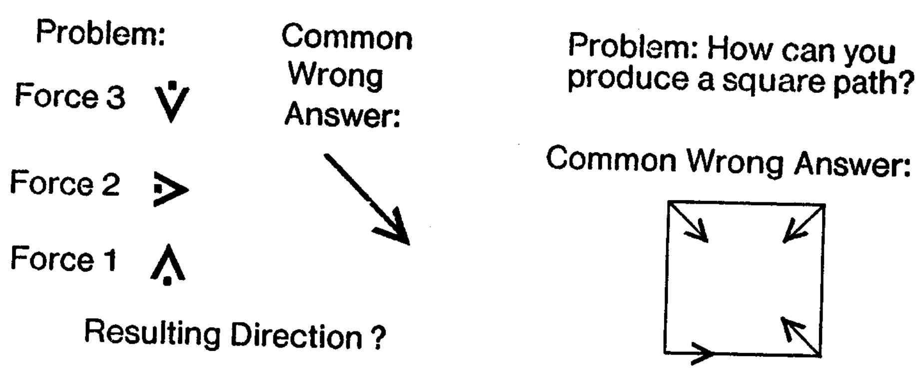
The students' answers to the questionnaire, which were described in the preceding chapter, uncovered many interesting examples of erroneous thinking concerning how forces affect the motion of objects.

One set of errors concerned controlling and predicting the direction of the spaceship's motion. A few of the students had the misconception that motion should always be in the direction of the most recently applied impulse force. Most of the students did, however, make some allowance for the momentum of the spaceship. They combined the current direction of motion with the direction of the impulse when determining the new direction of the spaceship's motion. However, many of these students did not quantify the situation properly. They failed to take into account the current speed of the spaceship when determining the new direction of motion. This factor has to be considered because the faster the spaceship is travelling, the less the direction of motion will change following a fixed sized impulse.

Another class of errors uncovered by the questionnaire had to do specifically with predicting and controlling the speed of the spaceship's motion. One third of the students said that applying a second impulse in the same direction as the first impulse, would not alter the speed of motion. They are not considering the momentum of the spaceship. Also, three quarters of the students could not correctly explain how to slow down the motion of the spaceship. In other words, they cannot suggest how to use the direction of the impulse to control the speed of motion. A related result is that the majority of students who drew vector diagrams to help themselves solve the problems on the questionnaire, did not appear to be paying attention to the length of the vectors which they drew.

One further finding of interest concerned the students' preference for answers given at 45 degrees, i.e. ↘. When the impulse forces being added or being subtracted were at 90 degrees relative to one another, the students incorrectly chose answers at 45 degrees relative to the velocities or impulse forces displayed in the problem. Examples of such problems and incorrect solutions are illustrated in figure 1.

Fig. 1.



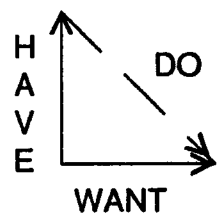
The students' justifications for their answers yielded suggestions as to the cognitive origins of these difficulties. These origins will be outlined briefly here and then discussed in more detail in the next section of this chapter.

Everyday experiences with forces and motion in a world where friction is an implicit factor has led to the derivation of erroneous intuitions concerning how forces affect the motion of objects, e.g., *when you kick an object, it goes for a ways and then stops.*

Also, there is confusion caused by applying knowledge from other domains, such as scalar addition and subtraction, which seems relevant but which has properties that are inconsistent with vector addition.

Further, the examples given in physics class itself may be a source of difficulties. The students are not given illustrations where there are restrictions placed on the size of forces. For instance, the example of vector subtraction they are given is shown in figure 2.

Fig. 2.

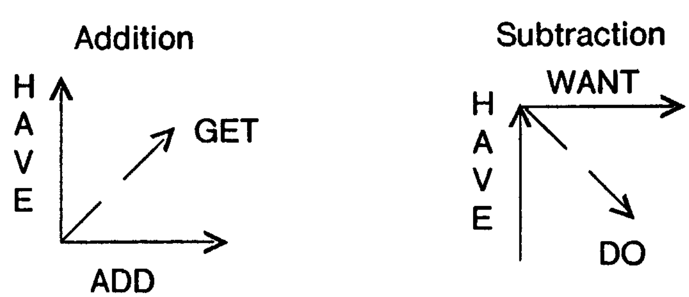


They then erroneously apply this reference example to a situation where the size of the impulse force is fixed and, therefore, needs to be taken into account.

Also, the vector representation itself is problematical. Vectors look like arrows which usually only represent direction, not speed and direction. Further, what the length of a vector represents can vary depending on the situation.

Another major source of difficulty is the kinds of descriptors and explanations which the students spontaneously apply to force and motion problems. For instance, when describing the resultant of two forces being applied to the same object, many of the students said that the motion will be *in the middle or in between* the two forces and employed the representations for vector addition and subtraction displayed in figure 3. These representations portray the *in the middle* relation very effectively. They focus on

Fig. 3.

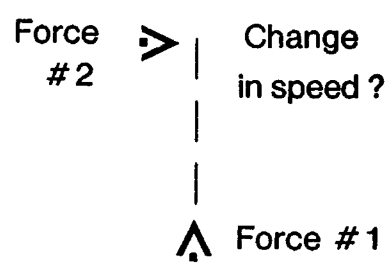


displaying the change in the direction of motion and have the feature of not encouraging the students to pay attention to the lengths of the vectors. These representations which the students devised thus reflect and reinforce misconceptions relating to the speed of motion.

Finally, many students think about force and motion problems in terms of speed and direction instead of velocity and velocity components. This introduces two complexities for these students. Firstly, they have to pay attention to two factors, speed and direction, instead of just velocity as captured by a vector. Secondly, there is an interaction between speed and direction which the students sometimes fail to realize. The change in the speed of motion is dependent on the direction of the impulse and the change in the direction of motion is dependent on the current speed of motion. This dependency makes it difficult to compute correct answers to force and motion problems without using a representation such as vectors.

Students sometimes give several diverse explanations for an erroneous answer and different students often give different justifications for producing the same wrong answer. Thus even a single error can have multiple and diverse cognitive origins. For example, in response to the question of *what happens to the speed of the spaceship when an impulse is applied at right angles to the current direction of motion (illustration given - see figure 4),*

Fig. 4.



the most common wrong answer was that the speed of the spaceship would remain the same. This wrong answer had several cognitive origins. The students justifications yielded all of the following sources of difficulty.

Even students who took into account momentum when predicting changes in the direction of motion, sometimes gave the following justification for the wrong answer that the speed of the spaceship remains the same. *Its the same size impulse so the speed is still the same.* This justification would only work if the spaceship were stopped when the impulse is applied. Living in a world with friction, where objects that one hits are usually stopped, has caused the person to believe that the same size impulse will always produce the same speed. They are unaware of the implicit assumption that the object is in the stopped state when one applies the impulse.

Another source of the error of believing that the spaceship's speed will remain the same, has to do with the fact that the impulse is applied at 90 degrees with respect to the direction of motion. The student regards ninety degrees as the zero point in the relationship between the direction of the impulse and its effect on the speed of the spaceship.

Another view expressed by some students is that once the spaceship is moving, impulse forces just change the direction of motion and do not effect the speed. Force is seen as a changer of direction. The energy of the force gets used in making the spaceship turn.

A further source of misconception comes from a misapplication of Newton's first law. This law implies that an impulse force produces constant eternal motion. The students, having heard Newton's first law in physics class, apply the idea of constant motion. However, the knowledge of the law is not accompanied by the appropriate understanding. This lack of understanding is indicated by the students saying that the speed of the spaceship remains constant, even after a second impulse, and giving Newton's first law as a justification.

