

UNIVERSITY OF ILLINOIS  
DIGITAL COMPUTER

ILLINOIS CODE M 9 - 152

**TITLE** Square Root Factorization

**DURATION** Input, output, and tape-handling; computing time is negligible

**TAPES USED**

- 1) Program tape: same for all problems
- 2) Parameter tape: 00 9K  
00 pF 00 pF  
00 1F 00 1F  
00 (P+S)F 00 (P+S)F  
00 (P)F 00 (P)F  
26 987N

where p = number of variables  
P = number of factors previously extracted  
S = number of factors to be extracted on this run

- 3) Column sums tape: Column sums are punched in the form required for D.N.I., scaled down by two decimal places; an 'N' follows the last column sum; it is assumed that each column sum is reduced by unity before being punched.<sup>1</sup>
- 4) Problem tape: The data is punched in the form required for the D.N.I., scaled down by  $10^{-2}$ ; an 'N' is punched at the end of each column of correlations; the columns should be numbered, starting with zero to facilitate the locating of columns later.

**PROBLEM** This program uses the square root method to find a factorization of the given correlation matrix.<sup>2</sup>

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<sup>1</sup> The basis for this assumption is discussed in Wrigley, C. F., and McQuitty, L. L. The square root method of factor analysis: a re-examination and a shortened procedure. Unpublished manuscript on file at the University of Illinois Library, p. 8.

<sup>2</sup> Op. cit. This paper gives a full discussion of the method, its advantages, its disadvantages, and the detailed computational procedure.

OPERATING PROCEDURE

- 1) To begin a run tapes are fed to the computer in the following order: Program, Parameter, Column sums.
- 2) The computer then punches ten hexadecimal characters in the form 000AB000AB

The two place hexadecimal number AB converted to decimal form, say, CD, indicates the column to be "pivoted" on. The computer stops after this number is punched.

- 3) The column CD is read in.
- 4) The computer then punches a factor followed by another ten hexadecimal digits and stops.
- 5) Read in the previous factor (s). The computer stops after each factor.
- 6) Read in the column of correlations indicated by the set of ten hexadecimal characters just punched. Steps 4, 5, 6 are repeated until all factors to be calculated on the run have been punched. After the last factor the computer automatically punches the column sums of the residual correlation matrix (in place of another set of hexadecimal digits) and then stops on an OF order. To begin a second, or later, run, the master tape, a new parameter tape, and the just-obtained column-sums tape are read in. Operation then proceeds to cycle through steps 4, 5, 6 as on first run. Be sure to read in all previous factors before each indicated column of correlations.

COMMENTS

The capacity of the routine without magnetic storage

drum is 250 variables. All factors may be extracted if desired. Six factors can usually be computed in a half-hour machine session.

At the end of each session the tapes of the factors that have been obtained during the session should be consolidated onto one tape. This reduces tape-handling on the next computer session. In making this consolidation one should be careful not to include the pivot numbers that have been punched at the end of each factor tape on the consolidated tape.

A pivot may be made on a column other than the one indicated by the computer. The steps which are necessary to accomplish this are as follows:

a) Disregard the ten sexadecimal digits which the computer has just printed (step 4 of the regular procedure).

b) Place a tape in the reader which has the following twenty sexadecimal digits

26 014 00 000 00 0(PQ) 00 0(PQ)

where (PQ) is the two-place sexadecimal equivalent of the number of the column that you want to pivot on next. Remember that the columns are numbered from zero.

c) Have the computer operator clear  $R_3$  and the Order Counter and then read in the tape described in (b) with the white switch.

d) Proceed with step 5 and continue as on regular pivot.

#### CHECKING ROUTINES

There are several checks built into the above program. As long as there are no errors, these codes do not affect the operating procedure. At the same time the user must be prepared to act if any of the deviations from normal operating sequence that are outlined below do occur. The deviations, the mistake each indicates, and the courses of action to be taken are listed below.

Program Tape Read-in. A ten-place sexadecimal number and some line-feed characters are punched just before

the last order on the Program Tape is read. This output indicates an error in the reading of the Program Tape.

Action: Have the Illiac operator clear the computer and read the Program Tape in again.

Division Hang-up. The computer stops on an improper division at memory location 04N. This will occur only when the diagonal entry is negative in the column of residual correlations being used. A residual diagonal entry can be negative in case of either a non-gramian correlation matrix (e.g., a matrix of tetrachoric correlations) or a mistake in the punching of an earlier factor.

