UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE M 23 - 246

TITLE: Closed Eigenvectors and/or Eigenvalues by Jacobi Method (SADOI or DOI)

TYPE: Closed Routine

NUMBER OF WORDS: 185

TEMPORARY STORAGE: 0 thru 12 and 2n or 4n locations given by pre-set parameter S3. 2n are required to obtain eigenvalues only; 4n if eigenvectors are also desired. n is the order of the matrix.

DRUM STORAGE: A block of 65 n words on the drum, given by pre-set parameter S4, is used to store the given matrix. If the eigenvectors of the given matrix are desired, the block of 65(n-1) + n words immediately following this block is also used.

DURATION: Approximately $23n^3$ msec. for eigenvalues, $46n^3$ msec. for both.

LIMITATION: n (=order of matrix) $\leq$ 65.

PRE-SET PARAMETERS: S2, S3, S4

2 00F 00 cF	c is location of the Square Root Routine R1.
3 00F 00 bF	b is the location of 2n or 4n temporary locations in the Williams Memory.
4 00F 00 aF	a is the location of the matrix on the drum.

DESCRIPTION: Let A be a real symmetric matrix of order n $\leq$ 65 stored on the drum at a as described under Matrix Storage below (a is given by pre-set parameter S4). The sum of the squares of the elements of A must be less than 1/2. Let the Square Root Routine R1 be located as given by pre-set parameter S2. Then this routine located at q when entered by

p	x0 nF
	50 pF
p+1	26 qF
	-- --

will compute the eigenvalues of A if x=5 and will compute the eigenvalues and eigenvectors if x=J. The mathematical

method used is described in the write-up of routine MO. The eigenvalues λ_k will be placed in the storage locations originally occupied by the diagonal elements of A, namely at $\underline{a}$ + 66k, k=0, ... , n-1. The eigenvectors of A, if computed, will be stored as column vectors at locations beginning at $\underline{a}$ + 65n. That is, the elements of the eigenvector corresponding to λ_k will be stored at locations $\underline{a}$ + 65n + k, $\underline{a}$ + 65(n+1) +k, ... , $\underline{a}$ + 65(2n-1) +k for k=0, ... , n-1.

If it is desired to obtain, instead of the matrix E of eigenvectors, a matrix BE, then

(1) word 122 of this routine should be changed to 22 103L 00F,

(2) before this routine is entered, the elements of the matrix B should be stored in the locations corresponding to the respective elements of E (the element B_{ij} of the matrix B is stored at location $\underline{a}$ + 65(n+i) +j for i, j = 0, ... , n-1), and

(3) this routine should be entered with the order pair J0 nF 50 pF in the Q register.

(See second and third paragraph under Matrix Storage below).

MATRIX STORAGE: In order that reading from and recording on the drum always be done with minimum access time the first element of each row is stored at an interval of 65 locations from the first element of the preceding row. Since the matrix A is symmetric, only the elements on or below the main diagonal need be stored. Hence, the element A_{ij} of the matrix A is stored at location $\underline{a}$ + 65i +j for i=0, ... , n-1 and $0 \leq j \leq i$. This routine contains within it two subroutines which transfer columns of elements to the memory from the drum and vice versa. Since these might be helpful to the user if used directly they are described in some detail in the paragraph below.

Let b be the address given by pre-set parameter S3. Let c be the right hand address of word 114L (relative to this routine). Let a be the drum address given by pre-set parameter S4. If word 181L (relative to this routine) is the number m x 2^{-39}, then the subroutine beginning at 132L (relative to this routine) when entered using

p	x0 kF 50 pF
p+1	26 132(M23) -- ---

($0 \leq k \leq m-1$) will transfer the words from drum locations a + 65k, a + 65k +1, ... , a + 66k -1, a + 66k, a + 66k + 65, a + 66k + 130, ... , a + 66k + 65(m-k-1) in that order to m consecutive locations in the Williams memory beginning at b or c depending upon whether X = J or X = 5, respectively. Under the same conditions when entered in a similar manner the subroutine beginning at 158L (relative to this routine) does the opposite; that is, it transfers words from locations beginning at b or c depending upon whether X = J or 5 to the drum locations indicated above. Thus for k=0, 1, ... , n-1 in that order, if the elements a_{ki} ($o \leq i \leq k$) of the k^{th} row of the matrix A are stored in locations b+i and if location 181L contains n x 2^{-39}, then the subroutine at 158L when entered with J0 kF 50 pF in Q will store those elements correctly on the drum. After routine M23 has been entered m=n or 2n depending upon whether only eigenvalues or both eigenvalues and eigenvectors were computed, and c = b+m. Hence, entering the subroutine at 132L as indicated above will cause the k^{th} eigenvalue to be stored at

b + k or c + k and (if computed) the elements of the k^{th} eigenvector to be stored at n locations beginning at b + n or c + n, respectively. The subroutine at 132L uses only location 0 as temporary storage; the subroutine at 158L uses 0 and 1.

