A STUDY OF THRESHOLD DETECTION

by

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ABSTRACT

An extremely versatile, high speed, tunnel diode stabilized threshold detector is described. The device is d.c. coupled, and uses negative feedback to control the hysteresis. The hysteresis is adjustable from less than 1 millivolt to 5 volts. When the hysteresis is set below 1 millivolt, the device serves as a stable zero crossing detector. The existing experimental threshold detector operates from d.c. to 500 kc/s.; the switching speed at the output is 30 ns. Operation at frequencies above 1 mc/s. is feasible.

Two tunnel diodes set the thresholds of the device. The thresholds depend only on the peak currents and peak voltages of the tunnel diodes. The valley currents and voltages have no effect on the transfer characteristic. Since the peak currents and peak voltages have a high degree of stability, the transfer characteristic is very stable. A static analysis yields theoretical expressions for the transfer characteristic.

The tunnel diode stabilized threshold detector is the most sophisticated device presented in the thesis.

The report begins with a discussion of some basic threshold detectors. A Schmitt trigger, a flip-flop, a four transistor complementary flip-flop, and a differentially driven flip-flop are included. Analytical expressions yielding the static transfer characteristics of these devices are presented.

Another more complex threshold detector which utilizes negative feedback is included. This device uses two flip-flops in conjunction with a latching circuit to set the thresholds. It is not as stable as the tunnel diode circuit, and is intended to have a hysteresis of .1 volt to 10 volts. The experimental model can have as little as 1 millivolt of hysteresis; but, the thermal stability is not good. The device operates from d.c. to 200 kc/s.; the output is capable of switching 1 ampere in 30 ns. A static analysis of this device is presented.

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TABLE OF CONTENTS

CHAPTER I

Introduction

1.1 The General Switching Threshold Detector

Threshold detectors and limiters have been used for phase measurement and for analog to digital conversion. Now, with the development of new switching systems, there is an increased demand for fast, stable threshold detectors.

In general, a threshold detector is a two port device having an input and an output. The output has two states. There is a range of input voltages for which either output state may occur depending on the history of the input; this is the hysteresis range. For input voltages above and below the hysteresis range, only one output state is possible.

A special threshold detector, having an output voltage which changes states whenever the input voltage crosses zero, is called a zero crossing detector. Such devices are realizable with simple electronic circuits. However, all practical zero crossing detectors have a transfer characteristic which fluctuates with time, temperature, and frequency, so that they can achieve the ideal transfer characteristic only for short periods of time. Thus, practical zero crossing detectors are useful only if the input signal exceeds the long term fluctuations of their hysteresis and detection point. The fluctuations limit the useful sensitivity of a zero crossing detector.

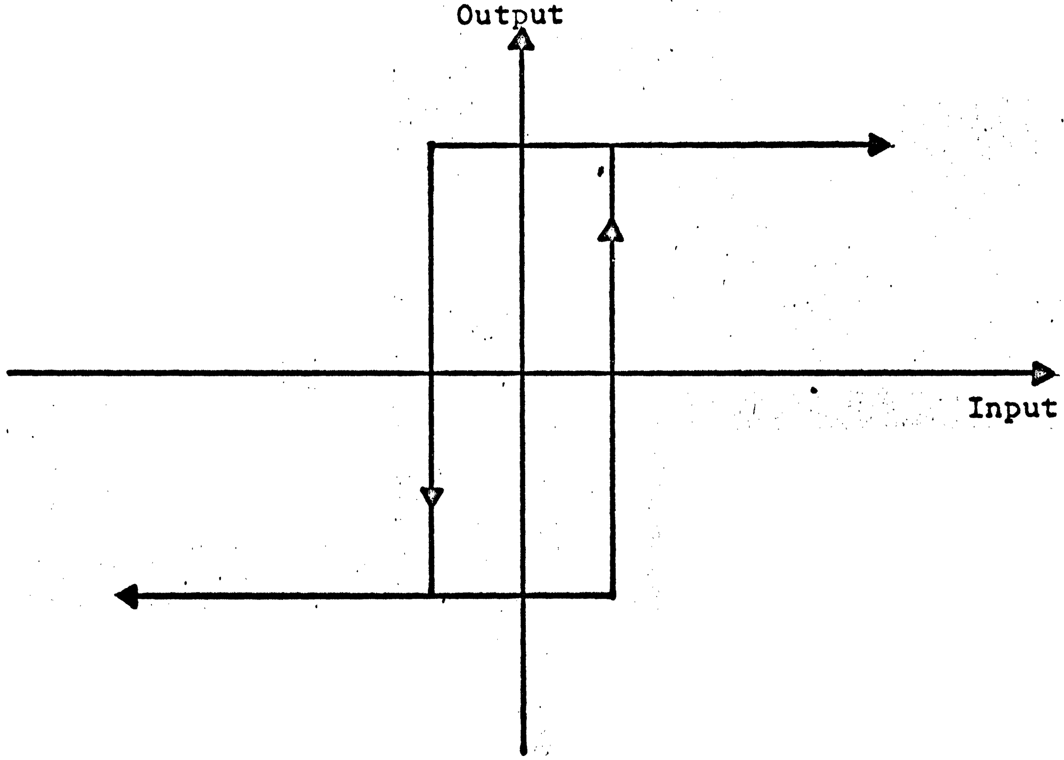


Fig. 1 Transfer Characteristic of a Threshold Detector

The purpose of this thesis is to describe some techniques and circuits which are useful for threshold detection. All the circuits are d.c. coupled; that is, all the circuits presented in this paper are designed to work at arbitrarily low frequencies. Zero crossing detection and feedback control of hysteresis are given special emphasis.

Chapter 1 is a collection of circuits and concepts used in threshold detection. The limiter and threshold detector are introduced and compared. A number of threshold detectors are discussed briefly. Expressions yielding their theoretical static transfer characteristics are included. The differentially driven flip-flop, a circuit which uses preamplification to reduce the hysteresis width, is discussed. Analytical expressions for the transfer characteristic are, again, included. Finally, the concept of feedback control of the hysteresis width is introduced.

Chapter 1 provides a background and extended introduction for chapter 2. Two more sophisticated circuits utilizing negative feedback are presented in chapter 2. The operation of these circuits is discussed in detail, and the theoretical transfer characteristics are calculated. The second circuit presented in the chapter, the tunnel diode stabilized circuit, is extremely versatile and stable. Th-s versatile and stable. This circuit has a hysteresis width which can be varied from 5 volts to less than 1 millivolt. It operates at speeds up to 500 kc/s; improved circuits may operate at speeds greater than 1 mc/s.

1.2 Switching Devices vs. Limiting Devices

A limiter is another device which is commonly used as a zero crossing detector. This device has a transfer characteristic which is similar to a threshold detector except that the hysteresis range is replaced by a region where the limiter acts as an amplifier.

Consider a threshold detector with a hysteresis region of width W which is centered about zero. Let the input signal be A sin ωt. The output of the threshold detector will have a phase lag, θ, where

$$\theta = \sin^{-1}\left(\frac{W}{2A}\right).$$

The switching time of the output of the threshold detector is constant regardless of the amplitude of the input signal.

If the same signal is used as the input to a limiter with an amplification region of width W which is centered about zero; then the output will have a rise time, T_r, where

$$T_r = \frac{2}{W}\sin^{-1}\left(\frac{W}{2A}\right)$$

The limiter does not have any phase shift from the input to the output.

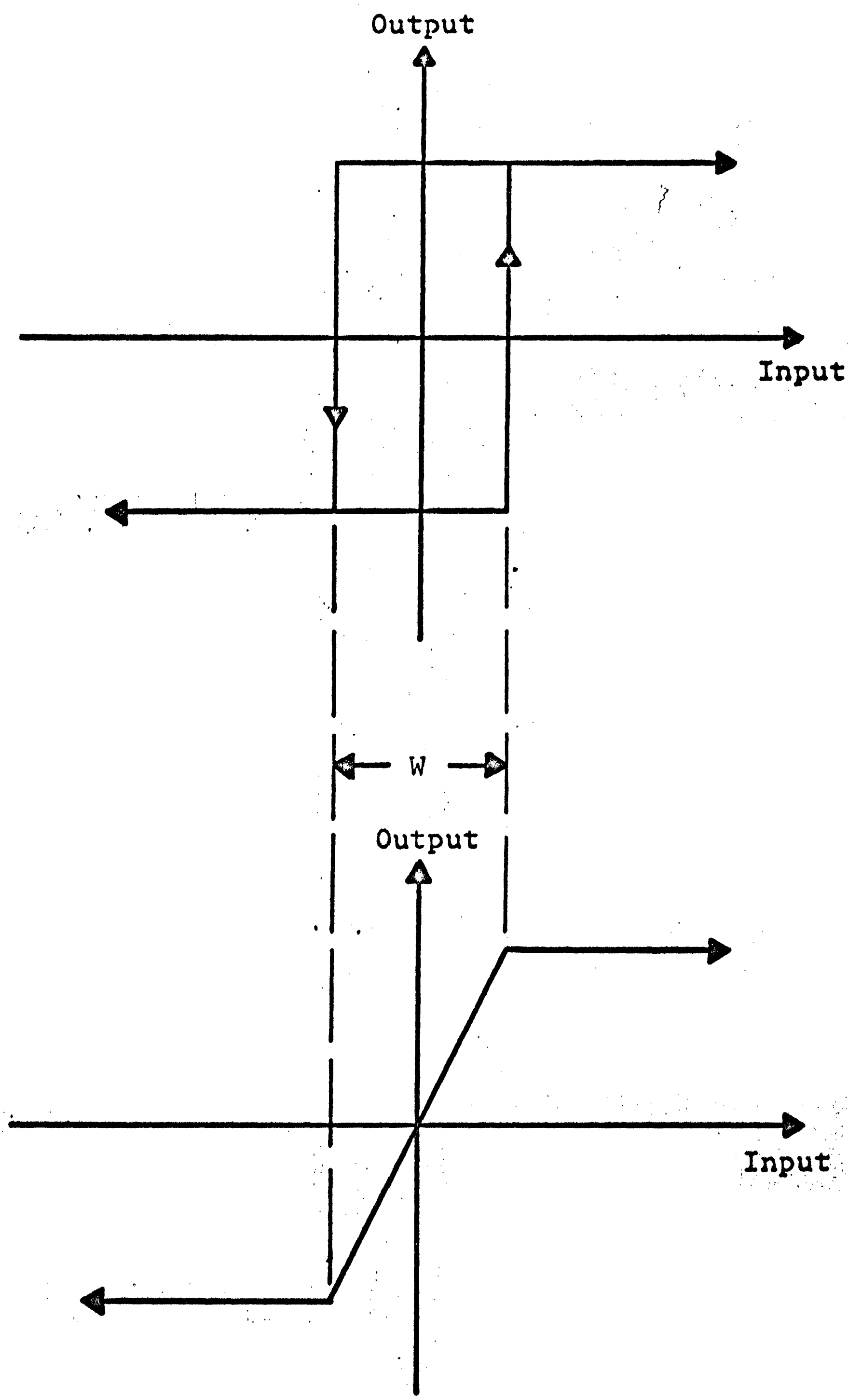


Fig. 2 Threshold Detector vs. Limiter

When the hysteresis width of the threshold detector is set to zero, the device becomes a zero crossing detector. Similarly, when the gain of the limiter in its amplifying region becomes infinite, it becomes a zero crossing detector.

In practice, stable limiters appear to be more difficult to realize than stable threshold detectors. Furthermore, limiters do not have the useful hysteresis region which is the identifying characteristic of threshold detectors. The hysteresis region is an integral part of certain two state modulation systems.[1]

1.3 Basic Threshold Circuits

1.31 The Schmitt Trigger

The Schmitt trigger or emitter coupled multivibrator is probably the most common threshold detector.[2] A typical circuit is pictured in fig. 3. The Schmitt trigger is useful whenever temperature stability is not important. It is usually acceptable when moderate or large hysteresis is required since temperature effects are not important for such operation. If, however, the hysteresis width is very critical, the Schmitt trigger cannot be used unless the temperatures of the transistors are regulated.

The circuit of fig. 3 has the transfer characteristic shown in fig. 4 where:

The transistor model used to obtain the theoretical expressions is discussed in the appendix.

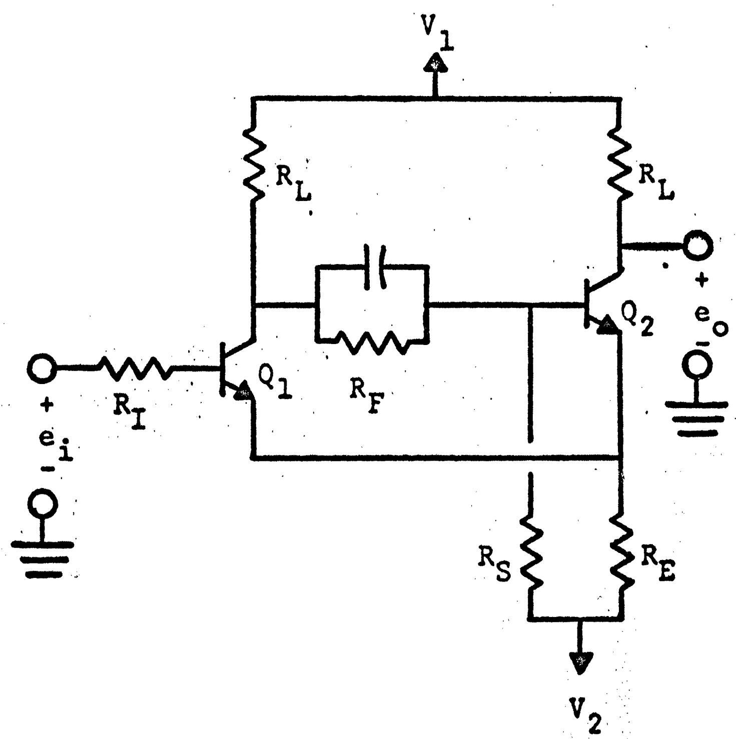


Fig. 3 A Typical Schmitt Trigger

$$e_1 = \frac{V_1\{R_S[(\beta_1+1)R_E+R_I][(\beta_2-1)R_L-R_F] + (\beta_2+1)R_E\beta_1R_LR_S - (\beta_2+1)R_ER_I(R_F+R_L)\}}{D}$$

$$+ \frac{V_2\{(R_F+R_L)[(\beta_1+1)R_E\beta_2R_L + (\beta_2+1)R_ER_I] + \beta_1\beta_2R_L^2R_S - R_IR_S[(\beta_2-1)R_L-R_F]\}}{D}$$

$$+ \frac{V_{BE}\{\beta_1\beta_2R_L^2R_S - \beta_2R_LR_I(R_F+R_L) - R_IR_S[(\beta_2-1)R_L-R_F]\}}{D}$$

$$D = (\beta_1+1)R_ER_S[(\beta_2-1)R_L-R_F] + (\beta_1+1)R_E\beta_2R_L(R_F+R_L) + \beta_1\beta_2R_L^2R_S + (\beta_2+1)R_E\beta_1R_LR_S$$

$$e_2 = V_2 + \frac{(V_1-V_2)[R_IR_S + (\beta_1+1)R_ER_S] + V_{BE}[\beta_1R_LR_S - R_I(R_L+R_F+R_S)]}{\beta_1R_LR_S + (\beta_1+1)R_E(R_L+R_F+R_S)}$$

$$e_3 = V_1$$

$$e_4 = \frac{V_1R_E + V_2R_L}{R_L + R_E}$$

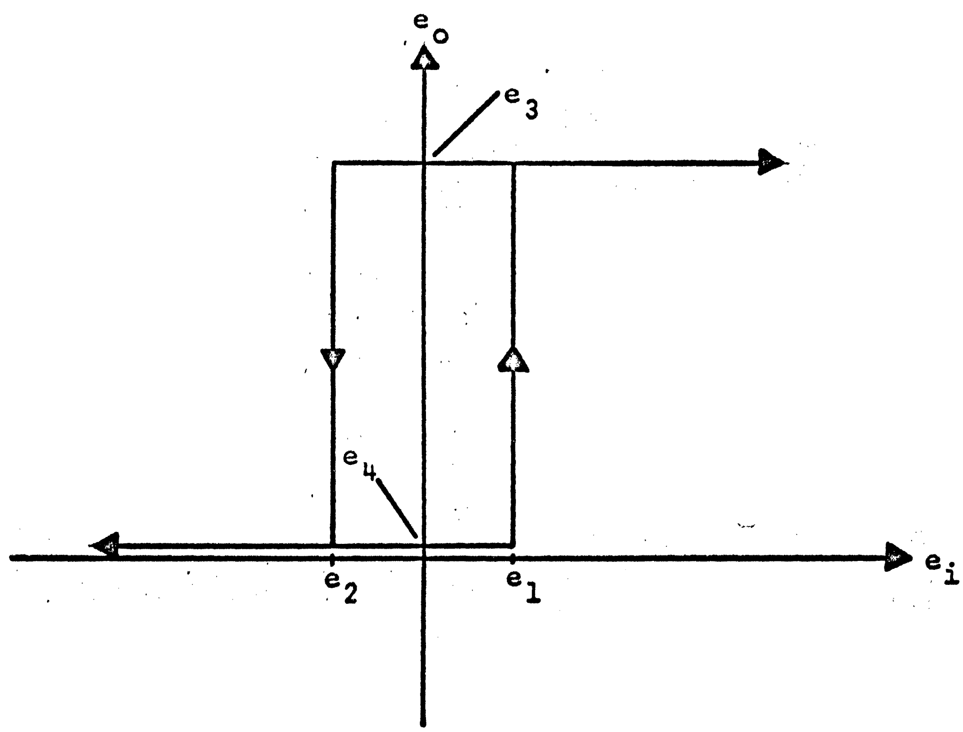


Fig. 4 Transfer Characteristic

If the voltage drop across R_I (due to the base current of Q_1) is small compared to the hysteresis width, e_1-e_2, then R_I can be neglected. Generally, R_I can be eliminated if:

$$R_I << \frac{(\beta_1+1)R_E R_L}{R_E + R_L}$$

If R_I is negligible, then:

$$e_1 = \frac{V_1\{(\beta_1+1)R_E R_S[(\beta_2-1)R_L-R_F] + (\beta_2+1)R_E\beta_1 R_L R_S\}}{D}$$
$$+ \frac{V_2[\beta_1\beta_2 R_L^2 R_S + (\beta_1+1)R_E\beta_2 R_L(R_F+R_L)] + V_{BE}\beta_1\beta_2 R_L^2 R_S}{D}$$

where D is the same as before.

$$e_2 = V_2 + \frac{(V_1-V_2)(\beta_1+1)R_E R_S + V_{BE}\beta_1 R_L R_S}{(\beta_1+1)R_E(R_L+R_F+R_S) + \beta_1 R_L R_S}$$

e_3 and e_4 are unchanged.