The remainder of this chapter examines these sources of difficulty in more detail.

8.2 Intuitions Derived from Living in a World with Friction.

The pilot work indicated that even people with no physics background could solve problems where two forces act on an object simultaneously. For example, they can determine the direction of motion of a ball being kicked by two people at the same time. They can also solve problems where two velocities of different origins combine together. For instance, they can determine the path of a swimmer attempting to cross a flowing river or the path of an airplane flying through a wind. However, when the results of two forces are separated in time, difficulties start to emerge. Suppose a ball is hit and starts to roll and then it is hit again in a different direction. People with no physics background will usually answer that the ball will go in the direction of the last hit. They are ignoring the momentum of the ball when they are giving their answer. Living in a world with friction could be one source of this error. In a frictional environment, objects in motion slow down unless a constant force is being applied to them. The net effect is that usually, in everyday life, when one applies an impulse force, such as a hit or a kick, the object is in the stopped state.[1] This results in the induction of the beliefs that *things go in the direction that you kick them* and that *the same size impulse will always produce the same resulting speed of motion.* There is an implicit assumption in these beliefs that the state of the object being kicked is stopped. However, people may be unaware of this assumption since they primarily deal with the stopped state so there is no need to differentiate it from other states.

Another source of confusion caused by living in a world with friction is the following. In a frictional environment, if there is constant motion, there must be a constant force producing that motion. Clement [1979] has documented the fact that students extend this belief to friction free environments. When the students see the result of constant motion while playing the games, they may erroneously believe that a constant force is producing the motion. Then, when they apply a force in a new direction, their expectation is that the object will go in the new direction since they have changed the direction of the constant force. From such a perspective, there is no momentum in the previous direction because they are mistakenly treating this as a friction-like environment. The friction counteracts the motion in the previous direction. To help avoid this false analogy, care was taken when presenting students with the games and questionnaire to emphasize that the spaceship was driven by an impulse engine as opposed to a continually acting engine.

Ignoring momentum, when predicting the new direction of motion following the application of an impulse force, was the most frequently observed error made by subjects during the pilot study and by the subjects of diSessa [in Papert et al. 1979]. Many of these subjects expected the dynaturtle to go in the direction that it was kicked, regardless of its current velocity. However, only a few subjects in this study consistently made this error when solving the problems in the questionnaire. It was also not observed very often in the strategies that subjects employed while playing the games. Thus ignoring momentum when controlling the direction of motion was less frequently observed amongst these students than amongst the subjects in the pilot work. This suggests that the physics course has had some effect on suppressing this misconception. There was one question, however, that consistently elicited the reemergence of this error. This was the circular motion question. The error probably reemerges in this context because the students do not know how to apply their knowledge concerning the additive effects of forces on the direction of motion to cases where the motion is changing continuously as opposed to discretely.

In contrast, at least one third of the students in this study did ignore momentum when predicting

1. Further, in the few instance where one hits something that is moving, the sequence of events is usually too rapid for analysis. The result is that one sees what one expects to see rather than what really happened.

changes in the speed of motion. They stated that a second impulse, applied in the same direction as the first impulse, would not alter the speed of motion.

8.3 Interference Caused by Knowledge of Scalar Arithmetic.

Many beliefs associated with scalar addition and subtraction conflict with the properties of vector addition and subtraction.

Vector addition has the property that the result of an impulse force can be added to the velocity of the spaceship with the result that the speed of the spaceship actually decreases instead of increases. Thus in the two dimensional vector world, addition can result in the decrease of a quantity. This is not the case in the one dimensional world of scalar addition. The result of a decrease in speed may thus be counterintuitive for students. Everyday experience may also be responsible for the decrease in speed being counterintuitive. When one adds liquid to an existing amount of liquid, one always gets more liquid not less. The idea of getting less, may thus be completely foreign to the students. In fact, the whole idea of addition in two dimensions is foreign to them.

Another feature of vector addition is that one can combine units of impulses and not necessarily gets whole multiples of the units as answers. For example, combining one fixed sized impulse with the result of another same sized impulse does not necessarily produce two impulses worth of speed. It can produce 1.414, for instance, if the impulses are orthogonal to one another. The idea of combining the equivalent of whole numbers and getting a fraction for an answer is inconsistent with what happens in ordinary arithmetic.

One final feature of the scalar arithmetic world that does not map onto the vector world is that the zero point of the number line is, loosely speaking, "in the middle", whereas, the zero point for vector addition is not. That is, if one is travelling at one impulse worth of speed and one applies an impulse in the same direction, the result is two impulses worth of speed. If one adds the second impulse in a direction that is opposite to the direction of motion, the result is zero impulses worth of speed. So far this is all analogous to what would happen with scalar arithmetic. However, consider the case where one adds the second impulse in a direction that is not the same nor the opposite, but, in the middle, that is, orthogonal to the current direction of motion. The expectation is that this is a zero point and hence does not alter the speed of the spaceship. However, the speed of the spaceship actually increases. In reality, an impulse fired at 120 degrees is the zero point which leaves the speed of the spaceship unchanged. A one hundred and twenty degree zero point would thus be counterintuitive from the point of view of the student's knowledge of scalar arithmetic. It may also violate his or her desire for symmetry.

8.4 Physics Class as a Source of Misconceptions.

Newton's first law which states that an impulse force produces constant, eternal motion is counterintuitive to someone brought up in a world with friction. The consequence of friction, is that motions resulting from impulse forces occurring in everyday life are neither eternal nor constant. When students first encounter this law in physics class, they probably pay attention to it because it is so counterintuitive. They can also understand it in the sense that they can interpret the phrase "constant motion". However, if they do not also understand that the results of impulse forces are additive with respect to speed, which, as has been described, many students do not, they may misapply the idea of constant motion to situations where additivity really applies. For example, in a situation where an impulse force is applied to a moving object, they may misapply Newton's first law and say that the speed of the object remains constant. Thus partial but not complete absorption of Newton's laws is one source of the error that students frequently made regarding what happens to the speed of the spaceship.

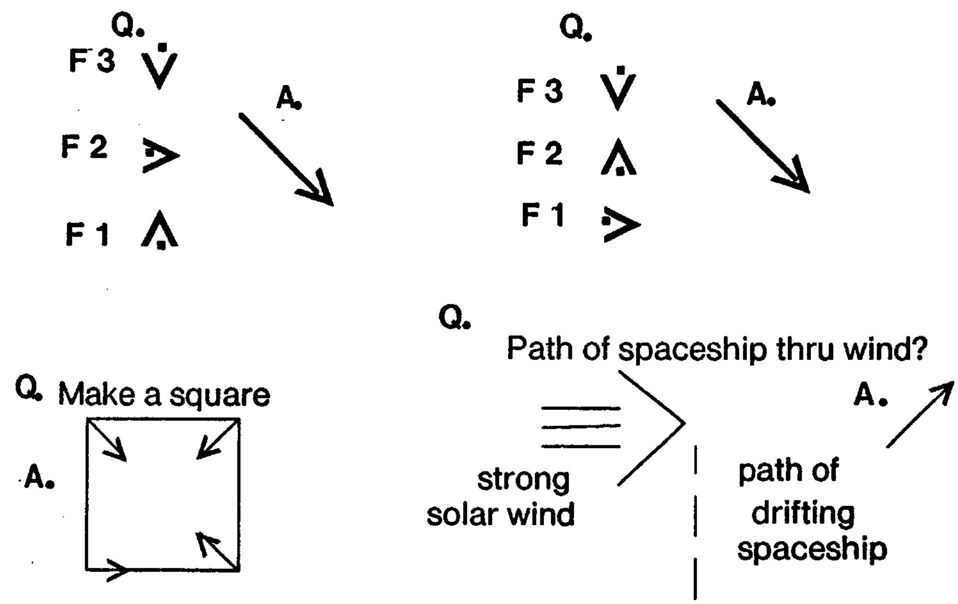
The particular approach of the PSSC physics textbook is the source of another error. When introducing vector addition, this textbook uses the length of the vector to represent the displacement of an object's position. Thus the length of the vector represents distance traveled not speed. This may be a source of difficulty when the student is then confronted with problems where the length of the vector should be used to represent speed not displacement and may partially explain why students had so much difficulty in determining what happens to the speed of objects.