ARITHMETIC CHECK: The arithmetic check described in the write-up of routine M0 is used by this routine. If this test is failed the difference between the new and old value of the sum of the squares of the elements is punched out in sexadecimal form. Then an FF stop from the right hand side of word 112 will occur. This failure can be caused by a drum, memory, or arithmetic error. The FF stop may be by-passed with the white switch.

DATE November 21, 1958
CODED BY C. Farrington
APPROVED BY J. N. Snyder

lgr

LOCATION	ORDER		NOTES	PAGE 1	M23
	00 K(M23)				
0	K5 F				
	42 108L		Set return address		
1	40 4F				
	46 15L				
2	44 53L				
	46 116L				
3	10 20F				
	42 117L				
4	00 3F				
	S5 F				
5	42 183L				
	10 3F				
6	36 7L		5 or J?		
	00 1F		J, double		
7	42 181L				
	44 117L				
8	42 117L				
	42 127L				
9	45 37L				
	44 181L				
10	42 114L				
	42 131L				
11	00 20F				
	46 113L				
12	41 11F				
	26 91L				
13	41 8F		Iteration		
	41 9F				
14	27 87L				
	47 80L				
15	50 F		address = n		
	50 15L				
16	26 132L		To read drum subroutine		
	47 17L				

LOCATION	ORDER	NOTES PAGE 2	
17	J0 F		
	50 17L		
18	26 132L		To read drum subroutine
	L5 F		
19	L4 37L		
	42 23L		
20	42 79L		
	L4 181L		
21	42 22L		
	42 76L		
22	49 12F		
	L5 F		
23	40 4F		
	L5 F		
24	40 1F		
	80 1F		
25	40 3F		
	L3 1F		
26	32 80L		
	50 1F		
27	L5 9F		
	74 1F		
28	L4 8F		
	40 8F		
29	S5 F		
	40 9F		
30	L5 F		
	40 5F		
31	L0 4F		
	40 2F		
32	41 F		
	50 F		
33	L7 3F		
	L2 2F		

LOCATION	ORDER	NOTES PAGE 3
34	36 36L	
	L1 1F	
35	66 2F	
	2L 38L	
36	L1 2F	
	10 1F	
37	66 3F	
	S5 S3	
38	40 7F	
	S5 F	
39	40 6F	
	7J 6F	
40	L4 119L	
	50 40L	
41	26 S2	To R1, Square Root
	40 10F	
42	L7 7F	
	10 1F	
43	66 10F	
	KJ F	
44	36 47L	
	L5 115L	
45	40 1F	
	40 2F	
46	L5 6F	
	22 57L	
47	SJ F	
	50 47L	
48	26 S2	To R1, Square Root
	L5 7F	
49	L0 12F	
	36 54L	
50	L5 7F	
	36 52L	

LOCATION	ORDER	NOTES PAGE 4	M23
51	L1 119L		
	22 52L		
52	19 1F		
	66 10F		
53	S5 S3		
	26 56L		
54	50 6F		
	75 1F		
55	00 1F		
	L0 6F		
56	40 6F		
	66 2F		
57	S5 S3		
	40 7F		
58	L5 114L		
	22 67L		
59	50 2F		
	7J F		
60	40 10F		
	50 7F		
61	7J F		
	50 F		
62	L4 10F		
	40 F		
63	7J 7F		
	40 10F		
64	50 F		
	36 65L		
65	7J 2F		
	L0 10F		
66	40 F		
	L5 61L		
67	F4 118L		
	40 61L		

LOCATION	ORDER	NOTES PAGE 5	M23
68	42 59L		
	42 62L		
69	46 64L		
	46 66L		
70	LO 113L		
	36 59L		
71	50 4F		
	7J 1F		
72	40 10F		
	50 1F		
73	79 5F		
	L4 5F		
74	L4 10F		
	40 10F		
75	50 3F		
	7J 6F		
76	L4 10F		
	40 F	k + c + S3	
77	40 10F		
	L5 4F		
78	L4 5F		
	LO 10F		
79	40 F		
	41 F	k + S3	
80	J2 F		
	50 80L		
81	26 158L	To write drum subroutine	
	L5 17L		
82	L4 118L		
	46 80L		
83	46 17L		
	LO 15L		
84	32 17L		
	36 85L		