If $R_S >> R_F$, then the expressions can be simplified in another way. If R_I is <u>not</u> neglected, the expressions for e_1 and e_2 become:

$$e_1 = \frac{V_1\{[(\beta_1+1)R_E+R_I][(\beta_2-1)R_L-R_F] + \beta_1 R_L(\beta_2+1)R_E\}}{D'}$$
$$+ \frac{(V_2+V_{BE})\{\beta_1\beta_2 R_L^2 - R_I[(\beta_2-1)R_L-R_F]\}}{D'}$$

where

$$D' = (\beta_1+1)R_E[(\beta_2-1)R_L-R_F] + \beta_1\beta_2R_L^2 + \beta_1R_L(\beta_2+1)R_E$$

$$e_2 = \frac{V_1[(\beta_1+1)R_E+R_I] + (V_2+V_{BE})[\beta_1R_L-R_I]}{(\beta_1+1)R_E + \beta_1R_L}$$

e_3 and e_4 are, again, unchanged.

The expressions, above, are correct if the equilibrium states of Q_2 are the cut off state and the saturated state.

The temperature dependence of this threshold detector is introduced through $\beta_1(T)$, $\beta_2(T)$, and $V_{BE}(T)$. β_1 and β_2 are the current gains of the transistors Q_1 and Q_2 respectively. V_{BE} is the base to emitter voltage of the transistors when they are active or saturated. V_{BE} has been approximated as a constant; it is in reality a logarithmic function of the emitter current.(3) For a fixed emitter current,

$$\frac{dV_{BE}}{dT} = -\frac{0.65}{T} \text{ mv./°C for T in °K.}$$

For T = 300 °K,

$$\frac{dV_{BE}}{dT} = -2 \text{ mv./°C.}$$

Another problem associated with the Schmitt trigger is the input impedance. the input impedance is not constant; it depends on the state of the circuit. If Q_1 is off, the input impedance is very large; if Q_1 is saturated, the input impedance is much lower. If the impedance of the source which is driving the Schmitt trigger is not sufficiently low, then the varying input impedance may cause a d.c. level shift at the input. This, of course, results in a shift of the thresholds.

1.32 Flip-Flops

Another circuit which can be used as a threshold detector is the flip-flop. Fig. 5 illustrates a typical circuit. Using the notation of fig. 4, the transfer characteristic of the flip-flop is:

$$e_1 = V_2 + V_{BE} + (V_1-V_2-V_{BE})\,\frac{R_E[(2\beta_1\beta_2+\beta_2-1)R_L-(\beta_1+1)R_F] + R_I[(\beta_2-1)R_L-R_F]}{R_E[(2\beta_1\beta_2+\beta_2-1)R_L-(\beta_1-1)R_F] + \beta_1\beta_2R_L^2}$$

$$e_2 = V_2 + V_{BE} + (V_1-V_2-V_{BE})\,\frac{R_E[(\beta_1+1)(R_L+R_F)] - R_I[(\beta_1-1)R_L-R_F]}{R_E[(\beta_1+1)(R_L+R_F)] + \beta_1R_L(R_L+R_F)}$$

$$e_3 = V_1$$

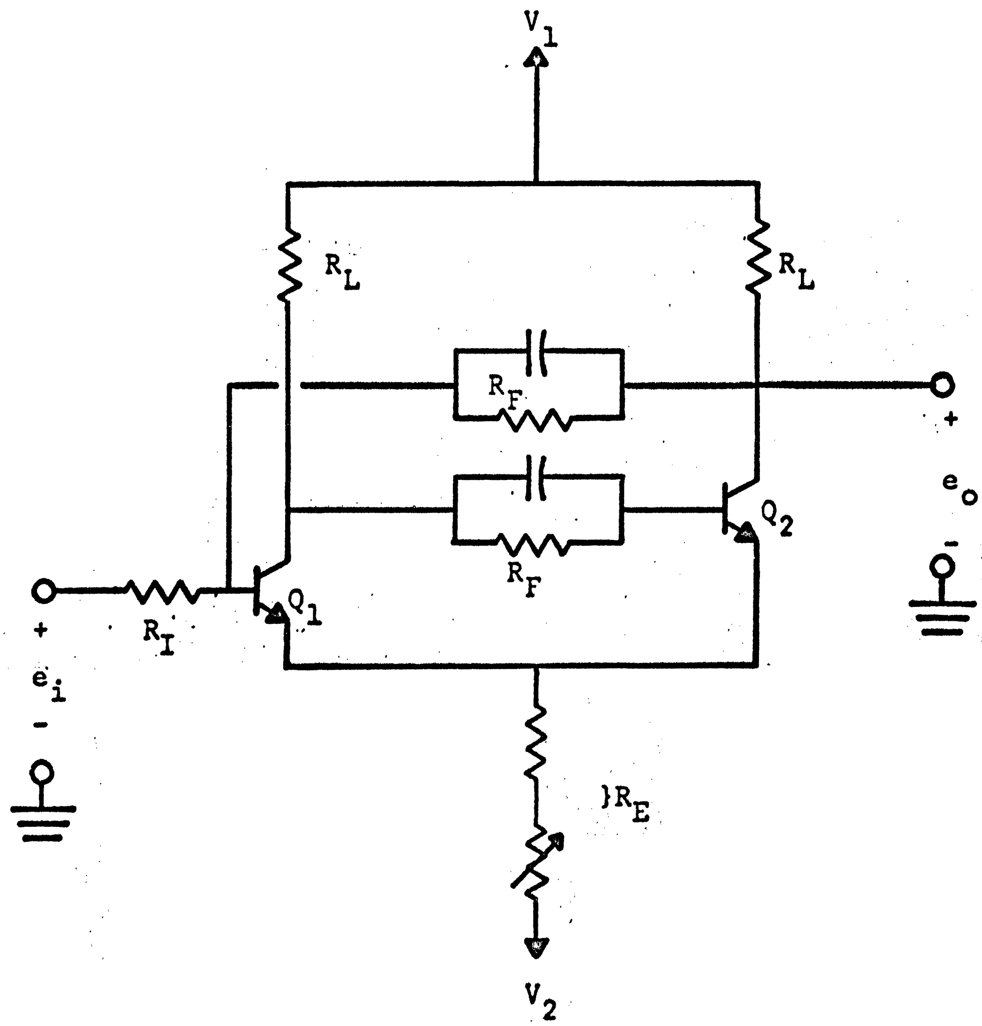


Fig. 5 A Typical Flip-Flop

$$e_4 \approx \frac{V_1 R_E R_I + V_2 R_L R_I + \beta_2 e_i R_E R_L}{R_E R_I + R_L R_I + R_E R_L} \qquad e_i > e_4$$

$$e_4 \approx \frac{V_1 R_E + V_2 R_L}{R_E + R_L} \qquad e_i < e_4$$

The expressions, above, are correct providing that the circuit is adjusted so that the equilibrium states of the transistor, Q_2, are the cut off state and the saturated state.

In fig. 5, a resistor running from the base of Q_2 to the lower supply, V_2, is necessary if germanium transistors are used. The resistor eliminates the problems caused by the leakage current flowing from the collector to the base of Q_2. If silicon transistors are used, the resistor is not needed.

The temperature dependence of this threshold detector is, again, introduced through β_1, β_2, and V_{BE}.

It is interesting to note at this point that R_E can have a temperature stabilizing effect on the threshold voltages. To illustrate this fact, let

(1) $\beta_1 = \beta_2 = \beta >> 1 \rightarrow \beta+1 \approx \beta$

(2) $R_E[(\beta+1)(R_L+R_F)] >> R_I[(\beta-1)R_L-R_F]$

(3) $R_E = R_L$

Then;

$$e_2 \approx \frac{1}{2}[V_1 + V_2 + V_{BE}(T)]$$

Since $V_{BE}(T)$ has the approximate temperature dependence of a diode, the effect of $V_{BE}(T)$ can be minimized by compensating V_1 and V_2 with diodes.

Unfortunately, e_1 is dependent on temperature in spite of R_E. Nevertheless, R_E does have a stabilizing effect.

Often the flip-flop circuit is used without an emitter resistor ($R_E = 0$). Such a circuit is useful because the emitters can be grounded so that the base potentials are only slightly different from ground potential. With $R_E = 0$ and $V_2 = 0$, the transfer characteristic of the flip-flop becomes, using the notation of fig. 4:

$$e_1 = V_{BE} + (V_1 - V_{BE})\,\frac{R_I[(\beta_1 - 1)R_L - R_F]}{\beta_1\beta_2 R_L^2}\,.$$

$$e_2 = V_{BE} - (V_1 - V_{BE})\,\frac{R_I[(\beta_1 - 1)R_L - R_F]}{\beta_1 R_L(R_L + R_F)}$$

$$e_3 = \frac{V_1 R_F + V_{BE} R_L}{R_L + R_F}$$

$$e_4 = V_{CE}(\text{sat.})$$

V_{CE}(sat.) is the collector to emitter voltage of a transistor when it is saturated. V_{CE}(sat.) is a function of the collector and base currents.

With $R_E = 0$, the flip-flop is very useful in constructing symmetric complementary circuits. It is an important "building block".

Like the Schmitt trigger, the flip-flop has a switching input impedance. When Q_1 is active or saturated, the input impedance is R_I ($R_E = 0$); when

Q_1 is off, the input impedance is infinite. When $R_E \neq 0$, the fluctuation of the input impedance is reduced.

1.33 Symmetric Threshold Detectors Using Complementary Flip-Flops.

Using complementary flip-flops as building blocks, a number of symmetric threshold detectors may be constructed. A typical circuit is illustrated in fig. 6. The advantage of this circuit is that the d.c. drift of the transfer characteristic will be very small if complementary transistors are used. Temperature fluctuations will not cause a d.c. offset in this device. However, a temperature change may cause the hysteresis width to vary. Another advantage of the circuit is that the input impedance is approximately constant.

Using the notation of fig. 7, the transfer characteristic of the device is given by:

$$e_1 = (V-V_{BE})\frac{R_I}{R_{L2}+R_{F2}}\cdot\frac{\beta_{1P}\beta_{2N}R_{L1}R_{L2}-(R_{L1}+R_{F1})(R_{L2}+R_{F2})}{\beta_{1P}\beta_{2N}R_{L1}R_{L2}}$$

$$- V_{BE}[1 + \frac{2R_I}{\beta_{1P}R_{L1}}\cdot\frac{(\beta_{1P}\beta_{2N}+1)R_{L1} + \beta_{2N}R_{F2} - R_{F1}}{\beta_{2N}R_{F2}}]$$

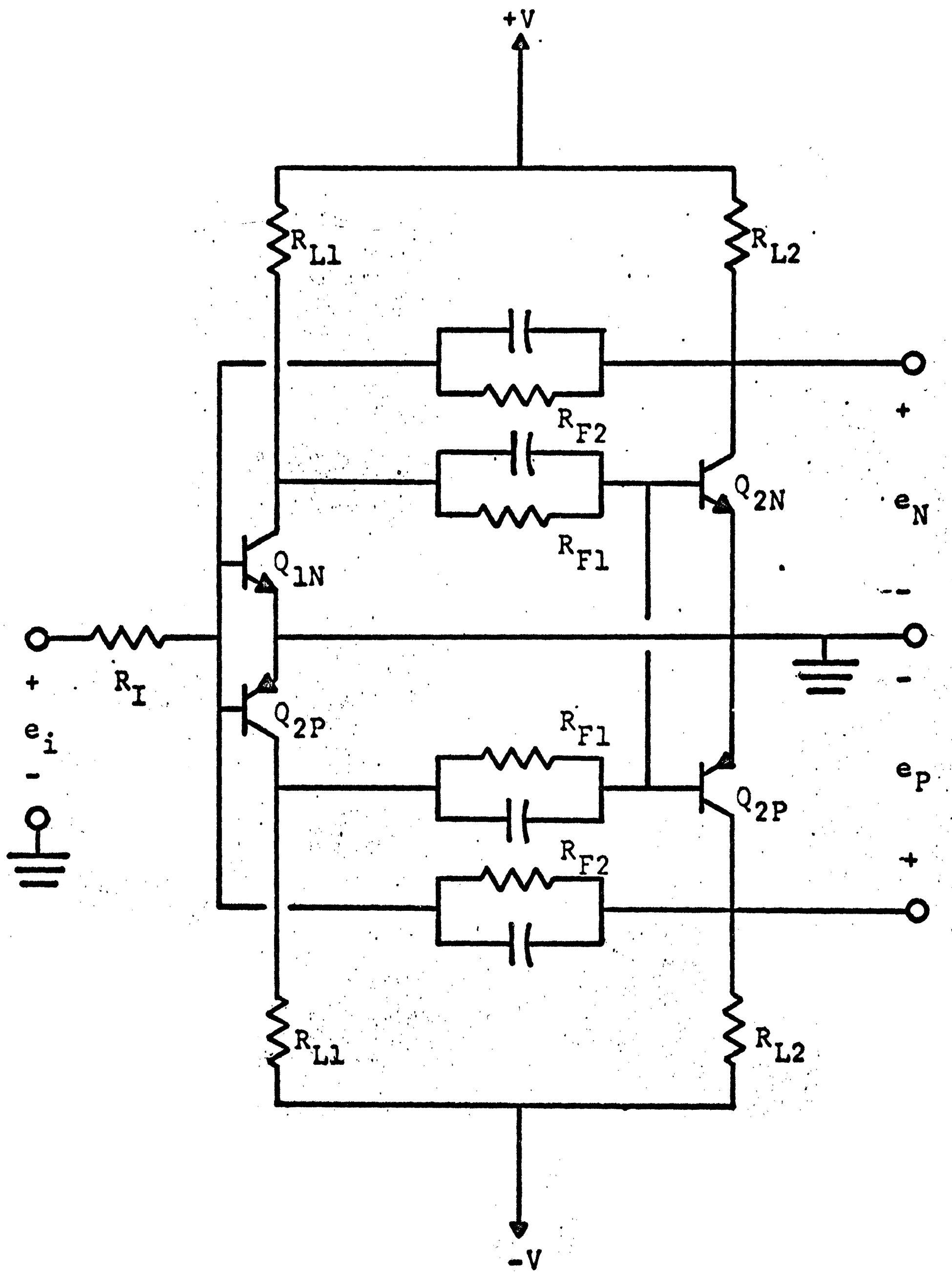


Fig. 6 A Symmetric Threshold Detector

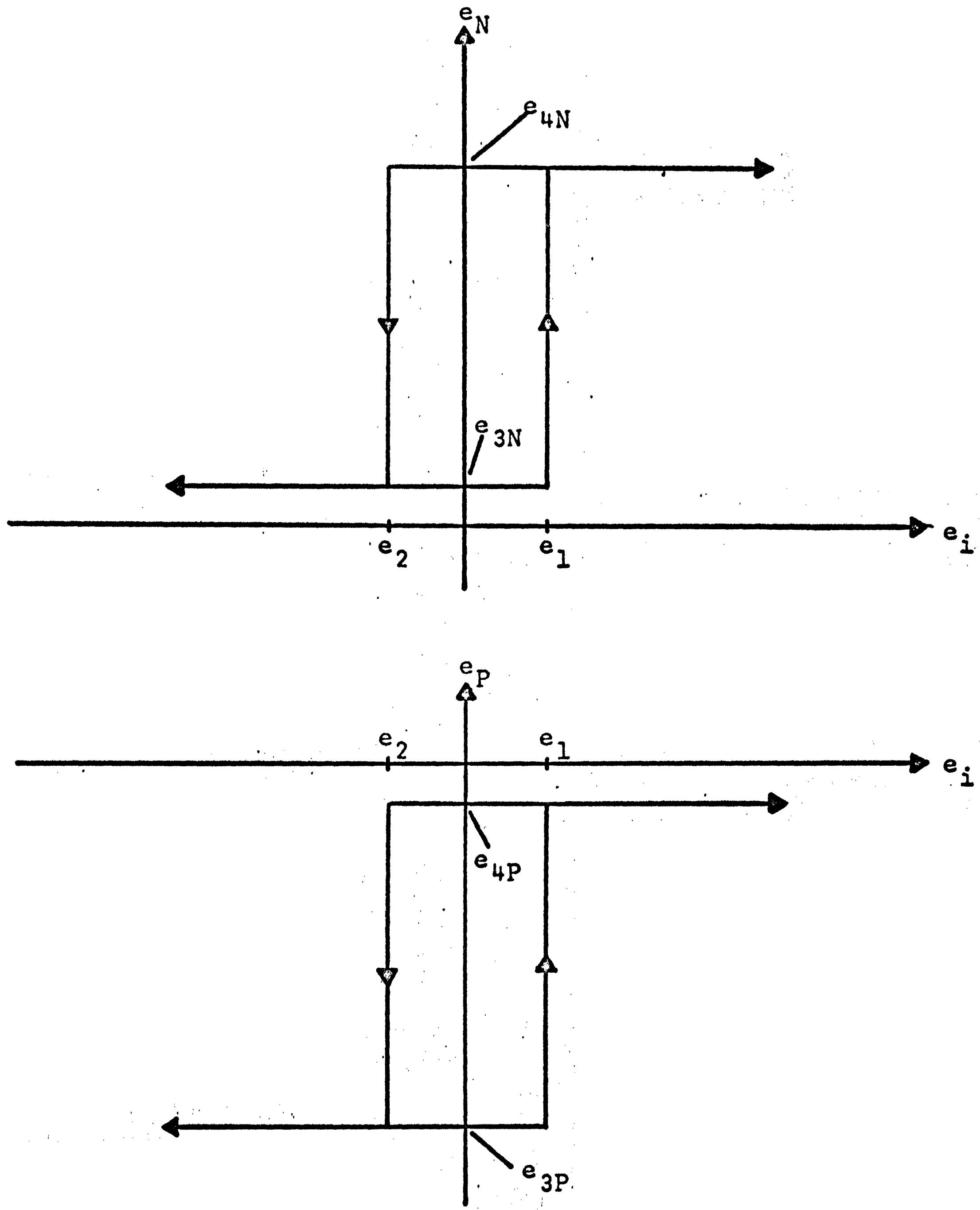


Fig. 7 Transfer Characteristic

$$e_2 = -(V-V_{BE})\frac{R_I}{R_{L2}+R_{F2}}\cdot\frac{\beta_{1N}\beta_{2P}R_{L1}R_{L2}-(R_{L1}+R_{F1})(R_{L2}+R_{F2})}{\beta_{1N}\beta_{2P}R_{L1}R_{L2}}$$

$$+ V_{BE}[1 + \frac{2R_I}{\beta_{1N}R_{L1}}\cdot\frac{(\beta_{1N}\beta_{2P}+1)R_{L1} + \beta_{2P}R_{F2} - R_{F1}}{\beta_{2P}R_{F2}}]$$

$$e_{3N} \doteq V_{CE}(sat.)$$

$$e_{4N} = \frac{VR_{F2} - V_{BE}R_{L2}}{R_{F2} + R_{L2}}$$

$$e_{3P} = -(\frac{VR_{F2} - V_{BE}R_{L2}}{R_{F2} + R_{L2}})$$

$$e_{4P} = - V_{CE}(sat.)$$

The expressions, above, hold if the output transistors are either cut off or saturated when the circuit is in equilibrium.

Switching occurs as follows in the circuit of fig. 6. Let e_i be positive and decreasing. Initially, Q_{1N} and Q_{2P} are saturated; Q_{2N} and Q_{1P} are off. As e_i decreases:

(1) Q_{1N} becomes active.

