UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE F 3 - 129

TITLE	Integration of n Simultaneous Second Order Differential Equations with Initial Conditions Specified (SADOI or DOI)	
TYPE	Subroutine 39 Parameter storage $\frac{1}{40}$	
TEMPORARY STORAGE	0, 4n locations specified by parameters located in 4 and 5.	
PARAMETERS	3 00 F 00 aF	a is the location of the first word of the auxiliary routine.
	4 00 F 00 (A_0)F	Storage positions A_i, $A_i = A_0 + i$ contain dependent variables $y_i (i = 0, 1, \ldots, n-1)$. Initial values of y_i must be placed here originally.
	5 00 F 00 (B_0-A_0)F	Storage positions B_i, $B_i = A_0 + (B_0-A_0) + i$, contain hy_i' where h is the increment. Storage positions C_i, $C_i = A_0 + 2(B_0-A_0) + i$ are used for temporary storage. Originally B_i and C_i must contain hy_i' and h^2h_i'', respectively. Locations D_i, $D_i = A_0 + 2(B_0-A_0) + i$ contain values of h^2y_i'' found by the auxiliary subroutine.
	6 00 F 00 nF	n is the number of differential equations to be integrated.
	7 00 F 00 eF	e is the address of the tolerance, ϵ. ϵ is a number such that if, for all i, the absolute value of the difference between two successive approximations to h^2y_i'' is less than $2^{-39}\epsilon$, the iteration process is assumed to have converged.

AUXILIARY SUBROUTINE The purpose of the auxiliary subroutine is to take the quantities y_i and hy_i' $(i = 0, 1, \ldots, n-1)$, whose locations are determined by the parameters A_i and B_i and compute the quantities h^2y^n. The quantity h is the increment of the independent variable. The auxiliary subroutine must be located in a sequence beginning with location a where a is specified by the parameter S3. In integrating over one step, the integration subroutine will call in the iterations necessary for convergence.

DURATION The time in milliseconds for one time through the subroutine is

$$0.54 + 2.17n + I(0.25 + 3.8n + D)$$

where I = number of iterations for an interval

D = time of auxiliary routine

n = number of equations.

DESCRIPTION This routine will integrate n second order differential equations of the type:

$$(1)\quad y_i'' = f_i(y_0, y_1, \ldots, y_{n-1}, y_0', y_1', \ldots, y_{n-1}')$$

$$i = 0, 1, \ldots, n-1$$

When entered with the entry

p	
	50 p
	26 integration routine,

the subroutine will carry out one integration step replacing values of $y_{i,\,j-1}$ and $hy_{i,\,j-1}'$ by $y_{i,j}$ and $hy_{i,\,j}'$.

THE INDEPENDENT VARIABLE If the independent variable x occurs in the functions f_i, or if it is required during an integration as an index, then it may be obtained by integrating the equation $y'' = 0$. The independent variable x is then treated as an additional dependent variable, for which no additional computing is required by the auxiliary subroutine. If the independent variable does not

appear in any of the f_i's, but is merely wanted for indication purposes, it is quicker to use a simple counter in the main routine.

REMARKS

The numerical procedure is given by equations 2, 3, and 4.

$$(2)\quad y'_{i,j,k} = y'_{i,j-1} + (h/2)\, f_{i,j-1} + (h/2)\, f_{i,j,k}$$

$$(3)\quad y_{i,j,k} = y_{i,j-1} + h y'_{i,j-1} + (h^2/3) f_{i,j-1} + (h^2/6) f_{i,j,k}$$

$$(4)\quad \left.\begin{aligned} y_{i,j} &= y_{i,j,k} \\ y'_{i,j} &= y'_{i,j,k} \\ f_{i,j} &= f_{i,j,k} \end{aligned}\right\} \text{ if, for all i, the relation}$$

$$h^2 \left| f_{i,j,k} - f_{i,j,k-1} \right| \leq \epsilon \qquad \text{is satisfied.}$$

In the above equations:

i indicates the variable of integration,

j indicates the step of integration,

and k indicates the number of iterations.

Sufficient conditions to guarantee that the procedure will converge and that numbers stay in bounds can be stated, but use of these conditions leads to extremely short intervals. In practice, if units of measure and a value for h are chosen so that numbers are within bounds within the auxiliary routine the value of h so chosen is usually satisfactory for convergence of the numerical procedure.

DATE	March 18, 1954 rt. 10/7/60
CODED BY	John Brooks
REVISED BY	Gene H. Golub
APPROVED BY	J. P. Nash

LOCATION	ORDER (OOK(F))		NOTES PAGE 1
0	K5 1F		
	42 35L		Plant link
1	L5 L	from 14	
	L4 28L		(+)i
2	L0 9L		Form i - (n+1)
	32 14L		Transfer if i = n+1
3	L4 9L		
	46 28L		Store i
4	L4 11L		Form A_i
	46 12L		
5	10 20F		
	42 11L		
6	L4 27L		Form B_i
	42 10L		
7	42 13L		
	L4 27L		Form C_i
8	42 9L		
	42 12L		
9	L5 S6		Dummy instruction used as constant
	L5 ( )F	by 8	
10	10 1F		
	50 ( )F	by 7	
11	S4 1022S4		Address digits used in computation
	L4 ( )F	by 5	of A_i
12	40 ( )F	by 4	
	L5 ( )F	by 8	
13	S4 F		
	40 ( )F	by 7	
14	26 1L		
	50 14L	from 2, 36	Enter auxiliary routine
15	26 S3		
	47 28L		Set i = 0
16	40 37L	from 34	Store , a yes or no mark
	L5 L	from 33	

LOCATION	ORDER		NOTES PAGE 2
17	L4 28L		(+)i
	L0 9L		Form i - (n+1)
18	32 34L		Transfer if i equals n+1
	L4 9L		
19	46 28L		Store i
	L4 11L		Form A_i
20	46 32L		
	10 20F		
21	42 31L		
	L4 27L		Form B_i
22	42 29L		
	42 30L		
23	L4 27L		
	42 26L		From C_i
24	00 20F		
	46 27L		
25	L4 36L		
	46 26L		Form D_i
26	L5 ( )F	by 25	
	50 ( )F	by 23	
27	40 ( )F	by 24	
	S0 S5		
28	50 ( )F		i by 3, 15, 19, 34
	40 F		
29	10 1F		
	L4 ( )F	by 22	
30	50 F		
	40 ( )F	by 22	
31	7J 38L		Multiply by equal 1/6
	L4 ( )F	by 21	
32	40 ( )F	by 20	
	L7 S7		
33	L2 F		
	32 16L		Question convergence

LOCATION	ORDER	NOTES	PAGE 3
34	26 16L		
	47 28L	from 18	Set i equal to zero
35	15 37L		
	32 ( )F	by 0	Iteration finished if equal zero
36	50 S5		
	22 14L		
37	00 F		
	00 F		
38	00 F 00 1666		
	6666 6667 J		