Another potential problem with the textbook is that in many of the examples where the combinations of forces are illustrated with vector diagrams, the forces and their results are given at 90 and 45 degree angles. This may lead the students to erroneously conclude that there is something special or "more right" about answers involving 45 and 90 degree angles. When the givens in a problem on the questionnaire were at 90 degrees relative to one another, wrong answers given at 45 degrees were very prevalent amongst the students (see the examples in figure 5).

8.5 Difficulties Caused by the Interaction Between Speed of Motion and Direction of Motion.

There are at least two ways to think about the motion of objects. One can think in terms of speed and direction or one can think of a unified concept of velocity as represented by a vector. While it is natural to perceive motion in terms of speed and direction, there are difficulties caused by determining the effects of forces according to what happens to the speed and what happens to the direction of motion

Fig. 5.



separately of one another. To elaborate, speed and direction can be referred to independently. For example, one can say that the speed gets faster and that the direction changes by ninety degrees. Thus they are like x and y components of velocity in that one can describe them separately. However, they are very different from x and y components in an important respect. Suppose one is thinking about what happens to the velocity of the spaceship in terms of x and y orthogonal components of velocity. The final velocity can be computed by adding up all the x components first and then adding up the y components and then combining the two resultants at the end. The x and y components can thus be worked with separately because they do not affect one another. This is not the case with speed and direction. When combining velocities, the change in speed is affected by the direction of the force and the change in direction is affected by the current speed.

Failing to consider the interdependence between how fast an object is travelling and the effect of an impulse force in altering the direction of motion was a common error. In addition, some of the students also thought that changes in the speed of motion were independent of the direction of the impulse. For example, many of the graduate students interviewed during a psychology seminar said that the speed would increase linearly after each impulse. They thus failed to consider the effect that the direction of the impulse would have on altering the speed of motion.

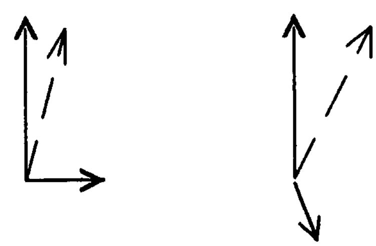
The following illustrates the kinds of reasoning employed by the students who do not take into account the interaction between changes in the speed of motion and changes in the direction of motion. While playing the games, students came up with some simple rules to help themselves control the direction of the spaceship's motion. For example, *when you want the spaceship to turn, you have to fire an impulse in a direction that is turned more than the amount you want the spaceship to turn.* This rule often fails because it does not take into account the fact that the current speed of the spaceship affects the direction in which one must fire an impulse in order to make the spaceship turn a given amount. Another rule often used was that *if you want to go in a given direction, start firing impulses in that direction.* This only works if the spaceship is stopped or going slowly. The rule needs the proviso that the faster the spaceship is going, the more impulses one must fire in the direction that one wants to go in and, if the spaceship is going very quickly, one must work on reducing the speed before firing impulses in the desired new direction of motion. In other words, this rule also fails because it does not take into account the current speed of the spaceship's motion.

The fact that the direction of the impulse and the current velocity of the object both affect the resulting speed and direction of the object's motion is the reason why all such simple rules fail. The easiest way of capturing the complexity of the situation is to use vector diagrams. An arrow, which is a familiar object to students, is used to represent velocity. The direction of the arrow represents the direction of velocity and the length of the arrow represents speed. Vectors can be combined according to simple procedures in order to predict the resulting speed and direction of motion. However, while most of the students said that they thought vectors were relevant to solving these problems, few actually used them. Further, those who did use vectors often did so incorrectly. The reason for this is that there are certain aspects of the vector representation that cause problems. There are also hidden assumptions about the way forces interact, which are implicit in the procedures for combining vectors, that violate many of the students' beliefs about the way forces alter motion. These will be discussed in the following section.

8.6 Consequences of Using the Pictorial Form of Vectors for Representing and Computing the Effects of Impulse Forces.

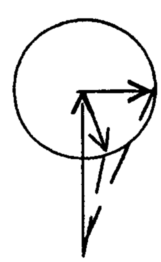
Learning to use vectors to solve Newtonian dynamics problems is very helpful. The head to tail formalism, when applied correctly, never gives wrong answers. Yet even students who know how to add vectors often use less reliable but qualitatively similar methods. To illustrate, consider an example of one way that students described the results of impulse forces to themselves. *When two forces are trying to influence the motion of an object, the result is a compromise, the object moves in a direction between the two forces. If the two forces are equal the result will be in the middle. If the two forces are unequal the result will* *be more towards the direction of the stronger force.* The problem with this seemingly reasonable idea is that it breaks down under some circumstances. For example, consider two instances where one has a large initial velocity. In case one, an impulse is fired at right angles to the initial velocity. In case two, an impulse is fired almost backwards with respect to the initial velocity. Both impulses are the same size. The compromise belief would imply that the final direction of motion should be closer to the initial velocity in the case of the orthogonal impulse than in the case of the almost backwards impulse (illustrated in figure 6.)

Fig. 6.



This, however, is not what happens. In reality, the orthogonal impulse changes the direction of motion more than the almost backwards impulse does as shown in figure 7.

Fig. 7.



Utilizing vectors would help avoid the error described above. It would also avoid errors caused by considering what happens to speed independently of what happens to direction. Vectors are a representational device which unify the speed and direction of motion into one entity, a velocity. They can be combined using the simple procedure of vector addition to predict how impulse forces will affect an object's velocity. However, this representation may be a source as well as a savior of such difficulties.

8.6.1 Vectors Focus Students on Direction.

One problem with vectors is that the direction of motion is represented by the direction of the vector, whereas, the speed of motion is represented by the length of the vector. Thus a transformation from speed to length is involved in working with vectors. Furthermore, length could be confused with representing distance or time. This is especially true for the students in this study because the PSSC textbook introduces vectors with length corresponding to displacement.

Vectors are a means of considering the speed and direction of motion in one entity. However, in drawing vector diagrams, one has to pay attention to the direction of the vector first. Once the orientation has been determined, then one pays attention to the length. This may be a partial explanation for why students have trouble with speed and why they forget to worry about speed. Another problem caused by having to pay attention to the direction of the vector first is that it implies that speed is somehow subordinate to direction. In reality, one is not subordinate to the other. Either one can be thought about first. However, when one comes to draw a vector, speed is subordinate to direction in the sense that the direction has to be computed first.

Thus the direction of motion is represented by the direction of the vector whereas the speed of motion involves a transformation into the length of the vector. Further, when drawing a vector, one has to focus on the direction of the vector before worrying about the length of the vector. The side effect is that when students are learning about how impulse forces affect motion, these factors operate to focus their attention on the direction of motion. This, combined with intuitions derived from everyday experience that contradict Newton's laws, is an explanation of why students have so much difficulty with problems involving the speed of motion.