(2) The magnitude of the collector voltage of Q_{1N} increases.

(3) Because the magnitude of the collector voltage of Q_{1N} is rising, the magnitude of the base current of Q_{2P} decreases.

(4) Q_{2P} comes out of saturation and becomes active.

(5) Regenerative switching occurs between Q_{1N} and Q_{2P}.

(6) Q_{1N} and Q_{2P} turn off.

(7) The by-pass capacitors cause Q_{1P} and Q_{2N} to become active.

(8) Regenerative switching occurs between Q_{1P} and Q_{2N}.

(9) Q_{1P} and Q_{2N} saturate. The switching is complete.

The symmetric device has three stable equilibrium states. Two of the states, the desired states, have two transistors saturated and two off. The third state has all the transistors off. This "off" state is undesirable. A relatively large swing in the input voltage may be required before the circuit "escapes" from this state. Fortunately, by-pass capacitors can be used to prevent the "off" state from occurring. When two transistors turn off, the capacitors bring the other two transistors into their active regions, and they eventually saturate.

In practice, the "off" state occurs when the circuit is energized by raising the supply voltages symmetrically. Once the circuit "escapes" from the state, it will not recur if the capacitors are large enough so that the two transistors which are initially off become active before the other transistors turn off. This is verified in practice; when the capacitances are increased beyond a certain critical value, the "off" state does not recur.

1.34 The Tunnel Diode Pair

Another basic threshold detecting circuit, built around the tunnel diode pair is illustrated in fig. 8.[4] The tunnel diodes are used to obtain an i-v characteristic with a negative resistance region which is symmetric about the origin. The analysis of this circuit is done graphically; it may be found in the reference.

The hysteresis of the tunnel diode pair can be made as small as 1 mv. However, the hysteresis varies due to temperature fluctuations. The circuit may oscillate; or, it may switch to an extraneous stable point near the origin. Furthermore, the source impedance is very critical to the operation of the device. The hysteresis is directly dependent on this impedance.

In order to avoid confusion later, it is necessary to explain why this device is not stable for fluctuating temperature. The non-linear i-v characteristic created by the tunnel diodes depends not only on the stable peak regions of the tunnel diodes, but also on the unstable valley regions. The i-v characteristic fluctuates with temperature changes because of its dependence on these valley regions.

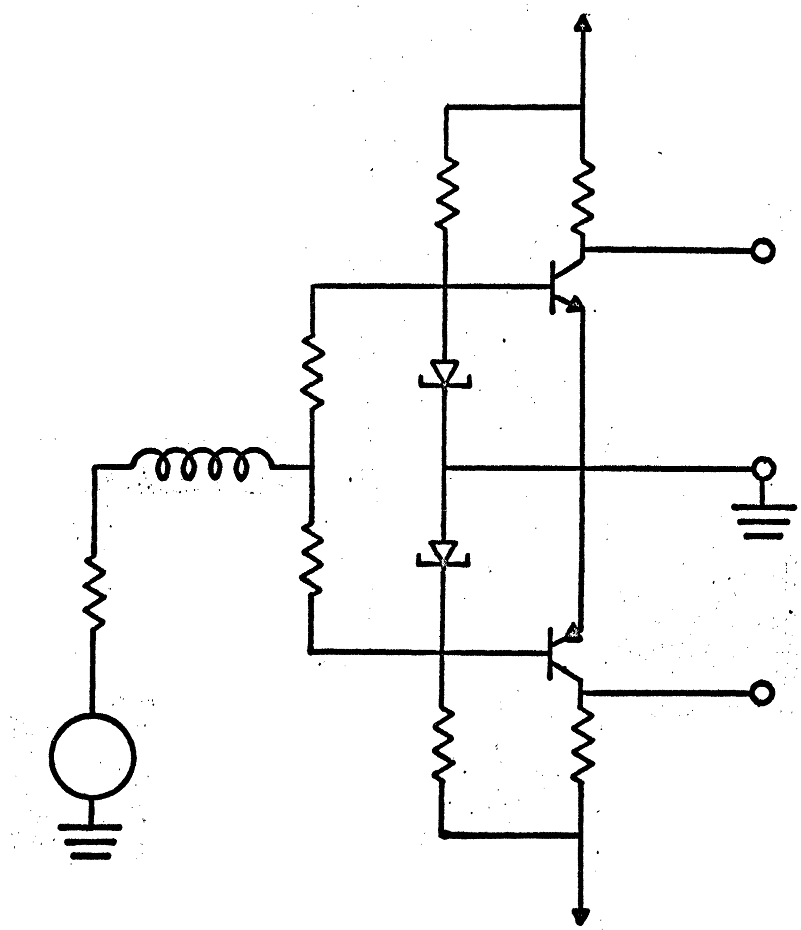


Fig. 8 The Tunnel Diode Pair

1.4 General Techniques for Reducing or Adjusting the Hysteresis

1.41 Preamplification

Preamplification is an obvious technique for hysteresis reduction. However, the design of a simple d.c. preamplifier is not so obvious. One possible circuit is the differentially driven flip-flop illustrated in fig.9.

The differentially driven flip-flop is a simple circuit. It is also inexpensive because complementary transistors are not required. However, if matched transistors in one package are used, the cost increases.

In order to analyze this circuit, the Thevenin equivalent of the differential amplifier is used. (Cf. fig. 10). The collector voltages of the transistors in the differential amplifier are modeled as a quiescent voltage, E_S, and an a.c. voltage $+e_s$ or $-e_s$. The drive resistance to the base of Q_1 and Q_2 is R_S. If the differential amplifier has some voltage gain, A, then $e_s = Ae_i$.

The circuit of figs. 9 & 10 has two distinct modes of operation. Assume that initially Q_1 is cut off and Q_2 is saturated. Switching may occur in two ways: If Q_1 becomes active before Q_2, then switching will be initiated when Q_2 becomes active. If Q_2 becomes

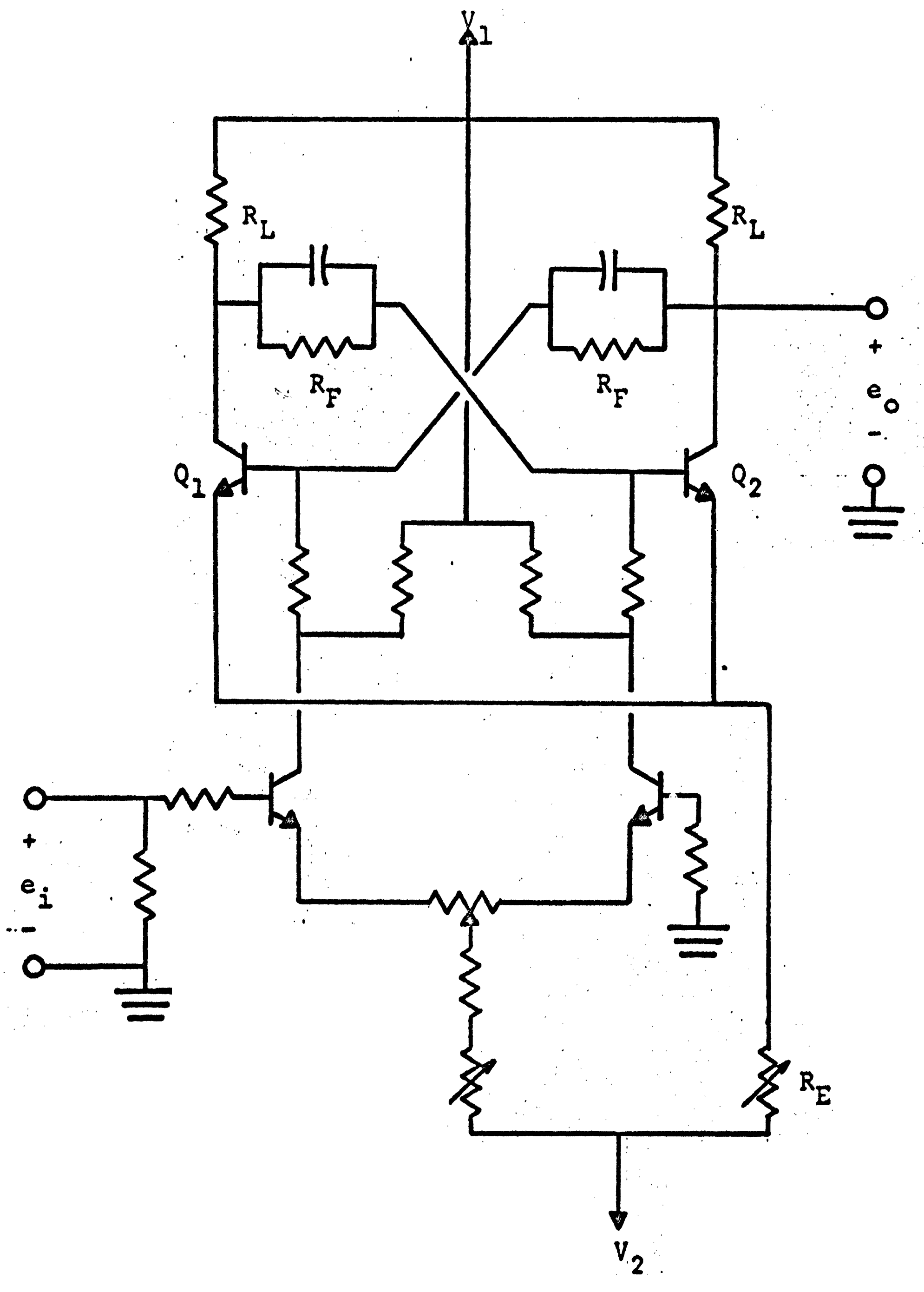


Fig. 9 A Differentially Driven Flip-Flop

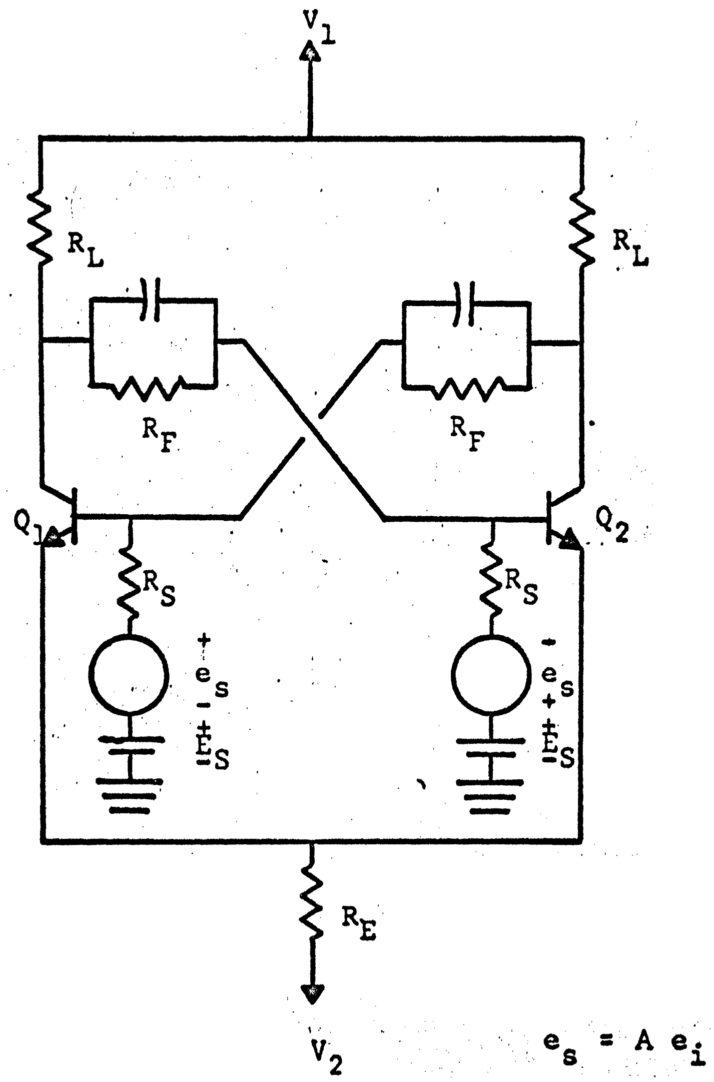


Fig. 10 Equivalent Circuit

active before Q_1, then switching will begin when Q_1 becomes active. In the first mode, the previously saturated transistor initiates switching. In the second mode, the previously cut off transistor initiates switching. The second mode is preferable because each switching threshold depends on the current gain of only one of the transistors.

Assuming that switching is initiated when the cut off transistor becomes active, the transfer characteristic is given by (Cf. fig. 11):

$$e_1 = \frac{[(V_1-V_2-V_{BE})R_S + (E_S-V_{BE}-V_2)(R_L+R_F)][(\beta_1-1)R_L-R_F]R_S}{A\{[2(\beta_1+1)R_E+(\beta_1+1)R_L+R_F]R_S+2(\beta_1+1)R_E(R_L+R_F)\}(R_L+R_F)}$$

$$e_2 = -\frac{[(V_1-V_2-V_{BE})R_S + (E_S-V_{BE}-V_2)(R_L+R_F)][(\beta_2-1)R_L-R_F]R_S}{A\{[2(\beta_2+1)R_E+(\beta_2+1)R_L+R_F]R_S+2(\beta_2+1)R_E(R_L+R_F)\}(R_L+R_F)}$$

$$e_3 \approx \frac{V_1R_E + V_2R_L}{R_E + R_L}$$

$$e_4 \approx V_1$$

The transfer characteristic depends on the current gains of the transistors. Thus, temperature instability is introduced through the current gains.

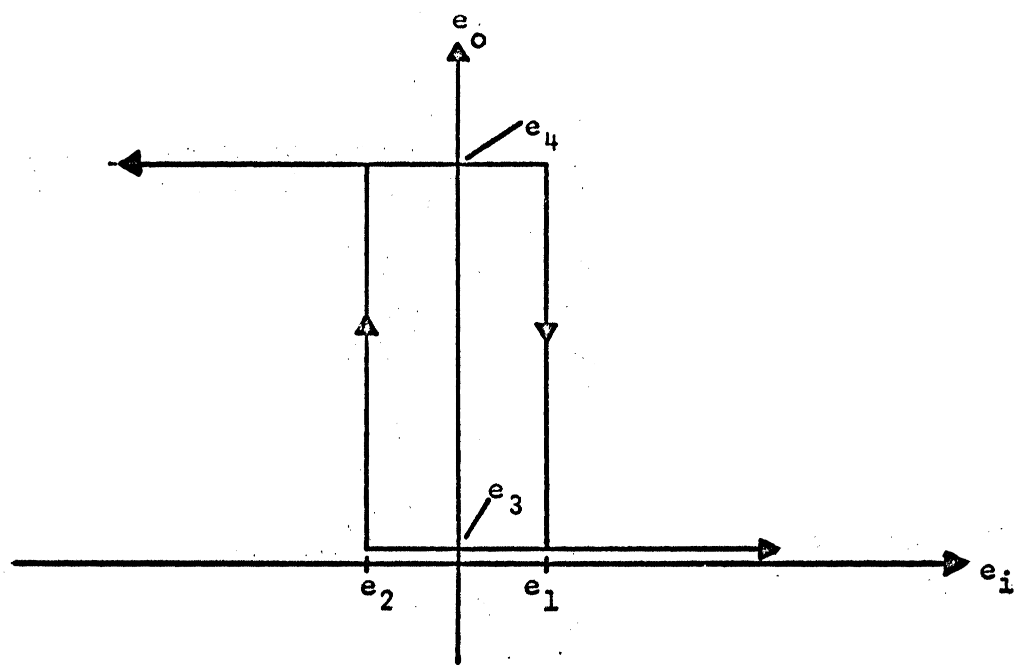


Fig. 11 Transfer Characteristic

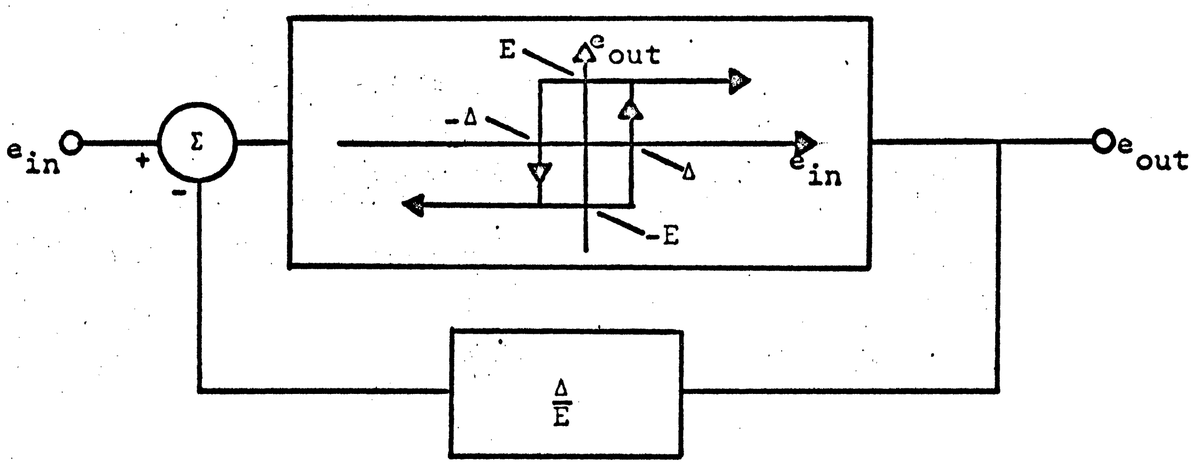
However, the gain of the differential amplifier decreases the hysteresis width and fluctuations by the same amount. So, the differential amplifier does not magnify the fluctuations relative to the hysteresis width. If the hysteresis is stable without the differential amplifier, it will be stable with the differential amplifier assuming, of course, that the differential amplifier is stable.

It is difficult to obtain a wide bandwidth with the differentially driven flip-flop and simultaneously maintain d.c. stability. In order to realize a better bandwidth, it is necessary to use higher quiescent currents; but, higher currents result in greater d.c. drift.

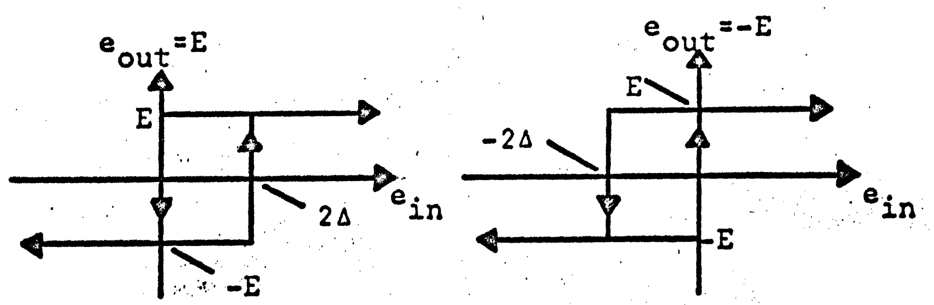
1.42 Feedback Control of the Hysteresis

Negative feedback reduces the hysteresis of a symmetric bistable device; positive feedback increases the hysteresis. Clearly, a threshold detector with fixed, stable hysteresis can have a variable effective hysteresis when feedback is used.