Action: Punch a tape as follows:

spaces 26OK7 00 000 spaces

Place this tape in the reader, clear  $R_3$  and counter, and read in on white switch. The residual column sums will be punched, ending the computer run.

Reduction to residuals. The computer punches (a) the residual column sum as just calculated, (b) 10 line-feed characters, (c) the residual column sum as reduced from the original column sum, and then the computer stops on the order "30 05N." This occurs instead of the punching of a new factor immediately after the pivot column of correlations has been read. One of the following mistakes is indicated:

- (1) an error in reading a factor or correlation tape
- (2) not using the indicated column of correlations
- (3) an error in the punching of an earlier factor
- (4) a computer error in reduction of column sums

Action: Repeat the pivot (i.e., repeat steps 5 and 6) as if the pivot number had just been punched. If this check fails a second time on the same pivot, the error most likely occurred on an earlier pivot.

Punch a tape as follows:

spaces 26 OK7 00 000 spaces

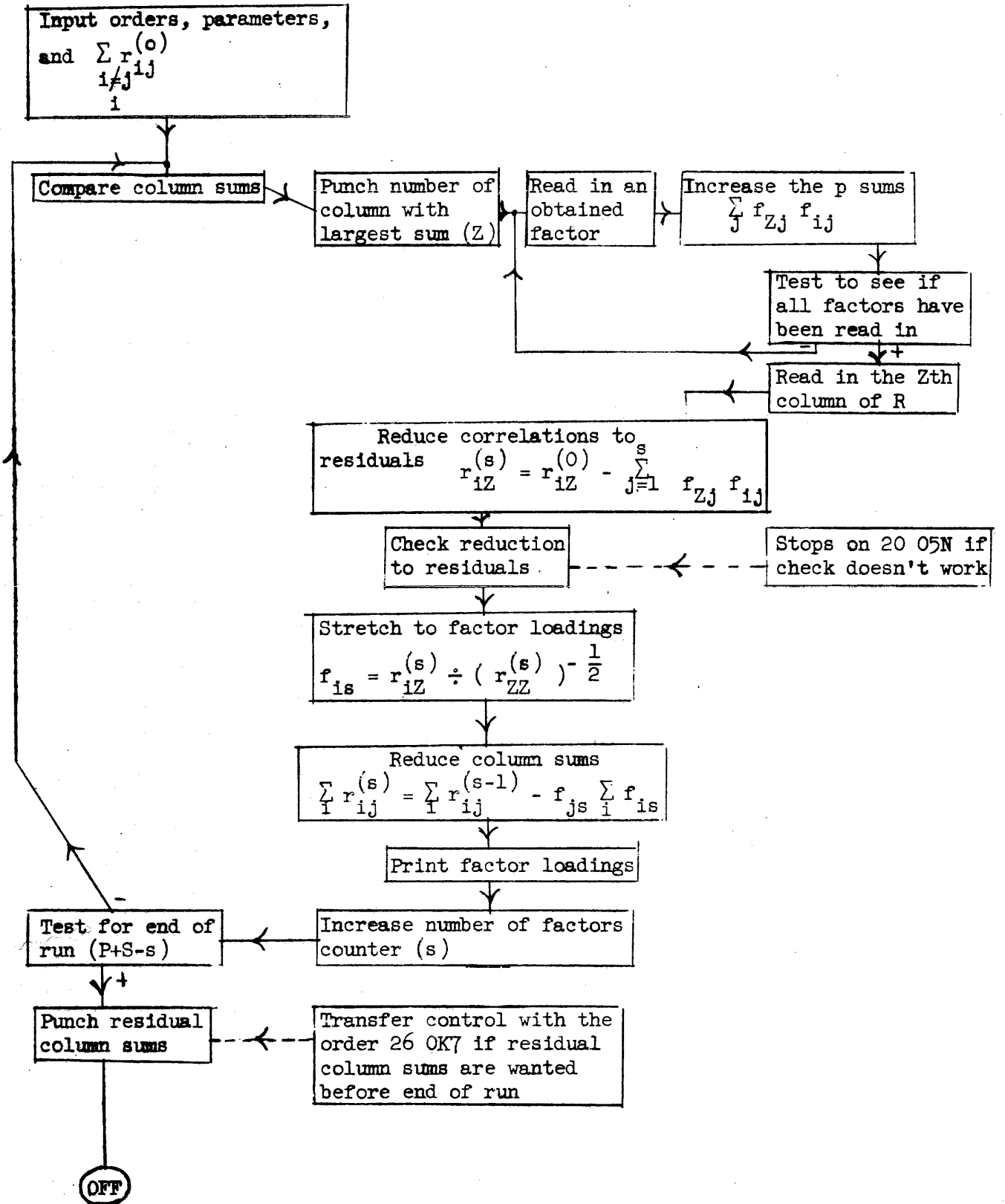
Place this tape in the reader. Clear  $R_3$  and the counter,