8.6.2 Difficulties with Representing Vector Addition.

The procedure used for adding vectors in the PSSC text places the tail of the second vector at the head of the first (see figure 8) as opposed to the procedure of placing the two tails together (see figure 9) which some texts use.

The procedure of placing the tail next to the head has the advantage of making it easier to compute speed and of making it easier to add a third velocity to the sequence. Unfortunately, it also has the disadvantage that describing the relationship of the resulting velocity to the original velocity becomes

Fig. 8.

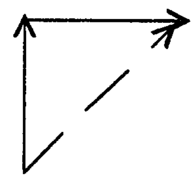
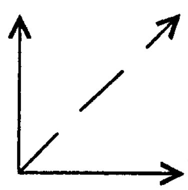


Fig. 9.

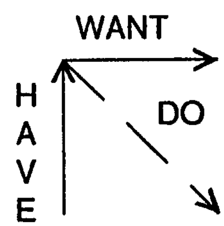


difficult in common sense terms. (See figure 8). Contrast this with the tail to tail procedure. (See figure 9). This second representation more readily allows the simple description of "in the middle" or "in between" to be applied than does the head to tail way of representing the problem. It was mentioned previously that some of the students spontaneously articulated an "in the middle" description about the relationship between the two component velocities and the resulting direction of motion. The tail to tail method is more consistent with such descriptors than the head to tail method.

However, there is a conceptual difficulty with the tail to tail method. It encourages the following kind of error. Students tend to wrongly generalize its use to vector subtraction problems. If they are given a subtraction problem where the motion is in a given direction and one wants them to determine the direction in which to apply a new force so as to get motion in another given direction, they will suggest applying a force in a direction that is in the middle with respect to the axis of the current motion and the axis of the desired motion.[1] This answer, illustrated in figure 10, would be appropriate if the size of the impulse force could be bigger than the original force. However, in the context of the problems the students were given, the impulses were of constant magnitude. Thus they have overgeneralized the "in the

1. Note that this "in the middle" way of doing vector subtraction by putting vectors head to tail is the way the PSSC text does vector addition. Also note that the "in the middle" way of representing vector addition by putting vectors tail to tail is the way the PSSC text does vector subtraction.

Fig. 10.



middle" principle to a case where it is not appropriate by ignoring the size of the force as a factor.

This raises the issue of criteria for a good representation - extendability and ease of computation (both things that physicists worry about) versus representing the situation in the way that corresponds best to other descriptors of the problem - in the middle, this direction then that direction (things the students spontaneous representations reflected). The preceding discussion has demonstrated the necessity of considering the cognitive aspects of the vector representation as well as the formal aspects when trying to interpret how the students interact with it.

8.6.3 Hidden Assumptions in Vector Addition.

A major problem with vectors is that they are presented as something very simple with a simple procedure for combining them. However, their use in solving Newtonian dynamics problems is carrying a lot of hidden assumptions. There are complexities that the student is not looking for.

In the Newtonian dynamics problems that were given to students, the first vector that should be drawn represents the current velocity of the object. However, as has already been discussed, not all of the students realized that one has to take into account the current state of motion when predicting the effect that an impulse force will have. They have adopted a simple cause effect view of the situation. From their perspective, the effect of an impulse depends upon the impulse not on the velocity of the object plus the effect of the impulse. The reason for this view is that in their everyday experience they primarily apply impulse forces to stopped objects, friction being being responsible for why most of the objects they deal with are stopped. Since they have only dealt with objects in this state, they have not learnt that variations in state cause variations in results. Thus they have not learned to consider current velocity as a factor when solving these problems.

The second vector drawn when solving these problems represents the component of velocity introduced by an impulse force acting on the moving object. The idea that an impulse forces translates into a component of velocity is crucial when solving these problems. It accounts for why a situation involving the combination of two velocities, such as a wind and an airplane, has the same vector diagram as two impulse forces acting on an object simultaneously and, further, why both of these situations can have the same vector diagram as an impulse force being applied to a moving object. Whether one is dealing with a velocity or a force is irrelevant because a force translates into a component of velocity. The fact that the students can solve problems given the first two types of situations but have difficulty with the third indicates that they do not regard them as analogous situations. Perhaps it is the idea that a force translates into a component of velocity that is partially responsible their difficulty.

Another assumption inherent in this use of vector diagrams, is that the component of velocity caused by a given impulse is always the same regardless of the velocity of the object. The behavior of the Newtonian computer microworld might actually cause some confusion because the result of an impulse is dependent on the velocity of the object. This may lead to the conclusion that the effect of the impulse is different in different contexts. However, it is not the effect of the impulse that is varying. That always remains the same. Rather, it is the variation in state that is responsible for the variation in the result when a given impulse is applied. This confusion might be alleviated by giving the student the facility to see a vector diagram of such situations so that they could see that it is the first vector representing the current state of motion that is responsible for the varying resultants.

One final assumption is that the result of the impulse force should be added to the current velocity of the object. Why should addition be the result of an interaction between a force and an object in motion? The results indicated that many student have difficulty with this idea especially with regard to what happens to the speed of the spaceship.

Some of the students see the effect of an impulse force as completely overriding the original motion of the object. This could be because they have not yet learned about momentum. Alternately, it could be because they see the role of the force as a changing the direction of motion. The "energy" of the force is used to redirect the motion of the object and hence override the original direction of motion. This is rather like the situation when one pushes a heavy object. One has to overcome the inertia of the object in order to get it to move. Here one has to apply a force in order to overcome the momentum of the object and get it to change direction. There was evidence for both of these ideas in the students answers to the questionnaire.

Other students saw the role of the force as additively changing the direction of motion but not the speed of motion. The force is used to interact with the momentum of the object and additively changes the object's direction of motion, but it does not affect the speed of the object.

Another view of the interaction could be described as a compromise. The momentum of the object fights with the impulse force for control over the object's motion. The result is a compromise, one gets some but not all of both. This view was expressed by some of the students who said that a second impulse applied in the same direction as the first would cause the spaceship to go faster but not two times faster.

In order to believe that the velocities should add, one has to believe that they both should be conserved so that one has all of both. This implies the change in direction of motion occurs without using anything up. However, this idea was counterintuitive for many students. Possibly because it appears to violate the general principle that you do not get something for nothing. Some students explicitly stated that energy must get used up in order to make the object turn. The relationship between the cause (the impulse force) and the effect (the change in velocity) is confused by misconceptions about the relationship between force, energy, and momentum. Thus, some students have ideas that conflict with the assumption that an impulse force introduces a component of velocity which should be added to the existing velocity of the spaceship.

The conclusion is that the students have difficulty utilizing vector diagrams to help themselves solve Newtonian dynamics problems because they have ideas that conflict with assumptions inherent in applying vector addition.[1] One of the pedagogical roles of the computer games should be to help the students realize these assumptions and to believe that they accurately describe the effects of impulse forces. The reason that believing these assumptions hold true is important is that they are crucial not only to the use of vectors in solving these problems but also to the understanding of Newtonian dynamics in general.

8.7 Summary.

The students have multiple and diverse knowledge components which they utilize when solving Newtonian dynamics problems. However, much of their existing knowledge which seems relevant to the solution of force and motion problems has properties which conflict with the formal physics. The result is that the students are able to only partially understand the implications of Newton's laws. For example, many students behaved as though the results of forces are additive with respect to the direction of motion

1. The fact that the assumptions inherent in the "simple" procedure of vector addition run counter to students' force and motion intuitions raises a more general point: when considering the learnability or understandability of a piece of knowledge, one must consider more than just the "simplicity" of the piece of knowledge taken by itself. One must also take into account how it relates to other components of the students' knowledge. DiSessa [1981] discusses this issue under the title of distributed encoding.

but are not additive with respect to the speed of motion.