LOCATION	ORDER	NOTES PAGE 6	
85	52 F		
	50 85L		
86	26 158L		To write drum subroutine
	L5 15L		
87	L4 118L		
	46 15L		
88	46 85L		
	L4 57L		
89	46 30L		
	46 79L		
90	L0 116L		
	32 14L		
91	41 7F		
	46 95L		
92	L5 114L		
	22 93L		
93	F5 6F		
	40 6F		
94	42 96L		
	42 97L		
95	J0 F		
	50 95L		
96	26 132L		To drum reader
	50 F		
97	L5 12F		
	74 F		
98	L4 7F		
	40 7F		
99	S5 F		
	40 5F		
100	F5 97L		
	42 96L		
101	42 97L		
	L0 117L		

LOCATION	ORDER	NOTES PAGE 7
102	32 96L	
	L1 11F	
103	32 121L	
	L5 95L	
104	L4 118L	
	46 95L	
105	L0 15L	
	36 93L	
106	L5 7F	
	50 11F	
107	40 11F	
	K0 F	
108	36 120L	
	22 F	exit M23
109	L0 8F	
	40 F	
110	L5 183L	
	L2 F	
111	36 13L	
	L5 F	
112	82 40F	
	FF F	
113	LJ F	2n + 3(S3) or n + 2(S3)
	26 13L	
114	7J S3	
	50 F	c = 2n + 3(S3) or n + 2(S3)
115	7L 4095F	
	LL 4095F	
116	85 F	
	00 F	
117	75 12F	
	74 F	(2 or 3) x (n + S3)
118	00 1F	
	00 F	

LOCATION	ORDER	NOTES PAGE 8	
119	20 F		
	00 F		
120	S1 F		
	36 13L		
121	26 184L		
	L5 4F		
122	32 103L		
	L5 117L		
123	42 124L		
	L5 95L		
124	46 129L		
	41 F		
125	F5 124L		
	42 124L		
126	L0 131L		
	L0 181L		
127	32 124L		
	40 F		(2 or 3)(n + S3)
128	F5 127L		
	40 127L		
129	J2 F		
	50 129L		
130	26 158L		To write drum subroutine
	22 103L		
131	N6 129L		
	41 F		2n + 3(S3) or n + 2(S3)
132	K5 F		Read drum subroutine
	42 154L		Set return address
133	32 134L		5 or J?
	L5 114L		J
134	26 135L		
	L5 37L		
135	42 145L		
	L4 181L		

LOCATION	ORDER	NOTES PAGE 9
136	42 156L	
	00 9F	
137	01 10F	
	40 F	Save k
138	J0 155L	
	00 6F	x 64
139	L4 F	+k = 65k
	L4 157L	+ drum read order
140	40 144L	Set to read matrix
	L4 F	+ k
141	40 149L	66k
	L5 145L	
142	L4 F	
	42 150L	
143	L1 F	
	36 149L	
144	85 11F	Read drum
	00 F	
145	50 F	b or c
	40 F	
146	F5 144L	
	40 144L	
147	F5 145L	
	42 145L	
148	L0 150L	
	36 144L	
149	85 11F	
	00 F	66k
150	J0 F	
	40 F	(b or c) + k
151	F5 149L	
	L4 155L	
152	40 149L	
	F5 150L	

LOCATION	ORDER		NOTES PAGE 10	M23
153	42 150L			
	L0 156L			
154	36 149L			
	22 F		Leave drum reader	
155	00 F			
	00 64F			
156	50 F			
	40 F		c+c or c+b	
157	85 11F		Matrix read constant	
	00 S4			
158	K5 F		Write drum subroutine	
	42 180L			
159	32 160L		J or 5?	
	L5 114L			
160	26 161L			
	L5 37L			
161	42 170L			
	L5 170L			
162	L4 181L			
	40 1F			
163	00 9F			
	01 10F			
164	40 F		k	
	J0 181L			
165	00 6F		x 64	
	L4 F		+k = 65k	
166	L4 182L		+ S4	
	40 171L		Set matrix write	
167	L4 F		+ k	
	40 176L		= 66k	
168	L5 170L			
	L4 F			
169	42 175L			
	L1 F			

LOCATION	ORDER	NOTES PAGE 11	M23
170	32 175L		
	L5 F		
171	86 11F		
	00 F		
172	F5 171L		
	40 171L		
173	F5 170L		
	42 170L		
174	L0 175L		
	32 170L		
175	S2 175L		
	L5 F		
176	86 11F		
	00 F	66k	
177	F5 176L		
	L4 155L		
178	40 176L		
	F5 175L		
179	42 175L		
	L0 1F		
180	32 175L		
	22 F	Leave drum writer	
181	00 F		
	00 F	c = 2(n+S3) or (n + S3)	
182	86 11F		
	00 S4		
183	00 F		
	00 F	8(n + S3)	
184	L4 7F		
	26 109L		