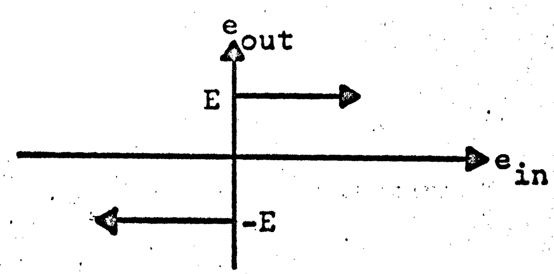
With sufficient negative feedback, a symmetric threshold detector can become a zero crossing detector. The effect of the feedback in this case is pictured in fig. 12. The feedback voltage shifts the fixed transfer characteristic along the input voltage axis. In the case of zero hysteresis, the threshold voltage required to cause the device to switch from the existing state to



Block Diagram of Zero Crossing Detector



Shifted Transfer Characteristics



Effective Transfer Characteristic

Fig. 12 Creating a Zero Crossing Detector with Negative Feedback the other state is always shifted so that it is positioned at zero input voltage.

The stability of a zero crossing detector, operating with negative feedback, depends, critically, on the stability of the hysteresis which exists initially without feedback. Negative feedback reduces the effective hysteresis; but, feedback has no effect on the fluctuations of the hysteresis. Thus, a small percent fluctuation of the hysteresis existing without feedback can cause drastic percent changes in the hysteresis of the zero crossing detector.

The problem of realizing a zero crossing detector by the feedback method really involves the design of a stable threshold detector. If a stable threshold detector can be constructed, then, when negative feedback is added, the smaller effective hysteresis will be stable. Some devices utilizing feedback to control their hysteresis will be discussed in chapter 2.

CHAPTER II

Feedback Control of Hysteresis

2.1 Introduction

Feedback control of hysteresis has been introduced in chapter 1. A number of devices working on the feedback principle have been constructed in the laboratory. Two devices will be considered here: a complementary latching flip-flop circuit, and a complementary latching tunnel diode circuit.

2.2 A Complementary Latching Flip-Flop Circuit

The schematic diagram of a practical complementary latching flip-flop circuit is shown in fig. 13. This device is designed to have a stable hysteresis which is adjustable from .1 volt to 5 volts. Although the circuit is not intended for use as a zero crossing detector, the hysteresis can be as small as 1 mv. However, the potentiometer settings become very critical at this level because this particular circuit is designed for higher levels. Operation of this or similar circuits at mv. levels is not recommended unless the ambient temperature is fairly constant because the hysteresis depends, partially, on the V_{BE}'s and β's of the transistors.

The emitter follower driver transistors are necessary in order to obtain enough drive for the output transistors. The device is capable of switching up to 1 amp. at 200 kc/s. or lower.

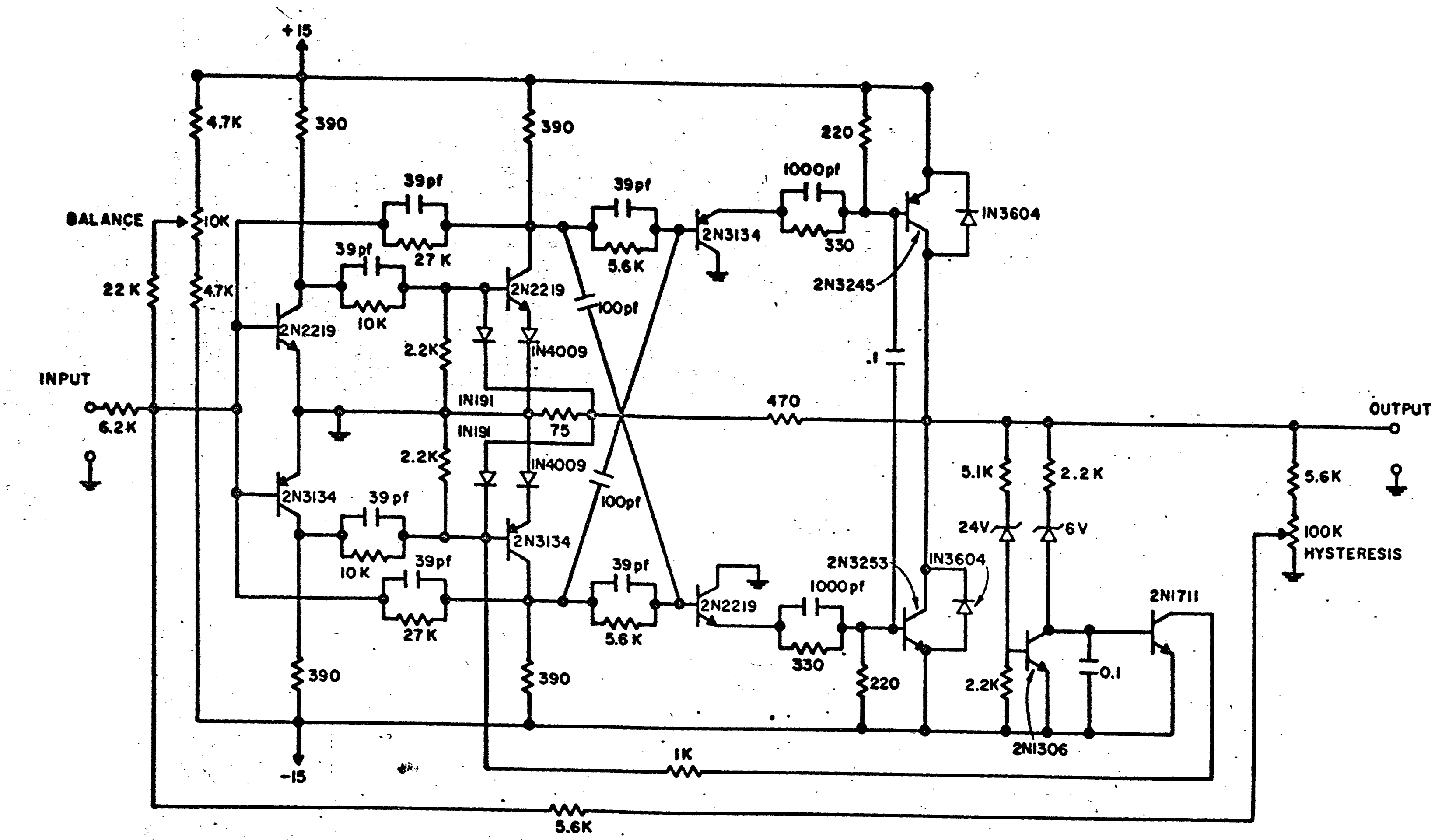


Fig. 13 An Experimental Complementary Latching Flip-Flop Circuit

The operation of the complementary latching flip-flop is indicated in the block diagram of fig. 14.

The output circuit has only one input on at a time; the other input is off. The output state determines which input is on. For example, when the output is positive, I_1 is on and I_2 is off. When the output is negative, I_2 is on and I_1 is off.

The output circuit serves two purposes. It selects two of the four thresholds present in the input circuits. This results in improved stability and reliability because otherwise the flip-flops would rely depend on a.c. coupling to achieve, in effect, two states. Furthermore, switching can be initiated at the output only by turning off the transistor which is initially on. Thus, the output transistors cannot be on at the same time.

The starting circuit is necessary to eliminate a state in which all the transistors are off. Theoretically, the starting circuit is only necessary when power is initially applied to the circuit. The starting circuit pulls the main circuit out of the "off" mode. Thereafter, the a.c. coupling in the output circuit should prevent any recurrence of the "off" mode. However, in practice, the a.c. coupling may fail occasionally causing the main circuit to switch to the "off" mode. The starting circuit is designed to detect when such a failure occurs and to automatically re-start the circuit.

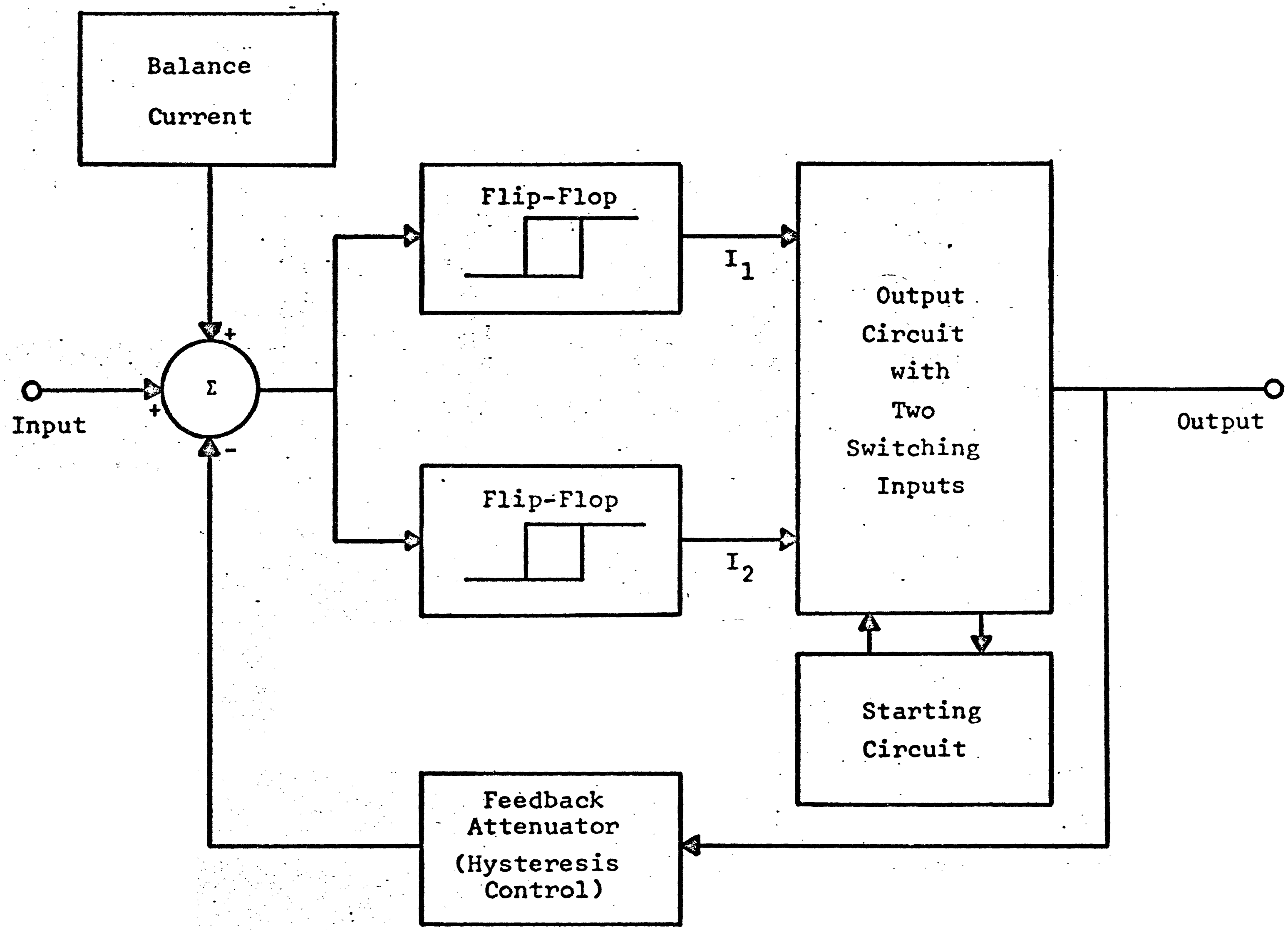


Fig. 14 Block Diagram of the Complementary Latching Flip-Flop Circuit

Referring to fig. 15, the switching of the main circuit normally occurs in the following sequence. Assume that the input signal is large, positive, and decreasing. Initially Q_1, Q_4, and Q_8 are saturated. Q_2, Q_3, Q_5, Q_7, and D_1 are off. Q_6 and D_2 are on. As the input signal decreases:

(1) Q_1 becomes active and eventually turns off. This does not cause the other transistors to switch because Q_3 is held off by the feedback from the output.

(2) Q_2 becomes active.

(3) The magnitude of the collector voltage of Q_2 decreases.

(4) Because the magnitude of the voltage at the collector of Q_2 is decreasing, the magnitude of the base current of Q_4 decreases.

(5) Q_4 becomes active.

(6) Regenerative switching occurs between Q_2 and Q_4.

(7) Q_2 turns on; Q_4 turns off.

(8) Q_6 turns off. Q_5 is pulled on by the a.c. coupling from the collector of Q_4.

(9) Q_8 turns off, and Q_7 turns on. The output voltage switches. At this point Q_5 and Q_7 are held on by the capacitors alone.

(10) The feedback from the output turns D_2 off. D_1 turns on so that Q_4 is held off.

(11) Immediately after D_2 turns off, Q_3 becomes active and then quickly saturates because of the current coming from Q_1 through R_D. (Q_1 is still off.)

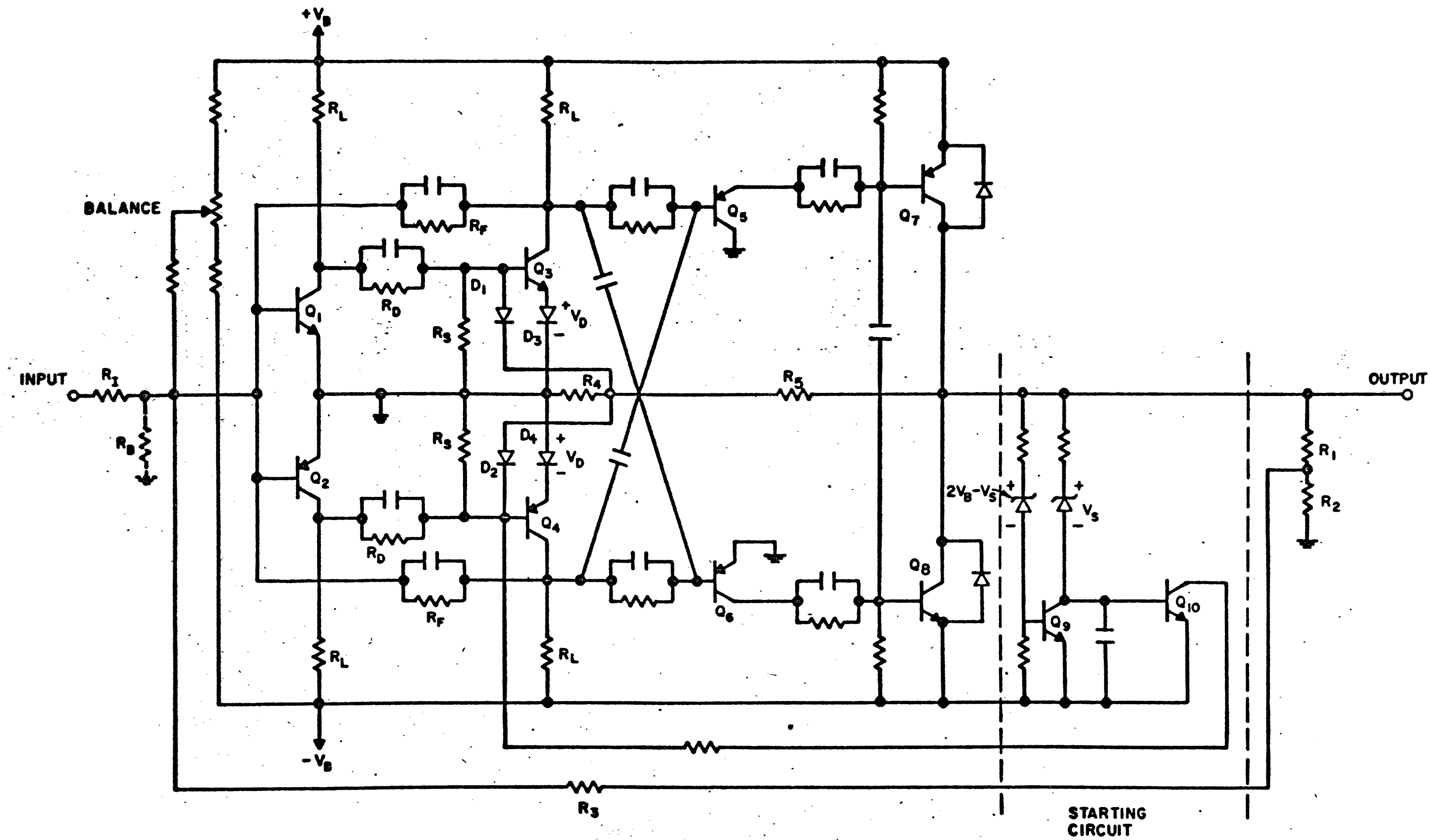


Fig. 15 A Complementary Latching Flip-Flop Circuit

(12) Q_3 holds Q_5 on.

(13) Q_5 holds Q_7 in saturation. The switching is complete. Switching in the other direction follows by symmetry.

The starting circuit affects the main circuit only when the main circuit is in the undesirable "off" mode; i.e., when all the transistors in the output circuit are off simultaneously. When the main circuit is in the off mode and when the input signals are small, Q_1 through Q_8 are off, and D_1 and D_2 are on. (Cf. fig. 15 for notation.) Starting occurs in the following manner:

(1) Q_{10} becomes active and eventually saturates. ($V_S < V_B$)

(2) Q_4 is pulled on by Q_{10}.

(3) Q_4 turns Q_6 on.

(4) Q_5 drives Q_7 on and into saturation so that the output switches to the positive state. The output is now in an allowed state.

(5) Q_{10} becomes active and then turns off. The starting circuit no longer affects the main circuit.

If the input signal is large positive or negative, then the main circuit may start without the starting circuit depending on the size of R_D and R_S relative to the feedback resistors R_4 and R_5. If the main circuit does not start automatically, then the starting circuit will start it although the switching sequence is not the same as it is with small input signals.

When the output is in the positive state, Q_9 is on. Q_9 shunts the base of Q_{10}; so that Q_{10} is off. When the output is in the negative state, Q_9 and Q_{10} are both off. Thus, the starting circuit does not affect the main circuit when the output is in either the positive or the negative state.

The diodes, D_3 and D_4, are used to lift Q_3 and Q_4 slightly above ground potential. These diodes inconjunction with D_1 and D_2 give short circuit protection at the output when the resistors are properly chosen. If the output is shorted to ground, then Q_3 and Q_4 are both held off by the diodes, D_1 and D_2. The diodes at the output protect the circuit from inductive loads.