In addition, the students' spontaneous ways of describing and solving force and motion problems have inherent difficulties. For instance, many students evolved separate rules for predicting what happens to the speed of motion and what happens to the direction of motion. These rules fail under many circumstances because of the interdependence between changes in speed and changes in direction. The students therefore need a better way of representing the effects of forces on the motion of objects.

A very effective representation used by physicists for this purpose is vector diagrams. However, the students have difficulty learning to use vectors because they have ideas that conflict with assumptions inherent in applying vector addition. For instance, the students' intuitive beliefs concerning how forces should affect the motion of objects conflict with the implications of Newton's laws. Further, they have learned about addition and subtraction in the context of scalar arithmetic. Scalar arithmetic has many properties which do not transfer to vector addition. Finally, the vector representation itself causes difficulties. There is confusion as to what the length of a vector represents. Also, when drawing a vector, one has to focus on the direction of the vector before worrying about the length of the vector. These factors operate to focus students' attention on the direction of motion and may partially account for the difficulty that students have with problems involving the speed of motion.

The students thus need a representation such as vector diagrams because spontaneous ways of solving force and motion problems run into difficulty. However, they have trouble understanding the vector formalism because many components of their existing knowledge (such as intuitions about force and motion and knowledge of scalar addition) which seem relevant, also have properties that conflict with Newton's laws and vector addition.

Chapter IX

Designing Interactive Learning Environments

The purpose of this chapter is to illustrate the principles of designing interactive computer learning environments that have emerged from this study. These principles are discussed in the context of describing improvements that could be made to the Newtonian computer microworld and to the sequence of games formulated within that microworld.

9.1 Learning in the Context of Interactive Computer Games.

An assumption inherent in this use of interactive computer games is that the students are capable of debugging their own knowledge provided one gives them appropriate feedback and designs the task so that it focuses their attention on areas where their knowledge needs revising. The fact that working with the games did improve the students' performance on very basic force and motion problems strongly supports this assumption. It also suggests two important design considerations. The first is the necessity of creating the microworld and games so that they provide good feedback. The second is the need to find out where the students exhibit erroneous thinking. These two design considerations are interrelated. The quality of feedback is dependent on the students' current focus of attention and on how the students are interpreting the behavior of the microworld.

To illustrate, many of the students did not notice changes in the speed of the spaceship following the application of impulse forces. However, the experimenter was able to notice even small changes in the speed. This apparent discrepancy in perception occurred for two reasons. Firstly, the experimenter knew from her knowledge of the formal physics that the speed should change following the application of an impulse. It was thus perceptible to her because she expected to see it. Many of the students, on the other hand, did not expect to see changes in speed and, so, did not perceive small changes in speed. Secondly, when the students were focusing on changing the direction of motion, they often did not notice even large changes in the speed of motion. The quality of the feedback is thus impaired, in this instance, by the students' misconceptions and by their focus of attention.

The problem of creating good feedback is, therefore, intertwined with problem of focusing the students' attention on areas where their knowledge needs revising. In order to achieve both of these design considerations, one has to do an empirical study to determine where the students have difficulty. The design process is thus an iterative one. Preliminary evidence as to the areas of difficulty was gained through a pilot study. The games and questionnaire were then designed and tried out on a group of students. This source of evidence led to new theories concerning the understanding and misunderstanding of Newtonian dynamics. The games and questionnaire are then redesigned and the process repeats. The design process could best be described as a series of successive refinements.

The students can achieve success with these games through a combination of general problem solving heuristics, partial knowledge of the relevant physics, knowledge from other domains (e.g. scalar addition), and feedback. Thus in addition to focusing the students' attention on areas where their knowledge needs revising and to providing good feedback, the designer also has to worry about facilitating the use of general problem solving heuristics and encouraging the application of knowledge which will help the students to better describe how forces affect motion. This aspect of the design process also requires iteration because one has to empirically determine what heuristics the students spontaneously use when playing the games and what components of their knowledge from other domains help them reason about force and motion problems.

When playing these computer games, the students utilize multiple aspects of their knowledge. They try one component of their knowledge (for example, the intuition that things go in the direction that you kick them), they see how it fails, and then invoke another idea. The feedback helps them to settle on the most useful ideas.

The games thus encourage what has been termed the scientific method: the process of forming a hypothesis, testing it, getting feedback, and modifying the hypothesis to fit the results. Sometimes the students approach a game with a specific hypothesis concerning how to achieve the desired result. At other times, the students may employ more general problem solving heuristics combined with feedback to help achieve the goal. In either case, learning occurs, since, if the students succeed at a game, even if they did so fortuitously, they need to evolve some theory or description of what they did in order to be able to repeat their success. If this description is inaccurate, they cannot repeat their success and they must revise it. Further, if their theory only works locally, one can encourage them to amend it by presenting a more complex task. Increasing complexity should encourage the students to evolve theories which more closely resemble those of the physicist.

The designer thus has two goals. Firstly, to get the students to recognize that their descriptions of force and motion are inadequate. Secondly, to get them to amend their ideas or replace them with better ways of reasoning about how forces affect the motion of objects. However, students can be very reluctant to give up erroneous thinking. This can occur for several reasons described as follows.

It could be that the erroneous idea has multiple sources of conceptual support. For example, when asked what the effect would be on the speed of motion of an impulse applied at right angles to the current direction of motion, many students said that there would be no change in the speed. One student supported her answer by saying, "impulses only change the direction of motion, not the speed and, besides, it is the same size impulse as the first, so the speed would be the same". This justification utilizes two misconceptions which happen, in this instance, to yield the same wrong answer. This multiple support for the answer that the speed remains the same, serves to make the student more certain of her answer. Since the student often has many ideas which are relevant to a particular problem situation, it is hoped that interacting with the microworld will give support to the more useful and appropriate ideas which the student has.

Another reason why students could be reluctant to give up erroneous thinking is that the idea could have been reinforced on many previous occasions. For instance, the belief that objects always go in the direction that you shove, push, or kick them is frequently supported in everyday experience. A function of the games is to provide experiences which contradict such misconceptions.

One further reason why students could be reluctant to give up wrong ideas is that they have nothing to replace them with. They will not discard an old idea unless they have something to put in its place. The students need to be given a new way of looking at force and motion. This is the role of the formal physics, specifically, in this situation, Newton's laws and the vector representation. Vectors are a good way of thinking about force and motion problems. They help students to see the force as an increment to velocity. However, the students need to believe the assumptions, discussed in the previous chapter, inherent in the use of vectors. The microworld and games should be designed so as to encourage such useful ways of representing and thinking about the domain.

9.2 Design Principles.

The remainder of this chapter will be devoted to a discussion of design principles in the context of describing improvements which could be made to the Newtonian computer microworld and to the sequence of games formulated within that microworld.

9.2.1 Focus on Aspects of the Students' Knowledge that Need Revising.

The first design principle was to create games which focused the students' attention on areas where their knowledge needs revising. For example, the game of Corner, where the student had to navigate the spaceship around a right angle, was meant to clearly contradict the intuitive belief that *objects always move in the direction of the most recently applied force.* One could imagine further means of helping the students to recognize the erroneous nature of such beliefs. It was suggested in the previous chapter that one of the sources of such "ignore-momentum" difficulties is that the students live in a world where friction affects the motion of objects. This leads to beliefs such as, *things go in the direction that you kick them* and *you need a constant force to get constant motion.* In order to make links between these intuitive beliefs derived from everyday experiences and the reality of Newton's laws, the students need to become aware of the role of friction and, further, that in a world without friction, things do not necessarily go in the direction that one kicks them and that one does not need a constant force to get constant motion. One way to perhaps achieve this awareness within the context of the Newtonian microworld is to introduce friction into the microworld and then gradually remove it. This could be done by having the spaceship travel through different mediums such as water, air, and outer space. The students could then try to hit the target, for example, while the medium is water, followed by air, and, finally, outer space. The amount of friction is thus being reduced each time so that the students can find out how friction affects motion.

This study has discovered additional misconceptions which the students must overcome in order to understand Newtonian dynamics.

For example, the students had difficulty predicting and controlling the speed of motion. In fact, the game of Target where the students had to hit the target with a low speed was the most difficult game of the sequence. In this game one has to achieve two goals simultaneously; hitting the target and achieving a low speed. Since the results of the questionnaire indicated that the students have a lot of difficulty figuring out how to reduce speed, having a game that required them to not only reduce speed but to hit the target as well was probably overly complicated. Further, having the additional goal of hitting the target may have diverted their attention away from thinking about how to reduce speed. Hence, a game where one just has to achieve a low speed would have been better. This could involve having the spaceship inside a large circle. The goal of the game would be to cross the edge of the circle, at any point, with a low speed. This would leave the student free to focus on reducing speed since all paths will eventually cross the edge of the circle.

Another frequent error that the students made was to ignore the interdependence between how fast one is going and the effect that an impulse has on changing the direction of motion. To help the students become more aware of the interaction between speed and direction and to learn to take it into account when solving Newtonian dynamics problems, there need to be more games which focus on simultaneously controlling the speed and the direction of motion. For instance, the Corner game could be extended into a maze involving the navigation around several corners. Then restrictions could be placed on the range of speed with which the spaceship can be travelling immediately prior to each of the corners. The acceptable range of speed would be different at the different corners. Thus the student has to cause the spaceship to turn the same amount at each corner under varying speed conditions. This should make the student aware that in order to navigate the corner, the direction of the impulse needs to be varied with the speed of motion.

A further error made by the majority of students was to over extend the additivity of velocities to a situation where it did not apply. When asked to predict the velocity of the spaceship while it was moving in the direction of a strong solar wind, most of the students said that one would add the speed of the spaceship to the speed of the wind. In reality, the spaceship would just end up travelling at the speed of the wind. The students thus need to be exposed to the effects of an impulse force interacting with a complex force. This could be done by introducing additional forces such as winds or gravity into the microworld.

9.2.2 Encourage Better Ways of Representing and Thinking About the Domain.

Even though the majority of the students said that they thought that vectors were relevant to the games and questionnaire problems, only one third of the students drew vector diagrams to help themselves to think about the games or to solve the problems of the questionnaire. Further, of the students who did utilize vectors, many did so incorrectly. Since vector diagrams are a very useful way to think about Newtonian dynamics problems, understanding and utilizing this way of representing the effect of forces on motion needs to be encouraged. The existing sequence of games was not particularly successful at achieving this goal. Some additional means of encouraging the use of vectors seems necessary. This could take the form of a pause feature which includes the facility to do vector addition in the paused state.

The following elaborates this proposed new feature of the microworld. When the student presses the pause button, the spaceship could disappear and a vector representing the current velocity of the spaceship could appear in its place. The student could then use the engine rotation button and the impulse button to add impulses in various directions in order to see the resulting velocities. The component of velocity introduced by the impulse force could be displayed and added to the existing velocity with the resultant velocity being displayed. If the student wishes that particular impulse to be implemented, he or she will hit the return button. Otherwise, they will hit an undo button and try to add a different impulse to see what change in velocity will be produced by that particular impulse.

By giving students a preformatted facility for doing vector addition within the context of this microworld, it is hoped that they will become familiar with the procedure for doing vector addition. Further, that they will come to believe that it works in the sense that one can accurately predict the motion of the spaceship using this technique.

However, as was discussed in the previous chapter, students had difficulty understanding vector diagrams. In particular, there was uncertainty as to what the length of the vector corresponded to, if anything. Also, some of the students held beliefs about the effect of forces on motion which contradicted the basic assumption behind vector addition. Mainly, that a force introduces a component of velocity which must be added to the existing velocity in order to predict the resulting motion of the object. How could demonstrating vector addition in this interactive, Newtonian environment help overcome these problems? The students may not realize that there is any significance to the length of the vector or be confused about what it represents. However, when they see that the existing velocities are represented by vectors whose lengths are not always the same, they may eventually correlate the current speed of the spaceship with the length of the vector. They will also observe that an impulse gets translated into a vector which is added to the vector representing the existing velocity and, further, that this addition produces a vector which accurately predicts the resulting velocity of the spaceship. Whether this interactive demonstration of vector addition combined with feedback from the microworld about how impulse forces affect the motion of the spaceship will be sufficient to help the students use vectors to solve these problems remains to be empirically determined. Possibly there will also need to be some discussion of the following issues:

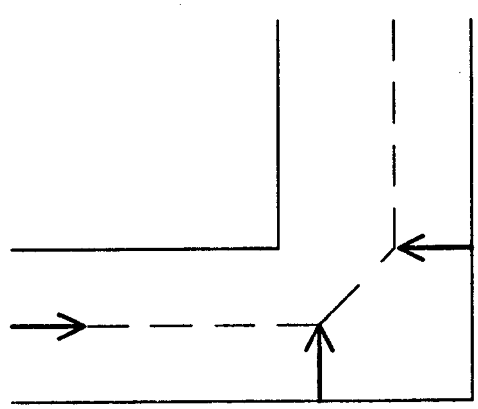
- what is being represented and how, especially with respect to the length of the vector,
- the fact that a force translates into a component of velocity,
- the idea of conservation of momentum. Both the original velocity of the spaceship and the velocity introduced by the force still exist and, since the spaceship can only move in one direction at once, the two velocities must add together.

These ideas are crucial to the use of vectors and to the understanding of Newtonian dynamics in general.

One further comment concerning the use of a pause feature combined with the facility for vector addition is that it represents a specific instantiation of the more general problem solving heuristic of freezing the state and analyzing the situation. This is a common technique employed when trying to solve physics problems. Thus giving students this facility should not only encourage the use of vector diagrams but might also encourage the use of this problem solving heuristic.

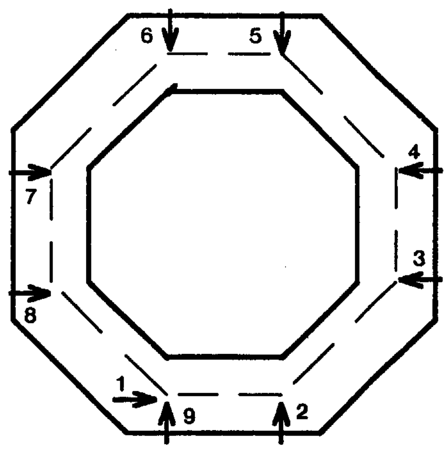
Another helpful way of thinking about how forces affect the motion of objects is to think of velocity in terms of orthogonal components. Physicists find this useful especially when solving complex force and motion problems. Many of the games in the sequence were played with the student having the facility to introduce only orthogonal impulses, that is, impulses at headings of 0, 90, 180, & 270 degrees. One of the objectives in having this facility was to help the student learn to think in terms of orthogonal components of velocity. However, the results of the questionnaire indicated that the games do not help students to think in terms of orthogonal impulses. This suggests that additional means are necessary. One possible approach is that when students utilize the delayed anti-impulse strategy (shown in figure 1) for going around the corner, as almost all of the students did, one asks them how this strategy works. In particular, ask them why do you end up going vertically upwards after the third impulse?