The static hysteresis of the complementary latching flip-flop is calculated with the aid of figs. 15 & 16. The resistive balance circuit has been replaced by a resistor, R_B. The switching thresholds occur when Q_3 or Q_4 is at the breakpoint between the off and the active state. If the output is initially in the positive state, switching will occur when:

$$\frac{V_B - e_c}{R_L} - \beta_1 i_{b1} - \frac{e_c - V_{BE} - V_D}{R_D} = 0$$

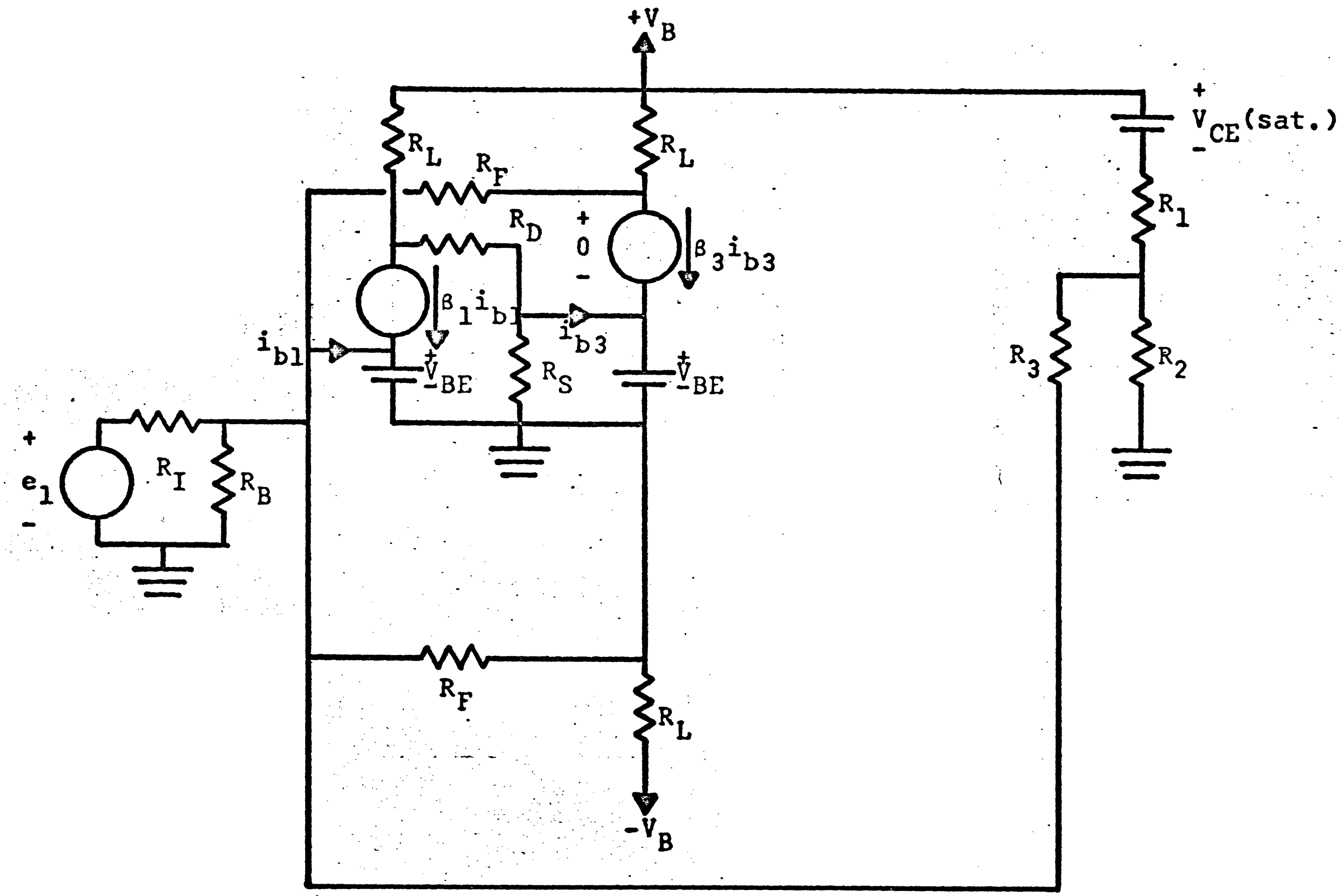


Fig. 16 Equivalent Circuit When Negative Switching Is Initiated

$$i_{b1} = \frac{V_D}{R_F} + \frac{V_{BE} - V_B}{R_F + R_L} - \frac{V_{BE}}{R_B} + \frac{e_1 - V_{BE}}{R_I}$$

$$+ (V_B - V_{CE}(sat.) - V_{BE}) \frac{R_2}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$\beta_3 i_{b3} = \frac{V_B - V_{BE} - V_D}{R_L}$$

$$i_{b3} = \frac{e_c - V_{BE} - V_D}{R_D} - \frac{V_{BE} + V_D}{R_S}$$

Solving for e_1:

$$e_1 \equiv V_B R_I \left\{ \frac{[(\beta_3 - 1)R_L - R_D]}{\beta_1 \beta_3 R_L^2} + \frac{1}{R_L + R_F} - \frac{R_2}{R_1R_2 + R_1R_3 + R_2R_3} \right\}$$

$$-(V_{BE} + V_D) R_I \left\{ \frac{[(\beta_3 - 1)R_L - R_D]}{\beta_1 \beta_3 R_L^2} + \frac{R_L + R_D}{\beta_1 R_L R_S} \right\}$$

$$- \frac{V_D R_I}{R_F} + V_{BE} \left\{ 1 - \frac{R_I}{R_F + R_L} + \frac{R_I}{R_B} \right\}$$

$$+ (V_{CE}(sat.) - V_{BE}) \frac{R_2 R_I}{R_1R_2 + R_1R_3 + R_2R_3}$$

By symmetry,

$$e_2 = -V_B R_I\left\{\frac{[(\beta_4-1)R_L-R_D]}{\beta_2\beta_4R_L^2} + \frac{1}{R_L+R_F} - \frac{R_2}{R_1R_2 + R_1R_3 + R_2R_3}\right\}$$

$$+ (V_{BE}+V_D)R_I\left\{\frac{[(\beta_4-1)R_L-R_D]}{\beta_2\beta_4R_L^2} + \frac{R_L + R_D}{\beta_2R_LR_S}\right\}$$

$$+ \frac{V_DR_I}{R_F} - V_{BE}\left\{1 - \frac{R_I}{R_F + R_L} + \frac{R_I}{R_B}\right\}$$

$$- (V_{CE}(\text{sat.}) - V_{BE})\frac{R_2R_I}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$e_3 = -e_4 = V_B - V_{CE}(\text{sat.})$$

The transfer characteristic is illustrated in fig. 17. The hysteresis is temperature dependent because V_{BE}, V_{CE}(sat.), V_D, and β_1 through β_4 all depend on temperature. If the unit is operated at a fixed ambient temperature, then the threshold is very stable. In fact, if the temperature is stable, a complementary latching flip-flop circuit can be designed to operate at millivolt levels.

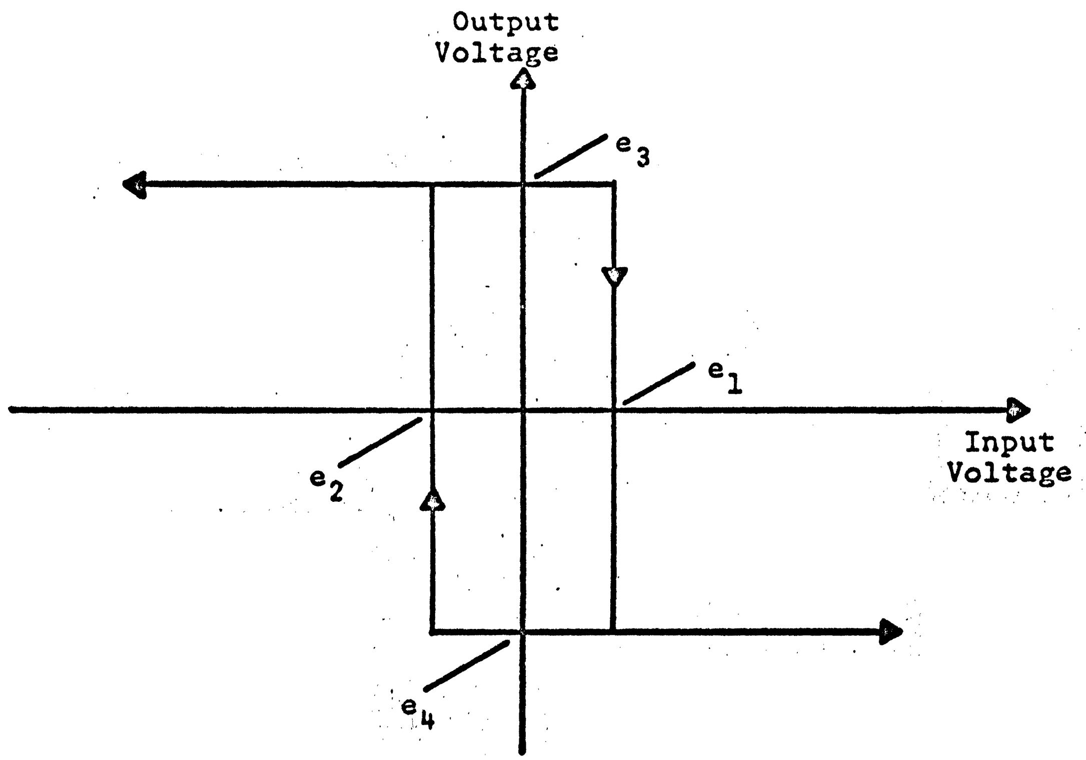


Fig. 17 Transfer Characteristic

2.3 A Tunnel Diode Stabilized Threshold Detector

A practical tunnel diode stabilized complementary latching circuit is shown in fig. 18. The hysteresis width can be adjusted from 5 volts to less than 1 millivolt by varying the negative feedback. The transfer characteristic of the device is quite stable for temperature fluctuations normally encountered in the laboratory. This device is an extremely versatile threshold detector because of the wide range of hysteresis widths. However, it is, primarily, intended for use as a zero crossing detector. Pictures of the transfer characteristic of the tunnel diode circuit are included in fig. 19. The device operates at frequencies below 500 kc/s.

The tunnel diode circuit operates on the same principles as the flip-flop device of section 2.2. However, the tunnel diode device has superior stability and speed.

The output circuit of the device is similar to the one in the flip-flop circuit. But, it serves a much more important purpose in this device. Each of the bistable tunnel diode circuits has one very stable threshold which depends on the peak current of the diode. The other threshold which depends on the valley current is not so stable. Because of the output circuit, only the stable threshold causes

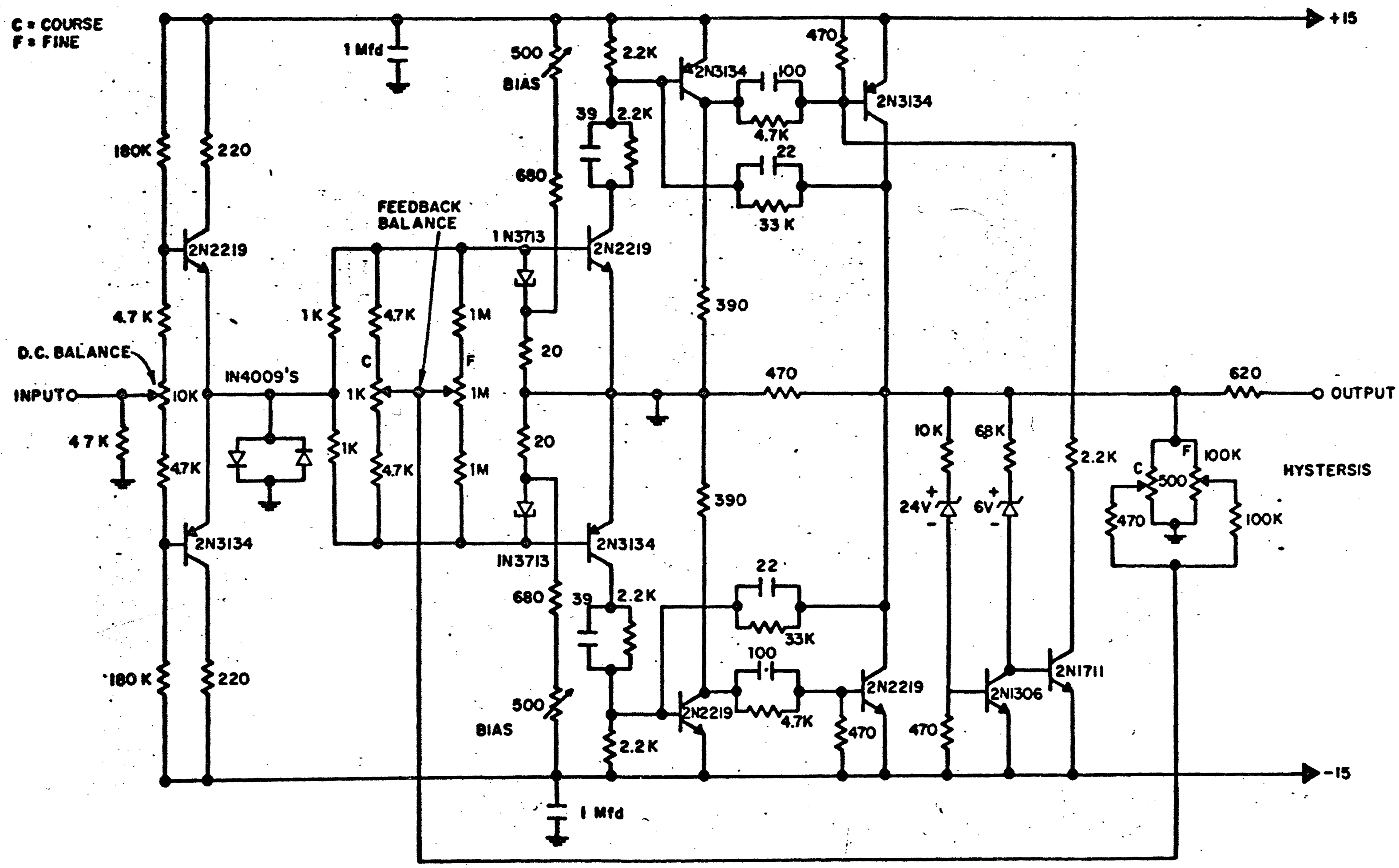
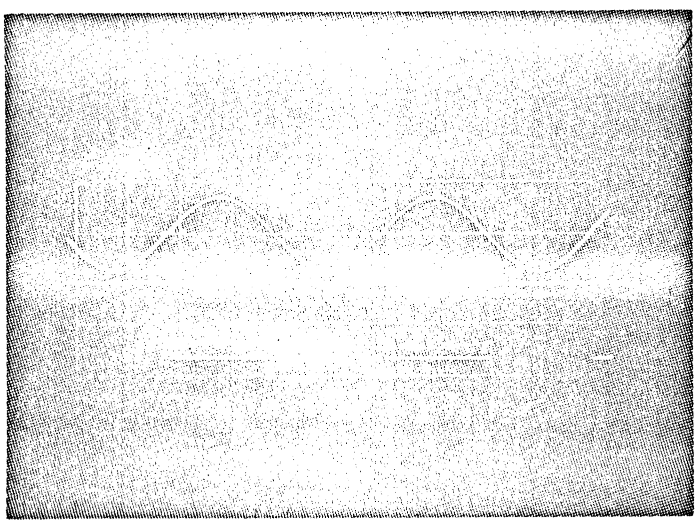


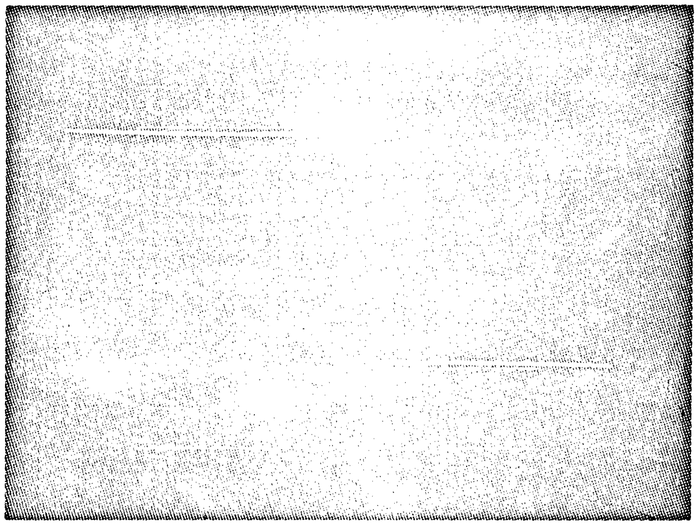
Fig. 18 An Experimental Tunnel Diode Stabilized Threshold Detector

Vertical: Output 5 v./div.
Horizontal: Input .5 v./div.

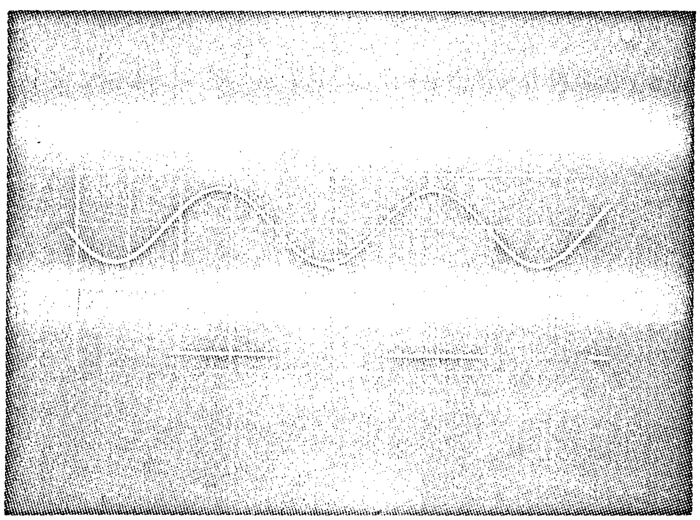


Upper Vertical: Input 5 v./div.
Lower Vertical: Output 10 v./div.
Horizontal: Time .5 ms./div.

Fig. 19(a) Experimental Transfer Characteristic
1 Volt Hysteresis

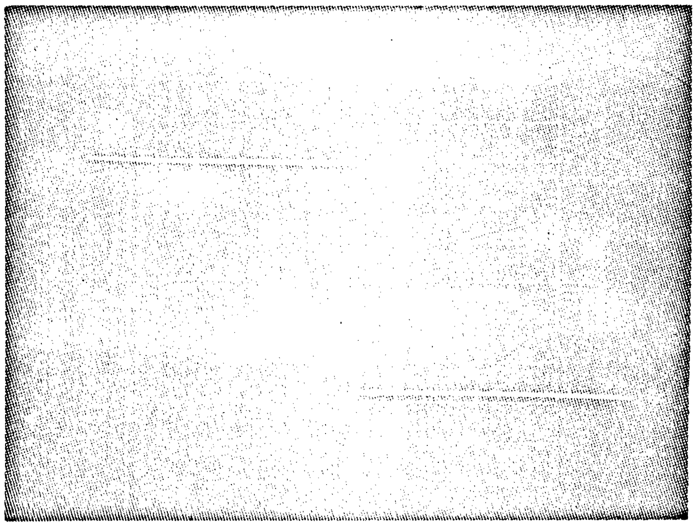


Vertical: Output 5 v./div.
Horizontal: Input 5 mv./div.