Fig. 1.



Hopefully they will explain why it works by referring to the cancellation of the horizontal component of velocity which still exists even though a vertical impulse was also fired. Further, games could be designed which encourage the derivation of strategies like the delayed anti-impulse strategy. For example, the students could be asked to navigate a hexagonal track where the most efficient strategy involves repeatedly canceling orthogonal components of velocity as illustrated in figure 2.

Fig. 2.



9.2.3 Facilitate the Use of Problem Solving Heuristics.

What often makes a game interesting or motivating is that it presents a new challenge. However, the students must have a basis for being able to master the game otherwise they will get frustrated and loose interest. This is especially true of the first few games in the sequence where the students need to build up some positive feelings towards their interaction with the microworld. Success with these games, as was described in chapters two and six, can be achieved through a combination of general problem solving heuristics, partial knowledge of the relevant physics, knowledge from other domains, and feedback. In the first few games it is particularly important that general heuristics achieve helpful results.

For example, consider the first game in the sequence, Race, where the students had to get the spaceship to cross a line with a high speed. Many of the students did not know that the results of impulse forces additively affect the speed of motion. However, employing the most general of heuristics, *just try something and see what happens*, works for this task. The simplest thing to try is just firing another impulse in the same direction as the first. This achieves the goal of the game by increasing the speed of the spaceship.

Contrast this with the second game in the sequence, Dock. In this game, the students had to stop the spaceship in a specified location. It was found to be either a trivial or a very frustrating task. The majority of students already knew how to make the spaceship stop. For those who did not, experimenting with impulses fired at separations of 90 degrees did not usually achieve the desired result. Giving the students the facility to fire only impulses separated by 180 degrees would be more appropriate for this game since stopping the spaceship would be a likely result of just experimenting with such impulses.

9.2.4 Encourage the Application of Relevant Knowledge From Other Domains.

The last chapter contained an analysis of why students have so much difficulty learning Newtonian dynamics. One of the main arguments presented was that there are components of the students knowledge which seem relevant to force and motion problems but which conflict with Newton's laws of motion and vector addition and hence cause confusion. Here the argument will be presented that there also exists components of the students knowledge which can help them understand Newtonian dynamics. One goal is to design games which invoke these helpful knowledge components.

The first two games in the sequence, Race and Dock, involved controlling motion in one dimension. They were restricted to one dimension in the hopes that it would invoke an analogy to scalar addition and thereby help the students to see the effects of impulse forces as additive.

What also emerged as a commonly applied and useful idea from another domain was the idea of cancellation. Initially one might think that cancellation is just a property of scalar addition. However, there were students who applied the idea of cancellation but behaved as though the effects of impulses were not additive under other conditions. For instance, they said that a second impulse applied in the same direction as the first would not affect the speed of motion, however, a second impulse applied in an opposing direction would stop the spaceship. This result implies that the concept of cancellation supersedes that of additivity in this domain.

Since nearly all of the students could apply the idea of cancellation to the behavior of impulse forces, it could be used as a basis for reasoning about the effects of other impulses. For example, applying impulse forces so as to slow down the motion of the spaceship could be viewed as a partial cancellation. However, the difficulty that the students experienced in solving problems of the form, *how could you reduce the speed of the spaceship without stopping it*, indicates that many students do not apply the idea of partial cancellation spontaneously. Games such as the one where students had to hit a target with a low speed should encourage the discovery of this useful idea. Possibly a sequence of questions such as, *how could you make the spaceship stop*, followed immediately by, *how could you make the spaceship almost stop*, would also encourage the idea of partial cancellation.

9.2.5 Represent the Phenomena of the Domain Clearly.

Another important design consideration is to represent the phenomena of the domain clearly. In this instance, the goal is to represent the implications of Newton's laws of motion.

For example, a potential source of confusion is that when students see the spaceship in constant motion, they may tend to erroneously infer that there must be a continually acting force producing the constant motion. However, the only forces being dealt with in this microworld were impulse forces and this was emphasized in the verbal introduction to the games. To help differentiate an impulse force from a continually acting force, a burst of flames could be displayed behind the engine whenever an impulse is fired. This was not done within the microworld used for this study because it caused the spaceship to slow down momentarily whenever an impulse was fired. With a faster computer this slowing down would not be perceptible. Distinguishing between an impulse forces and a continually acting forces will become even more important when the use of this computer microworld is extended to involve both continually acting and impulse forces.

Chapter three described how the microworld was created so as to represent Newton's laws clearly. This issue was addressed prior to the designing of the games. However, as with the other design factors, there is an empirical component to displaying the laws clearly. How well the phenomena is represented is dependent on how the students interpret the behavior of the microworld.

For instance, the students often did not notice changes in speed. In contrast, the experimenter readily noticed these same changes. Further, the students frequently gave incorrect answers to questions where they were asked to predict what would happen to the speed of the spaceship following a given impulse or to say how one would achieve a given change in the speed of the spaceship. The games did help the students to say how one would reduce the speed of the spaceship. However, they caused as many students to change to the wrong answer as changed to the right answer on a question where they had to predict the effect of an orthogonal impulse on the speed of motion. In the latter case, the speed increased by a factor of 1.4. In order to make such changes in speed more noticeable, a continuous, digital readout of speed could be displayed at the bottom of the screen. This would provide the student with an additional source of evidence about the speed of the spaceship. It might also serve to focus their attention more on what happens to the speed of motion when impulses are fired.

9.2.6 Eliminate Irrelevant Complexities from the Computer Microworld.

The final design principle is to eliminate sources of difficulty that have nothing to do with the conceptual task which the game is meant to be focusing on. In the context of a Newtonian computer microworld, one attempts to eliminate the need for the student to pay attention to features of the microworld that are either not relevant to Newtonian dynamics at all or, at least, are not relevant to the aspect of Newtonian dynamics which a particular game is trying to focus on.

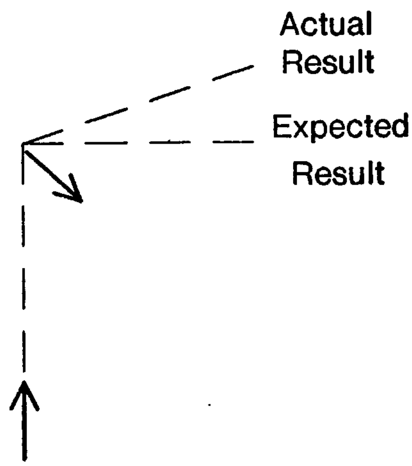
This aspect of the design process is dependent on observing students interact with the microworld. For example, determining the size of the unit impulse force is an empirical question. Many of the students fired several impulses at the start of each game in order to get up to what they considered to be a reasonable speed. This meant that whenever they wanted to alter either the direction or speed of motion, they had to account for several initial impulses instead of just one. Adapting a strategy for variations in speed of motion is an important thing to learn and was the focus of one of the games. However, it meant that these students always had to pay attention to this as well as to whatever else the game was trying to focus them on. Thus it introduced an extraneous factor into many of the games. In order to avoid this, the size of the impulse fired by the spaceship's impulse engine should be increased. This would have the effect of increasing the speed of the spaceship following the initial impulse given by the student.

At some point in their experience with this microworld, students were given the facility to change the heading of the spaceship's impulse engine by 30 degree intervals. It was concluded that this should have been 45 degree intervals, not 30, for several reasons:

- 30 degree separations in heading are too small. They are not always distinguishable. For example, a heading of 30 degrees is not easily differentiated from a heading of 60 degrees. If the students mistakenly perceive impulses applied in these two directions as being the same, the behavior of the spaceship will appear erratic. What is perceived as being the same impulse will have a different effect under identical conditions.
- many of the strategies employed by the students utilized anti-impulses, i.e. impulses applied at 180 degrees with respect to the current direction of motion. In order to turn 180 degrees with the facility for 30 degree turns, one has to press the engine rotation button six times. Thus, students often had to do this hurriedly in order to get six presses of the engine rotation button done in time for the anti-impulse to have the desired effect. The side effect was that sometimes students did not get it done in time or else they pressed the button once too often and overturned. They then had to go all the way around again which meant that the anti-impulse was usually applied too late. Increasing the amount of turning from 30 to 45 degrees would help reduce these errors which served only to frustrate the students.
- a final reason for changing from 30 to 45 degree engine rotation intervals is the following. The questionnaire revealed that the majority of students believe that after an initial impulse, a same sized impulse applied at 45 degrees backwards with respect to the direction of motion, will cause a right angle

turn in the direction of motion. (See figure 3).

Fig. 3.



In reality, an impulse fired backwards at 45 degrees would have to be larger than the initial impulse in order to achieve a 90 degree turn. The games did not help to overcome this misconception. Possibly giving the students the ability to apply such 45 degree impulses would give the students more direct evidence relevant to this common misconception.

In the original implementation of this computer microworld created by diSessa [in Papert et al. 1979], the student had the facility to turn the impulse engine either to the right or to the left. This was achieved by the student pressing either R or L on the keyboard. It was found during the pilot work that deciding whether to press R or L and locating it on the keyboard was time consuming and a had nothing to do with Newtonian dynamics. Thus the students in this study were only given the facility to turn the spaceship's engine to the right. This enabled them to keep the index finger of the left hand on the engine rotation button and the index finger of the right hand on the button which fired impulses. The problem of deciding whether to press R or L and of trying to locate them on the keyboard was thereby avoided.

9.3 Can the Same Sequence of Games Help Most Students?

The approach utilized in this research was to diagnose common difficulties and then present the same sequence of games to all students. The findings of this study suggest that this was a reasonable strategy in this instance. One of the most striking results was the common pattern of wrong answers which students gave to problems on the questionnaire. The students' justifications for these wrong answers yielded origins of difficulty in learning Newtonian dynamics which were shared by many of the students. Further, the students proceeded quickly through a game which they found easy. Thus, if they were given a game which focused on a difficulty that they did not possess, it did not hinder their progress significantly.

However, different students did have different sources of difficulty in learning Newtonian dynamics. Ideally, it would be best to use some diagnostic technique, such as the questionnaire or performance on the games, to determine what sources of difficulty plague the individual student. The sequence of games could then be customized to fit the learning problems of each student.

One potential research project could involve designing a computer program to perform the diagnostics and alter the sequence of games accordingly. This approach to intelligent computer coaching was the focus of the Wumpus Advisor project [Goldstein, 1977] at M.I.T.. The computer coach tried to teach a set of reasoning rules for a game of deductive reasoning called Wumpus. The coach was programmed to observe the student's Wumpus play, make inferences about which rules the student knew, and adjust its tutorial accordingly.

There are, however, difficulties with this approach [White, 1977]. Playing Wumpus or understanding Newtonian dynamics involves the application of many diverse components of the students knowledge. For example, there is the representation which the student uses to encode information relevant to solving the problem, there are the problem solving heuristics which the student utilizes, there is the application of knowledge from other domains which can both help and hinder the student's performance, and so on. Analyzing the results of a diagnostic tool, such as the questionnaire, in order to determine the origins of a student's difficulty is a complicated process. For example, one important skill, which computers do not currently possess, is the ability to observe drawings which the students make when solving the problems. There are thus sources of information concerning the student's reasoning process which the computer does not have access to. Nonetheless, creating an artificially intelligent diagnostician would be an interesting goal to pursue.

9.4 Conclusions.

Despite a good teacher[1] and a well respected physics textbook (PSSC), the students in this study

1. The students had nothing but praise for their physics teacher. They described him as knowledgeable, conscientious, and likable.

had difficulty with force and motion problems. Even though most of the students had recently studied a chapter on Newton's laws of motion, many could not correctly answer basic questions pertaining to Newton's second law of motion. For example, one third of them said that applying an impulse force to a moving spaceship would not affect its speed. Yet after less than one hour of playing the Newtonian computer games, there was a significant improvement in the students responses to such simple force and motion problems.

This result suggests that computer games designed according to the criteria described in this chapter could play a valuable role in the learning of physics. Games force the student to figure out how to achieve a given effect, such as, reducing the spaceship's speed or controlling its path. This form of problem solving appears to help students understand both the implications and the applications of Newton's laws of motion.

One probable reason for this success is that the computer microworld embodies Newton's laws in a way that makes them "more obvious". This is done by:

- eliminating confusing complexities like friction and gravity,
- making simplifications such as having quantized impulses being applied to point masses with the effect of instantaneous acceleration,
- introducing perceptual aids, for example, having the spaceship leave a trace of its path and having a digital readout of the spaceship's speed,
- introducing conceptual aids such as having the facility to freeze the motion of the spaceship and try various impulses using vector diagrams to represent the situation,
- and by designing games which focus attention on particular aspects of the implications of Newton's laws.

All of these features operate to make it easier to see the implications of Newton's laws.

This approach of using computer games to facilitate learning could be extended to other domains. It is relatively easy to see how this could be done for other dynamics situations. For instance, the principles of atomic and nuclear physics could be embodied in a microworld where particles move and interact with one another. The student could attempt to control the interactions of the particles in a game-like format by varying given parameters. The microworld would act to enlarge the atoms or nuclei and slow down their motion so that interactions would become visible. Further, the microworld could simplify the situation by introducing only one principle at a time and thereby making it easier to see the laws of physics.

One interesting speculation is that it might be simpler to design interactive computer microworlds for learning about domains like atomic and nuclear physics, than it was for a domain like Newtonian dynamics, because students may not have as many erroneous preconceptions. The designer would, therefore, not spend as much time diagnosing misconceptions and creating games which focus on these misconceptions. Instead the designer would have to concentrate on making the phenomena "easy to see" and on encouraging the students to apply aspects of their knowledge that would help them to understand this new domain.

9.5 Summary

This chapter illustrated the principles of designing interactive computer learning environments that emerged from this study. Among these principles were the following:

- focus the students on aspects of their knowledge that need revising,
- encourage better ways of representing and thinking about the domain,
- facilitate the use of problem solving heuristics,
- encourage the application of relevant knowledge from other domains,
- represent the phenomena of the domain clearly,
- and eliminate irrelevant complexities from the computer microworld.

Although these principles may seem straightforward, attempting to implement them is far from simple. One has to empirically determine where the students have difficulty in understanding the domain in order to design games which focus attention on these sources of difficulty. Further, one has to determine what components of the students' knowledge from other domains can help them to understand this new domain. In addition, one has to study how the students interpret and interact with the computer microworld in order to ensure that the phenomena of the domain are clearly represented and that the students are not being distracted by irrelevant complexities. The design process is, therefore, a series of successive refinements and is heavily dependent upon empirical investigations.

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