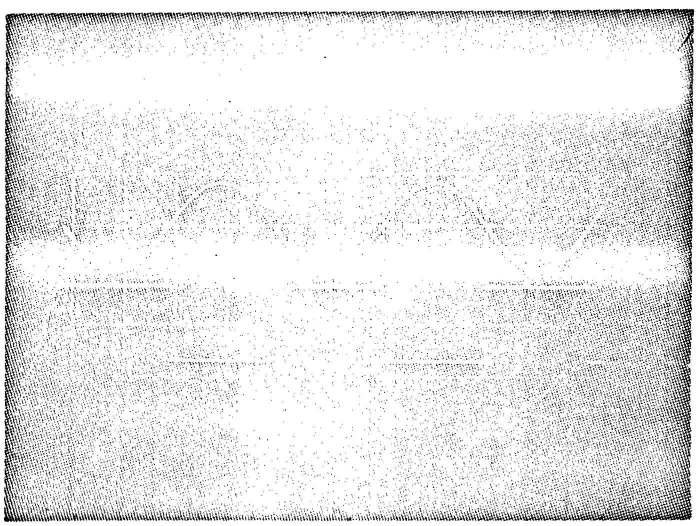


Upper Vertical: Input 50 mv./div.
Lower Vertical: Output 10 v./div.
Horizontal: Time .5 ms./div.

Fig. 19(b) Experimental Transfer Characteristic
10 Millivolts Hysteresis

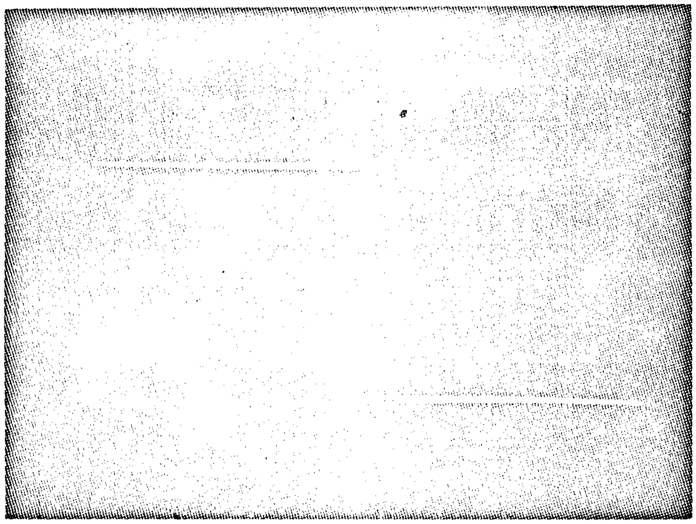


Vertical: Output 5 v./div.
Horizontal: Input 500 µv./div.

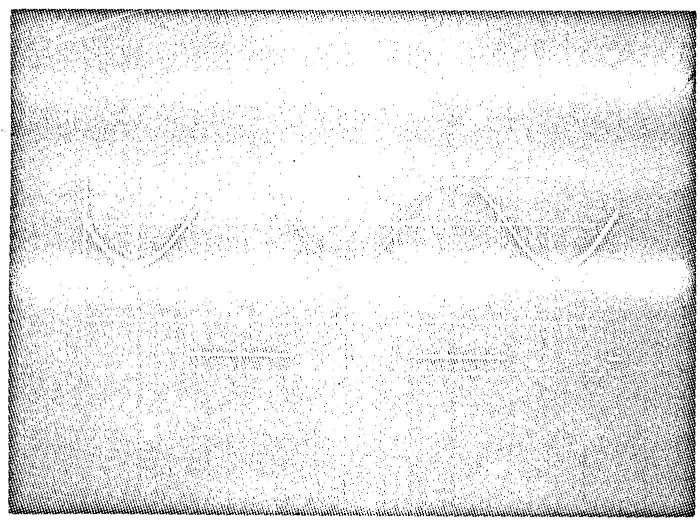


Upper Vertical: Input 500 µv./div.
Lower Vertical: Output 10 v./div.
Horizontal: Time .5 ms./div.

Fig. 19(c) Experimental Transfer Characteristic
Zero Crossing Detection



Vertical: Output 5 v./div.
Horizontal: Input 50 µv./div.



Upper Vertical: Input 50 µv./div.
Lower Vertical: Output 10 v./div.
Horizontal: Time .5 ms./div.

Fig. 19(d) Experimental Transfer Characteristic
Zero Crossing Detection

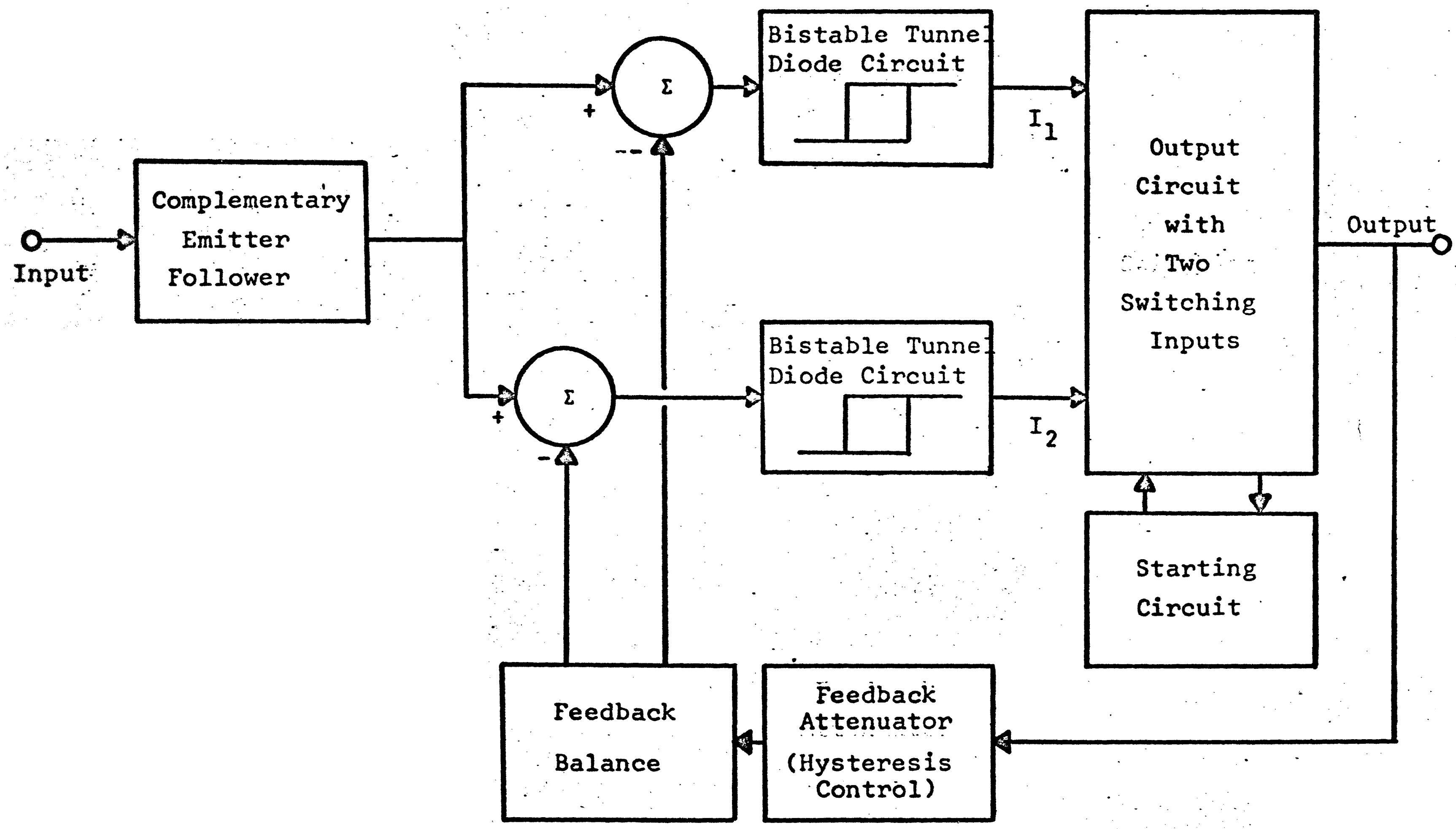


Fig. 20 Block Diagram of Tunnel Diode Stabilized Threshold Detector

switching at the output. The other unstable threshold is ignored. The output circuit also prevents both output transistors from coming on simultaneously.

The starting circuit is necessary to eliminate a state for which both output transistors are off simultaneously. The starting circuit acts only when such a state occurs; otherwise, it does not affect the main circuit.

The complementary emitter follower is used to obtain a high input impedance. The low output impedance of the emitter follower also improves the stability of the threshold detector by preventing interaction of the bistable tunnel diode circuits.

It may be helpful to review the static operation of a basic bistable tunnel diode circuit. Such a device together with a transistor amplifier is pictured in fig. 21(a). An equivalent circuit is included in fig. 21(b). The effect of the transistor is modeled as a diode. The base leakage current is neglected. It must be small compared with the peak current of the tunnel diode if the threshold depending on the peak current is to be stable. Silicon transistors which have leakage currents in the order of nanoamperes are used; so, the leakage current is certainly negligible in comparison with

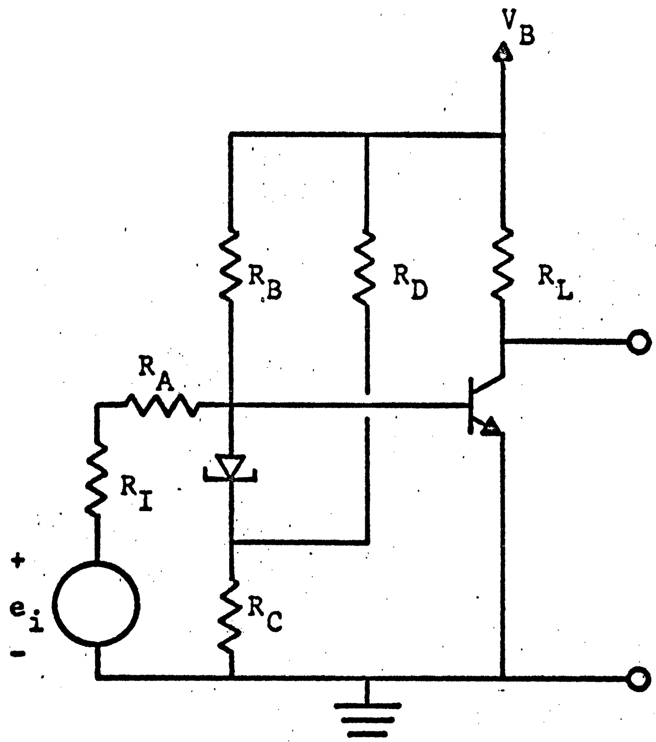


Fig. 21(a) Basic Bistable Tunnel Diode Circuit with Transistor Output

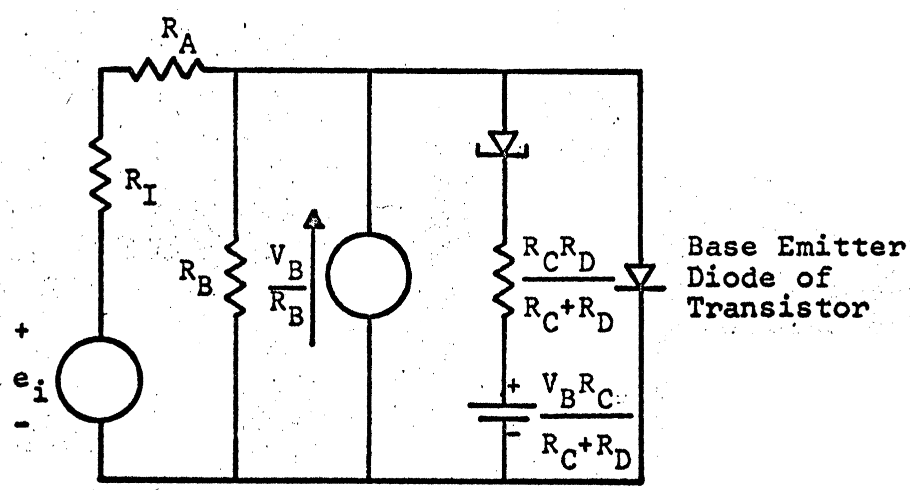


Fig. 21(b) Equivalent Circuit

the tunnel diode peak current of 1 milliampere. The R_D, R_C network is used to raise the tunnel diode about 300 millivolts above ground potential. This is necessary when a germanium tunnel diode is used in conjunction with a silicon transistor. Then the transistor is off when the tunnel diode is in the low voltage state and on when the tunnel diode is in the high voltage state. This bias voltage can be eliminated or, at least, reduced if silicon or gallium arsenide tunnel diodes are used instead of the germanium tunnel diodes.

The bias voltage is critical to the operation of the circuit. The voltage must be low enough so that when the tunnel diode is in the low voltage state, the transistor base current is negligible. For operation at millivolt levels, the base current must be 10^{-3} of the tunnel diode peak current. On the other hand, the bias voltage must be high enough so that when the tunnel diode is in the high voltage state, the base current is high enough to turn the transistor on. Preferably, the base current should be high enough to saturate the transistor. The use of silicon or gallium arsenide tunnel diodes is recommended since, then, the bias is not so critical and may even be unnecessary. The circuit of fig. 18 uses germanium tunnel diodes. The bias current and voltage must be changed if other tunnel diodes are employed.

Fig. 22(a) through (i) are self explanatory. The tunnel diode, transistor, R_B, R_C, and R_D have a nonlinear i-v characteristic which has a negative resistance region (Fig. 22(e)). The load line depending on e_i, R_I, and R_A is superimposed on the nonlinear characteristic (Fig. 22(g), (h), and (i)). e_i shifts the load line along the voltage axis. When $e_i = 0$, there are two stable states. When e_i is greater than some positive voltage, only one stable state, the high voltage state, exists. When e_i is less than some negative voltage, only the low voltage state is stable. If the circuit is initially in the low voltage state, and if e_i becomes so positive that the load line no longer intersects the low voltage region of the curve; then the circuit will switch to the high voltage state. Switching in the other direction occurs in the same manner.

The voltage at which the bistable tunnel diode circuit switches from the low voltage state to the high voltage state is dependent on the peak current of the tunnel diode. The peak current is very stable; for a selected diode, it may change by only a few percent for a temperature change of 100 °C. Therefore, the threshold which depends on the peak current is very stable.

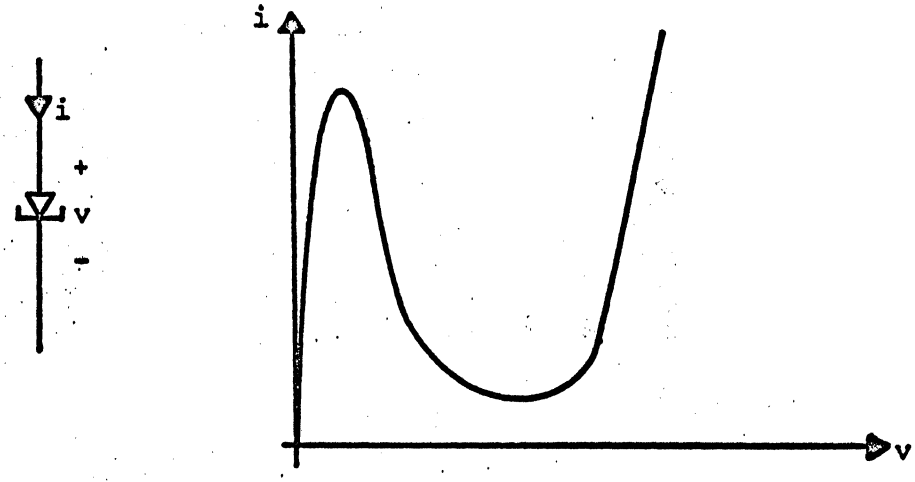


Fig. 22(a) Tunnel Diode

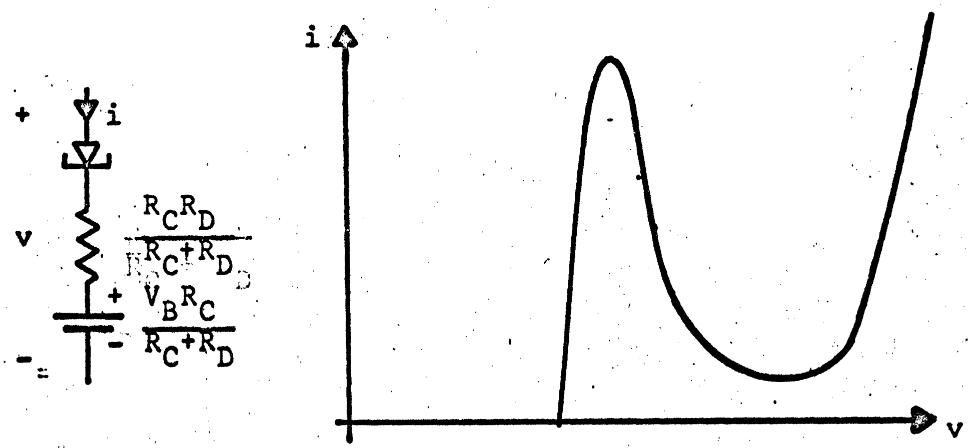


Fig. 22(b) Tunnel Diode & Voltage Bias

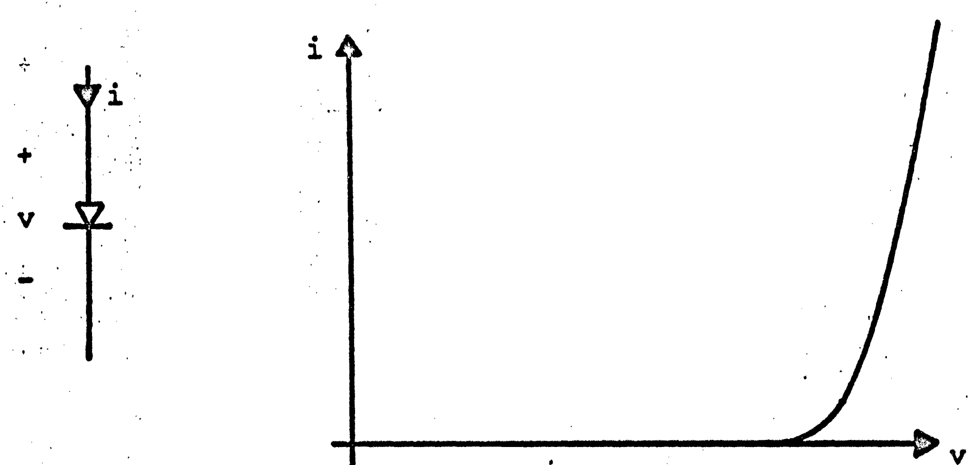


Fig. 22(c) Base-Emitter Diode of Transistor

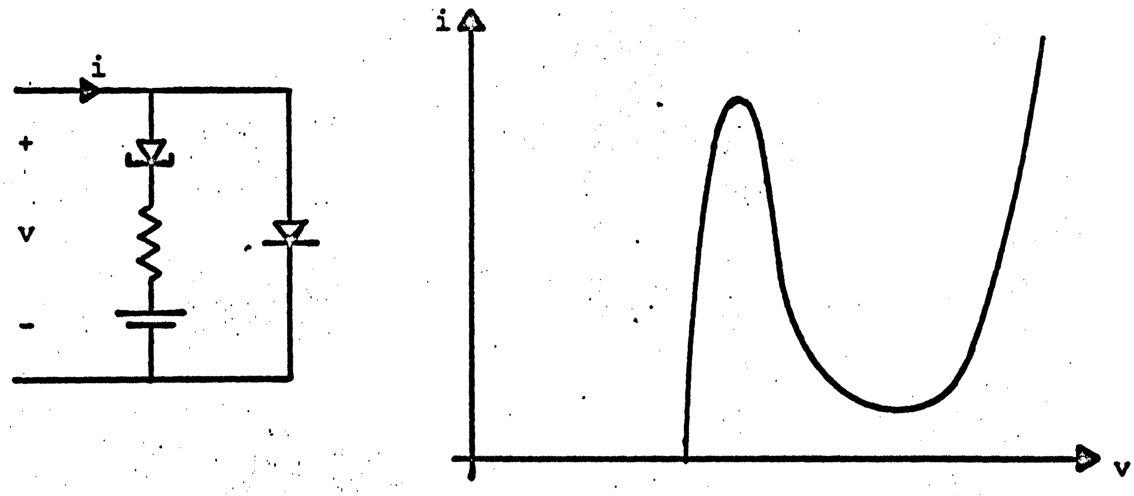


Fig. 22(d) Tunnel Diode, Voltage Bias, & Diode

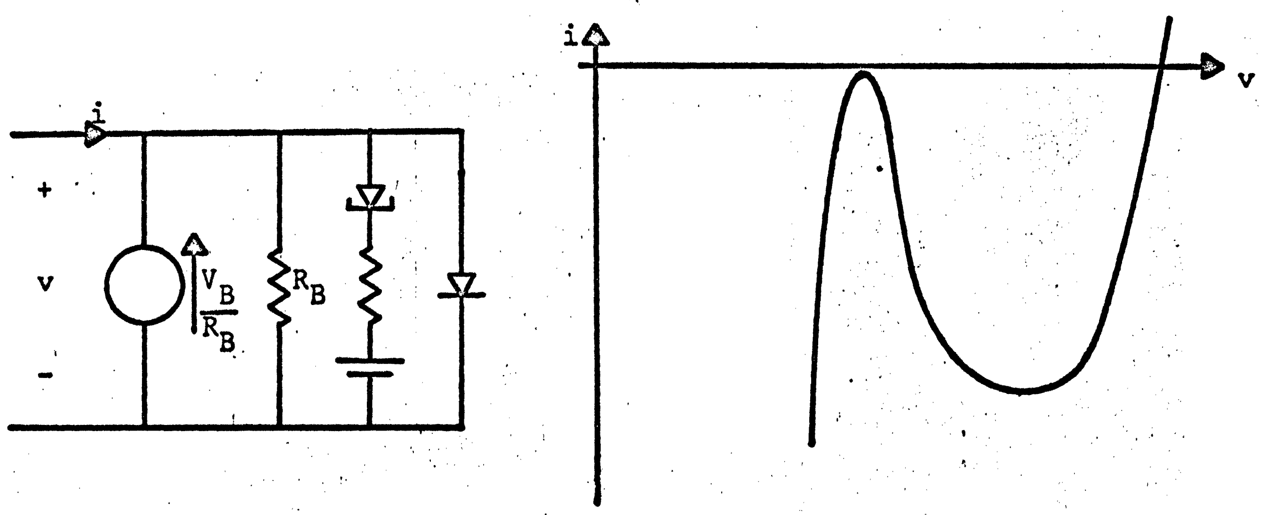


Fig. 22(e) Tunnel Diode, Voltage Bias, Diode, & Current Bias

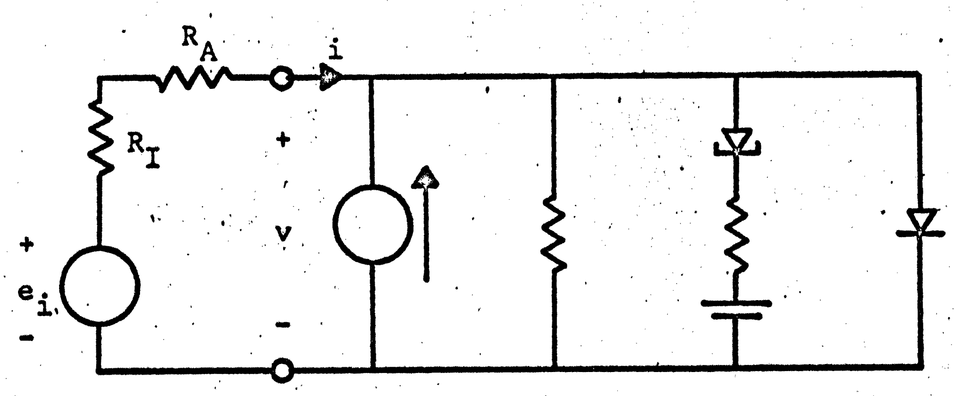


Fig. 22(f) Biased Tunnel Diode Circuit with Input Source

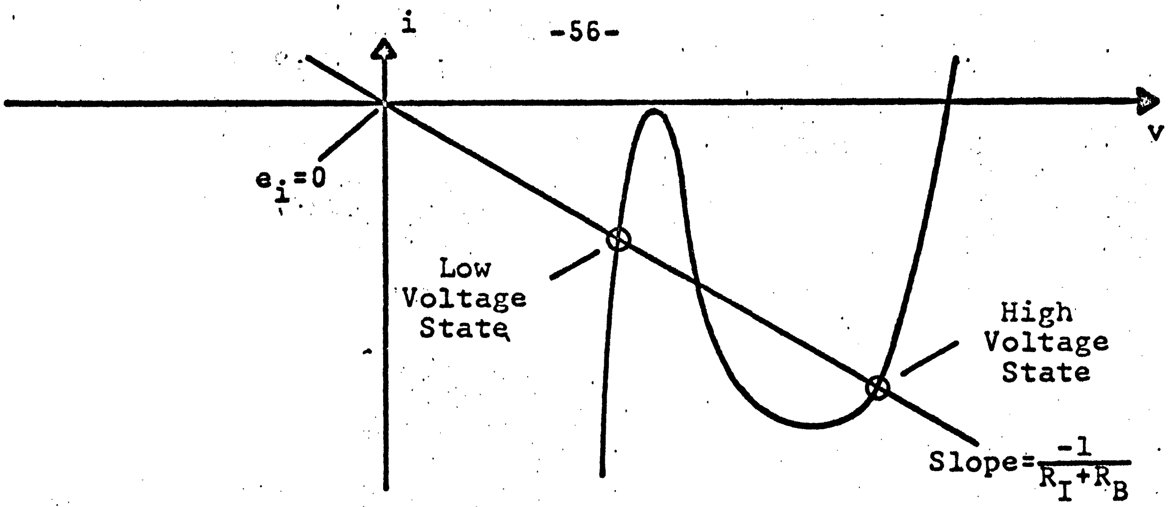


Fig. 22(g) Load Line for $e_i=0$ (Two Stable States)

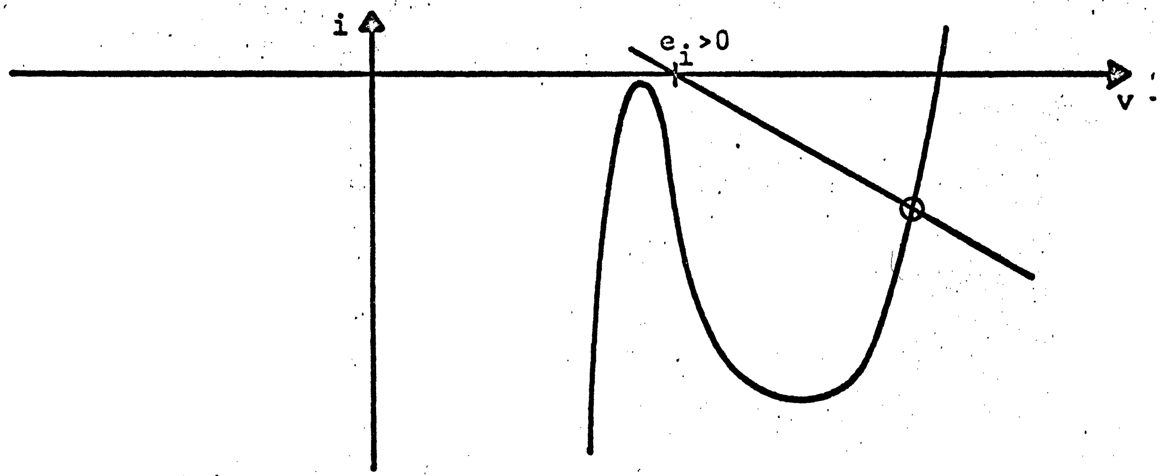


Fig. 22(h) Load Line for $e_i>0$ (Stable High Voltage State)

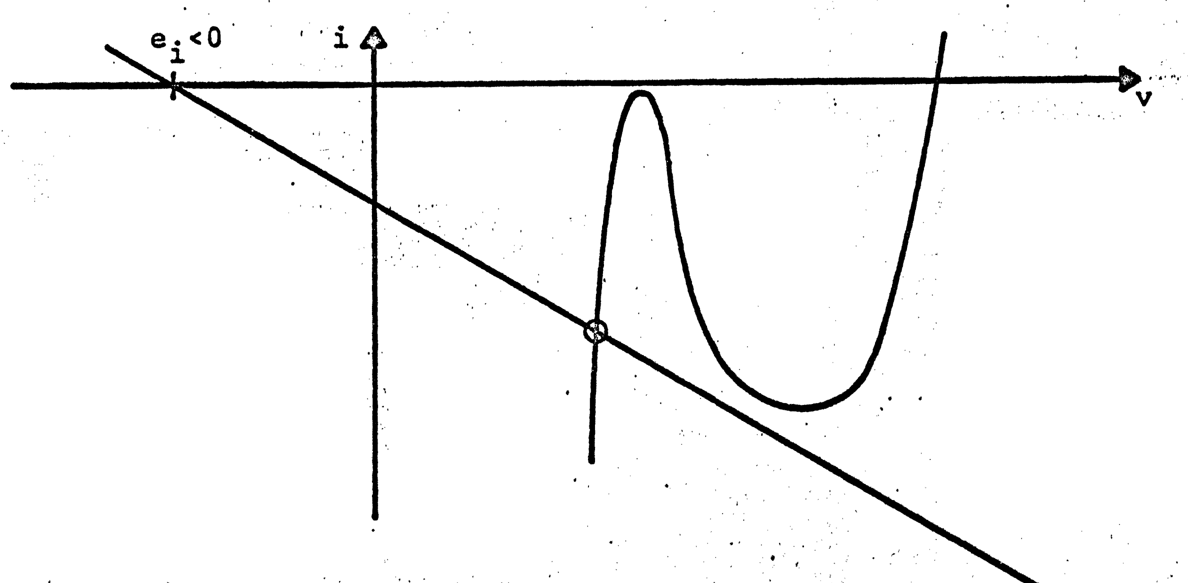


Fig. 22(i) Load Line for $e_i<0$ (Stable Low Voltage State)

The tunnel diode threshold detector is drawn in a different form in fig. 23. The emitter follower has been replaced by a voltage source, e_i, and a resistor, R_I. The clamp diodes have been eliminated since they do not affect the operation of the circuit at small signals.

Referring to fig. 23, the switching sequence is as follows: Assume that e_i is initially large, positive, and decreasing. Initially, TD_1 is in the high voltage state; TD_2 is in the low voltage state. Q_1, Q_3, and Q_6 are saturated; Q_2, Q_4, and Q_5 are off. As the signal decreases:

(1) TD_1 switches from the high voltage state to the low voltage state.

(2) Q_1 switches off. No switching is initiated in Q_3 or Q_5 because Q_3 is held in saturation by the feedback from the output.

(3) TD_2 switches to the high voltage state. This initiates switching.

(4) Q_2 switches on.

(5) Q_2 pulls Q_4 on and into saturation.

(6) Q_4 turns Q_6 off. The output voltage rises.

(7) The a.c. coupling from the output causes Q_3 to become active.

(8) The magnitude of the collector voltage of Q_3 increases causing Q_5 to become active.

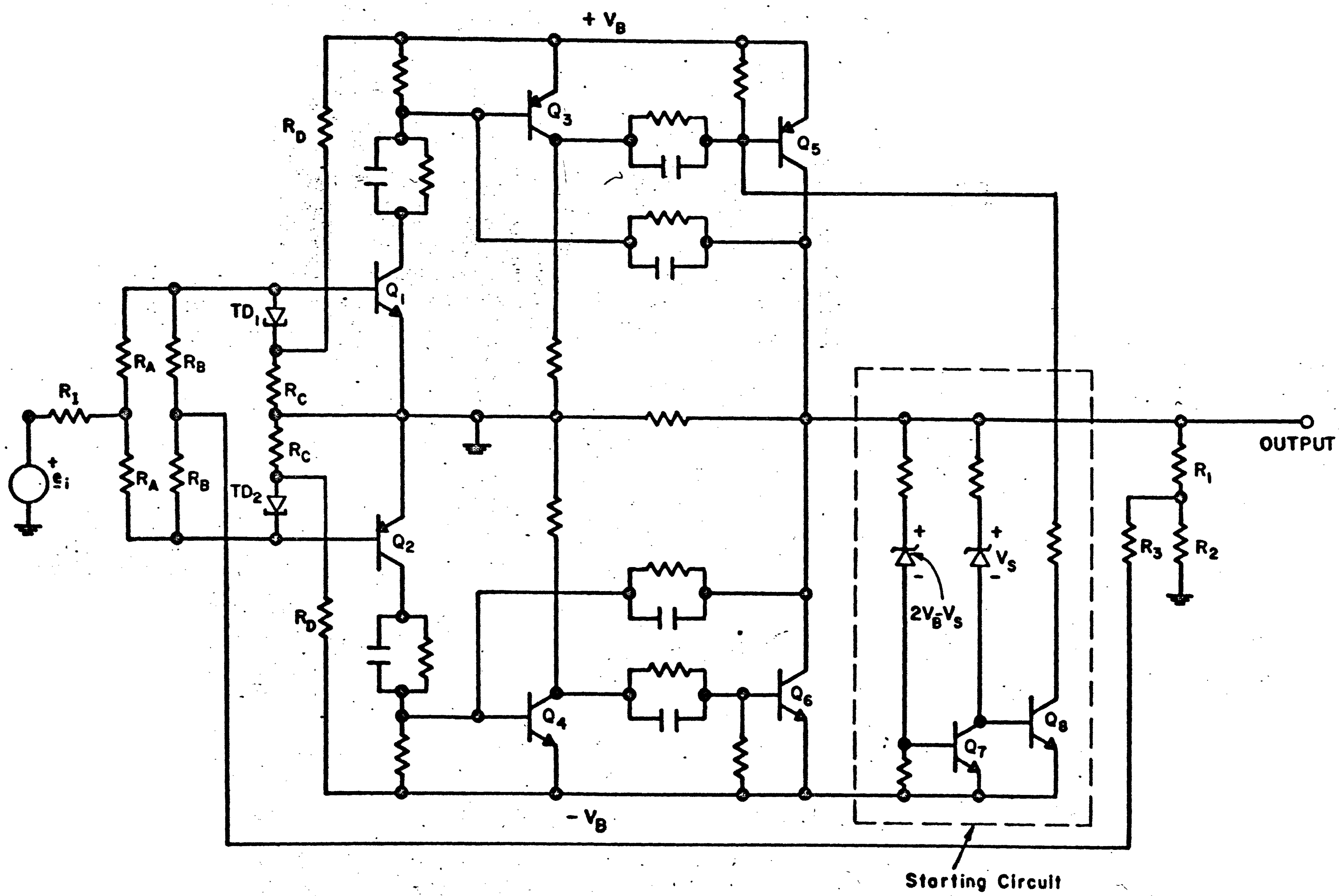


Fig. 23 Tunnel Diode Stabilized Threshold Detector

(9) Q_3 and Q_5 switch regeneratively. The output voltage reaches the positive state.

Switching in the other direction follows by symmetry.

The discussion, above, gives the impression that the switching occurs in rough steps. This is not true; switching at the output is rapid (30 ns.) and continuous.

From the discussion of the switching sequence, it is apparent that the operation of the output circuit depends, critically, on the a.c. coupling. Without the a.c. coupling, the output voltage would switch to ground potential and remain there. When the a.c. coupling capacitors are properly selected, the output switches smoothly between the positive voltage and the negative voltage.

The starting circuit affects the main circuit only when both output transistors are off simultaneously. Again, referring to fig. 23, the starting sequence is as follows. Assume that the input signal is zero. Initially, TD_1 and TD_2 are in the low voltage state. Q_1, Q_2, Q_5, Q_6, Q_7, and Q_8 are off. Q_3 and Q_4 are saturated.

(1) Q_8 becomes active and eventually saturates. ($V_S < V_B$)

(2) Q_8 pulls Q_5 on and into saturation. The output switches to the positive state.

(3) The feedback from the output causes Q_3 to become active and to eventually turn off. The circuit is now in an allowed state.

(4) Q_7 becomes active and eventually saturates.

(5) Q_7 shunts the base of Q_8 causing Q_8 to turn off. The starting circuit no longer affects the main circuit.

The hysteresis of the tunnel diode circuit is calculated with the aid of figs. 24, 25, 26, and 27. The complementary emitter follower has, again, been omitted because it has no effect on the switching and very little effect on the hysteresis. A simple piecewise linear approximation of the tunnel diode i-v characteristic can be used. The piecewise linear approximation is shown in fig. 24.

In the circuit of fig. 26,

$$V_{T1} = \frac{[V_B - V_{CE}(\text{sat.})]R_2}{R_1 + R_2}$$

$$V_{T2} = \frac{V_B R_C}{R_C + R_D}$$

$$R_{T1} = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1 + R_2}$$

$$R_{T2} = \frac{R_D R_C}{R_D + R_C}$$

A very accurate, but complicated, expression for the hysteresis can be obtained by solving the circuit of

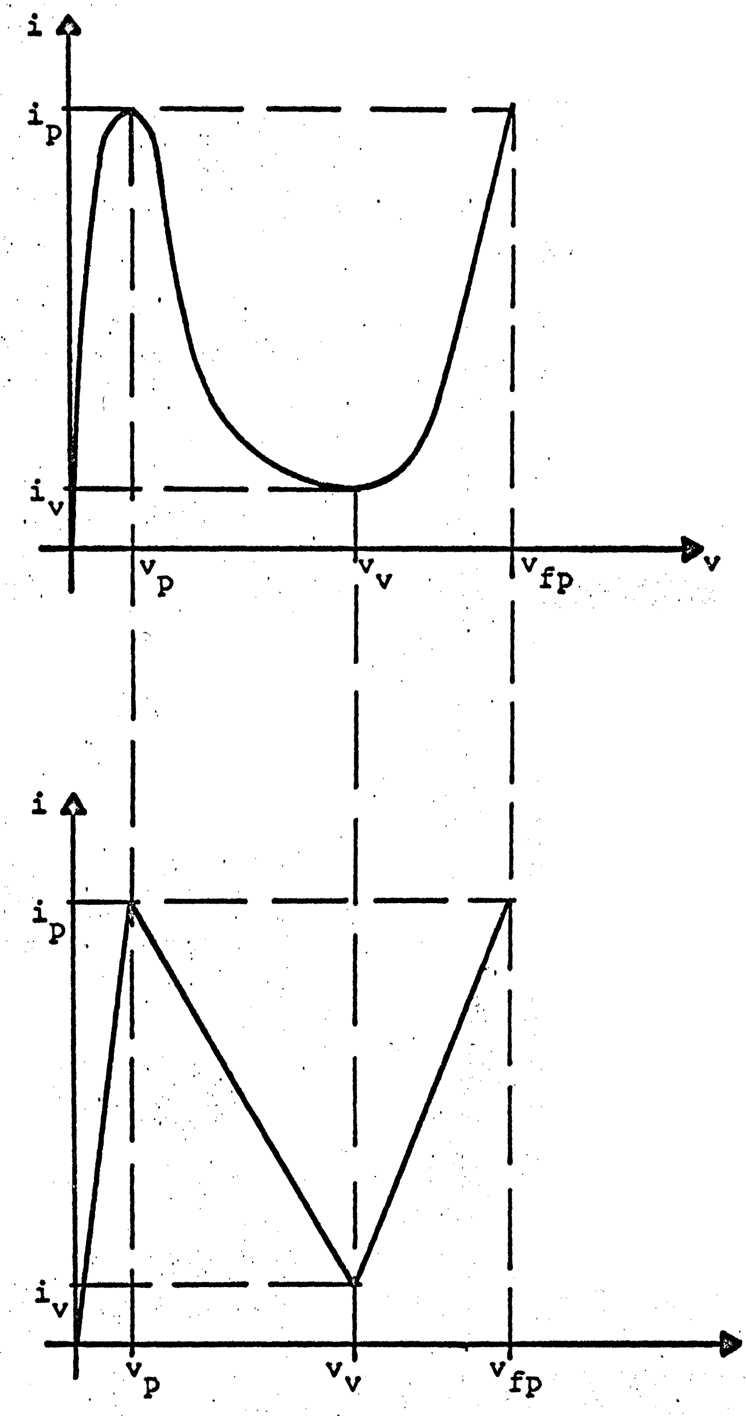


Fig. 24 Piecewise Linear Tunnel Diode V-I Characteristic.

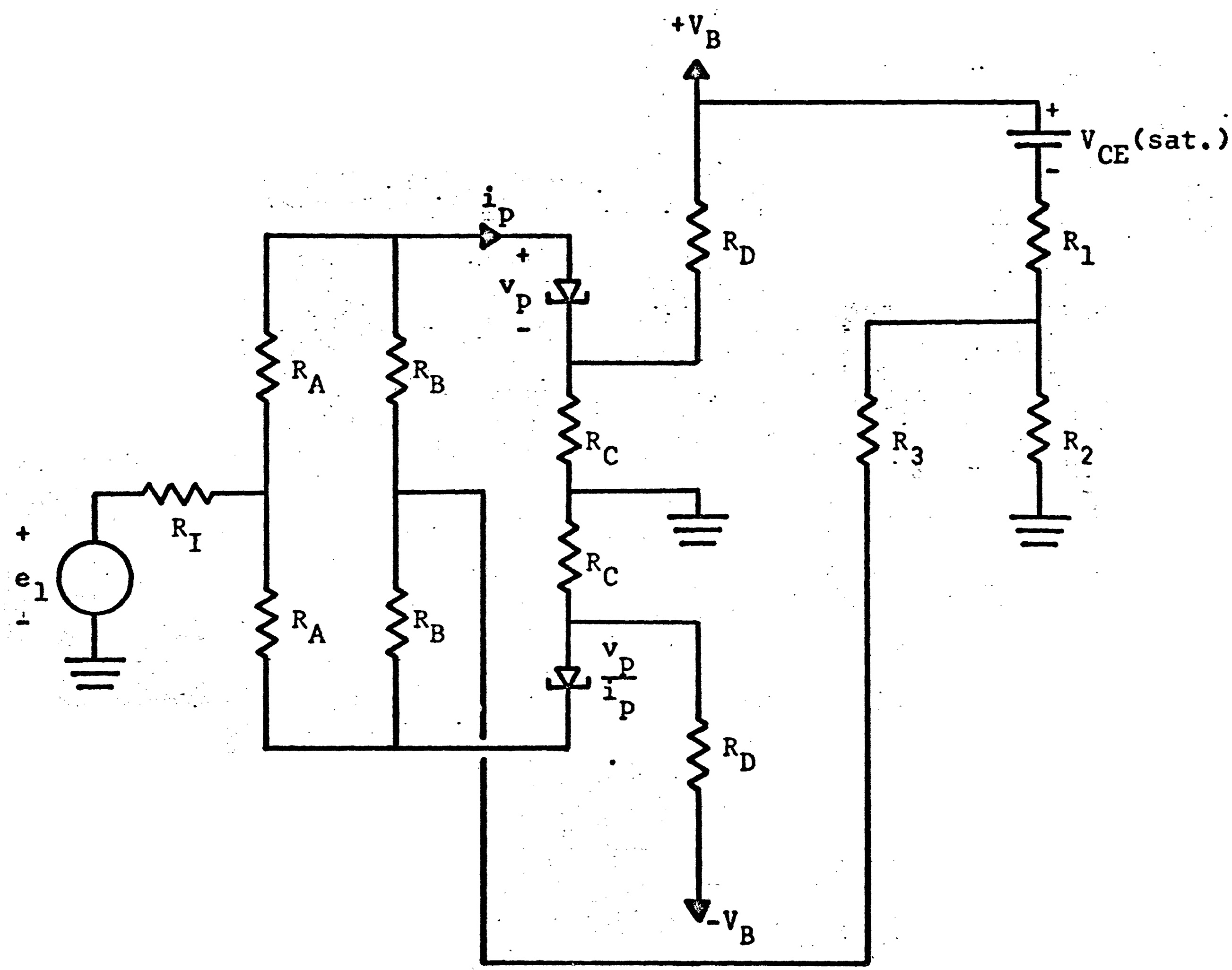


Fig. 25 Equivalent Circuit at Positive Threshold

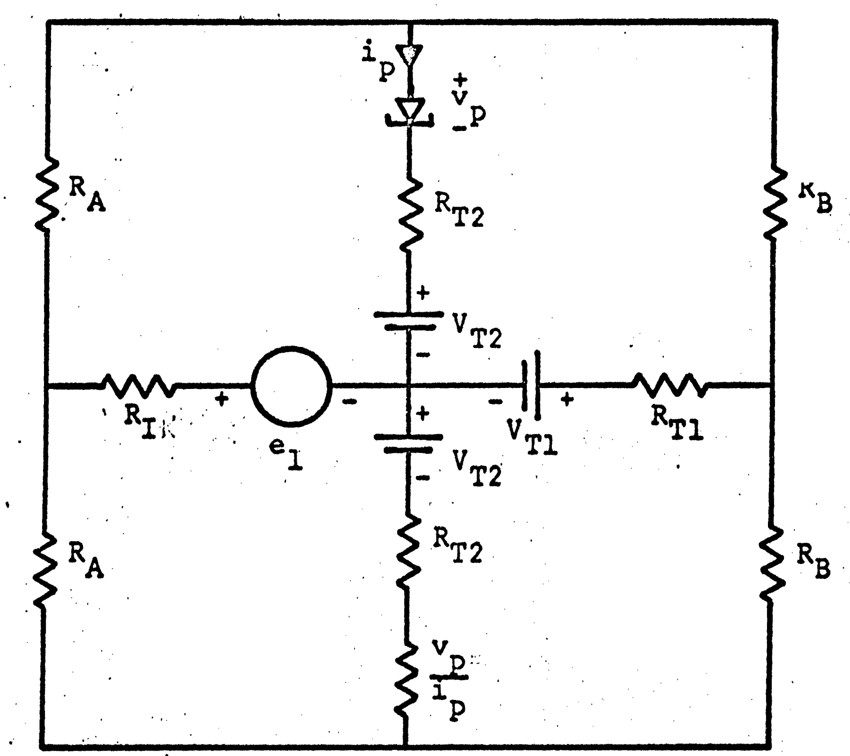


Fig. 26 Another Configuration of the Equivalent Circuit

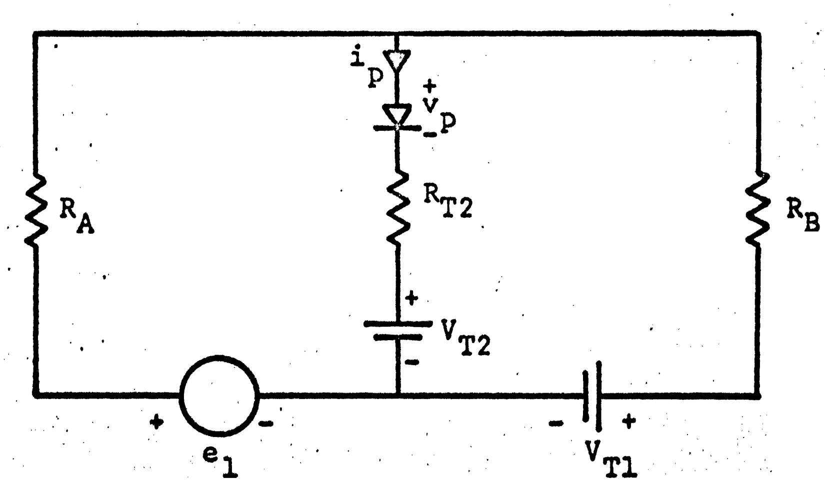


Fig. 27 Approximate Equivalent Circuit at Positive Threshold

figs. 25 & 26. However, a simpler approximate solution can be obtained from the circuit of fig. 27 if two conditions are met. If

$$R_I << R_A \quad \text{and}$$
$$R_{T1} << R_B,$$

then the coupling between the upper and lower parts of fig. 26 is weak. The effect of the lower part on the upper part can be neglected and vice versa. Then only the simple circuit of fig. 27 must be analyzed. In practice, the inequalities, above, are satisfied since it is desirable to minimize the coupling between the two tunnel diode circuits.

The approximate transfer characteristic calculated from fig. 27 is shown in fig. 28. Referring to fig. 28,

$$e_1 = -e_2 = (v_p + \frac{i_p R_D R_C + V_B R_C}{R_C + R_D}) \cdot \frac{R_A + R_B}{R_B} + i_p R_A - [V_B - V_{CE}(\text{sat.})] \cdot \frac{R_A R_2}{R_B(R_1 + R_2)}$$

$$e_3 = -e_4 = V_B - V_{CE}(\text{sat.})$$

Although v_p, i_p, and $V_{CE}(\text{sat.})$ all depend on temperature, the thermal stability of the tunnel diode threshold detector is very good. The temperature dependence of v_p and i_p is extremely weak. The

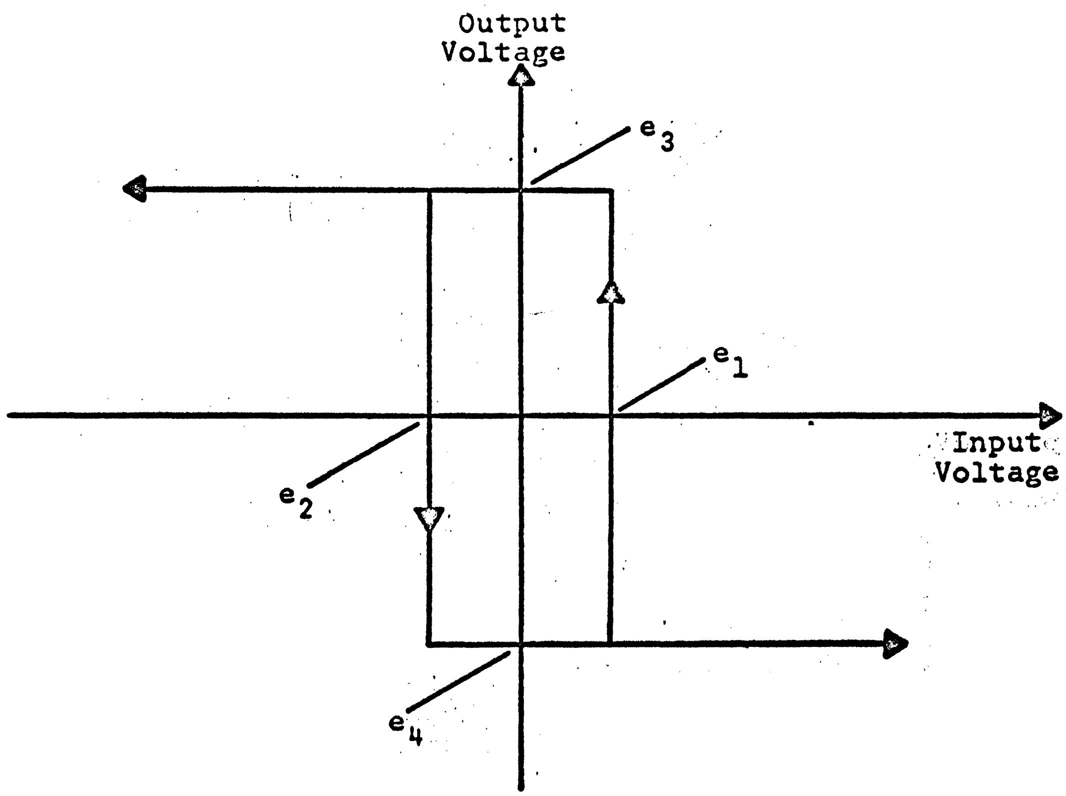


Fig. 28 Transfer Characteristic

fluctuation of these parameters with temperature is generally less than 0.1%/°C. Specifically, the peak current and voltage of a germanium tunnel diode, 1N3713, have the following temperature coefficients:

$$\left.\frac{dv_p}{dT}\right|_{T=25°C} = 65\ \mu v/°C$$

$$-1\ \mu a/°C < \left.\frac{di_p}{dT}\right|_{T=25°C} < .3\ \mu a/°C$$

typically,

$$\left.\frac{di_p}{dT}\right|_{T=25°C} = -.3\ \mu a/°C.^{(5)}$$

Temperature variations have a greater effect on V_{CE}(sat.); however, fluctuations in V_{CE}(sat.) are not important. As long as $V_B >> V_{CE}$(sat.), fluctuations in V_{CE}(sat.) do not have a strong effect on the transfer characteristic. A millivolt variation of V_{CE}(sat.) might cause a 10 microvolt variation in the transfer characteristic.

The tunnel diode stabilized threshold detector can be adjusted to have a hysteresis of less than a millivolt. Since the series input resistors are 1000 ohms (Cf. fig. 18), the signal currents are of the order of microamperes when the input voltage is in the millivolt range. When the circuit is operating with a millivolt input signal, the tunnel diodes are switching when their current changes by a microampere out of a milliampere. In other words, the tunnel diodes are switching consistantly from the low voltage state to the high voltage state when their current changes by 0.1 %.

The tunnel diode stabilized threshold detector is a sophisticated device. It is also an expensive device. But, the experimental circuit has less than 1 millivolt of hysteresis from d.c. to 500 kc/s even though the circuit is by no means optimum. The hysteresis of the device is directly dependent on the supply voltages; so, the supplies must be regulated. Since the tunnel diodes set the switching thresholds, one would expect the device to work at frequencies well above 500 kc/s. However, the transistors limit the frequency range. Faster transistors should alleviate this problem. Some form of compensation in the feedback path might, also be useful.

The most important point is that the stable thresholds from two separate tunnel diode circuits can be selected by a special output circuit and used to form a stable threshold detector. Negative feedback can, then, be used to adjust the hysteresis.

APPENDIX

The transistor model used in the calculation of the theoretical expressions is a simplified version of the Ebers-Moll model. The simplified model is shown in fig. 29. Without such a simple model, the algebra becomes prohibitively complex. Furthermore, results calculated with more accurate models often yield complicated expressions which do not give much insight into the basic operation of the circuit.

Referring to fig.29, the diodes are assumed to be ideal switches. V_{BE} represents the voltage drop across the base to emitter diode. V_{BE} is approximated as a constant although it is in reality a logarithmic function of the emitter current. A useful equation, predicting the behavior of V_{BE}, is

$$dV_{BE} = -25\left(\frac{kT}{q}\right)\frac{dT}{T} + \left(\frac{kT}{q}\right)\frac{dI_E}{I_E}$$

where I_E is the current flowing out of the emitter. The equati β is also assumed to be constant. Actually, β drops at low and high current levels; however, in many transistors there is a wide range of current levels for which β is constant. The β of silicon transistors usually increases with temperature.

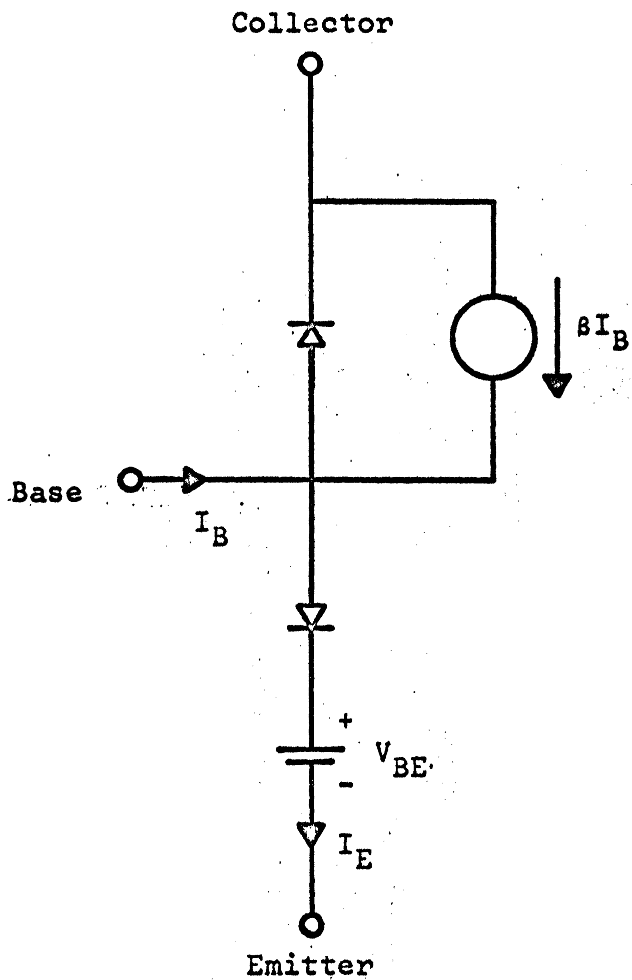


Fig. 29 A Simple Transistor Model

Generally, $$1\ \%/^\circ C < \frac{d\beta}{dT} < 5\ \%/^\circ C.$$

When the transistor is saturated, the collector to emitter voltage is approximated by a constant voltage, V_{CE}(sat.). V_{CE}(sat.) is a function of the collector current and the temperature. At moderate currents,

$$V_{CE}(\text{sat.}) \approx \frac{kT}{q}\left[\ln \frac{\beta}{\beta_r} + 2\right]$$

where β is the forward current gain as before, and β_r is the reverse current gain. β_r is the current gain when the collector and emitter are interchanged. In general, $\beta >> \beta_r$.

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