and read the tape in on the white switch. The residual column sums will be punched and the computer will stop on an OF order, ending the computer run.

Coded by R. J. Twery

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Flow Chart for Square Root  
Factor Analysis



LOCATION	ORDER		NOTES	PAGE 1
	Code X 1		Decimal Order Input	
	00 3K			
	00 F			
	00 6F		Print constant	
	00 6K			
	00 250F		Beginning of storage	
	00 250F			
	00 22K	from 95F, 100F		
	Code N 3		Decimal Number Sequence Input	
	00 43K			
	Code P 6	from 153F, 170F	Single Column Print	
		216F, 218F		
	00 177K			
0	40 ( )F	by 994F		
		from 994F		
	50 177F		Read in column sums	
1	26 22F			
	26 80F			
	00 80K			
0	41 14F			
	L5 10F			
1	40 4F	from 9		
	L4 8F			
2	46 83F			
	L5 ( )F	by 993F 5, 12		
3	L0 ( )F	by 2		
	36 86F			
4	L5 4F			
	40 14F		Compare residual column sums	
5	L4 8F			
	42 2L			
6	00 63F	from 3		
	L5 4F			
7	L4 10F			
	40 4F			
8	L0 9F			
	36 90F			

LOCATION	ORDER		NOTES	PAGE 2 M 9
9	L5 4F 26 81F			
10	26 17F 00 F	from 8		
11	82 12F L5 8F	from 19F		
12	42 82F L3 12F	from 222F from 21F	Print number of column with highest sum	
13	34 94F 22 96F			
14	40 ( )F 50 94F	by 988 from 13, 31		
15	26 22F 22 112F		Read in the pivot column of correlations	
16	32 32L 41 5F	(waste) from 13		
17	L5 6F I4 14F			
18	46 24L 24 19L		Read in factors	
19	40 ( )F 50 19L	by 987F from 18, 32		
20	26 22F 41 4F			
21	I4 6F 42 24L	from 59F		
22	L0 6F I4 13F			
23	46 25L 42 25L			
24	50 ( )F 7J ( )F	by 18 by 21	Accumulate sums of cross-products of products of factor loadings to be subtracted from correlations	
25	I4 ( )F 40 ( )F	by 23 by 23		



LOCATION	ORDER		NOTES	PAGE 3 M 9
26	L5 4F L4 10F			
27	L0 9F 32 29L			
28	L4 9F 40 4F			
29	26 57F L5 5F	from 27		
30	L4 10F 40 5F			
31	L0 12F 34 94F			
32	24 19L 41 4F	from 15		
33	L4 13F 42 37L			
34	L0 13F L4 7F			
35	46 37L 46 38L			
36	00 63F 00 63F			
37	L5 ( )F L0 ( )F	by 35 by 33	Make subtractions, reducing correlations to residuals	
38	40 ( )F L5 4F	by 35		
39	L4 10F L0 9F			
40	36 200F L4 9F		Go to subroutine checking reduction to residuals	
41	40 4F 26 33L			
42	L5 7F L4 14F	from 214F		
43	46 44L 41 15F			

LOCATION	ORDER		NOTES
44	L5 ( )F 50 44L	by 43	Find square root of diagonal residual
45	26 70F 40 5F		
46	41 4F L4 7F	from 55	
47	46 48L 42 49L		
48	L5 ( )F 66 5F	by 47	
49	S5 F 40 ( )F	by 47	Stretch residuals to factor loadings and form sum of loadings
50	40 F 50 F		
51	7J 16F L4 15F		
52	40 15F L5 4F		
53	L4 10F 40 4F		
54	L0 9F 32 59L		
55	L4 9F 22 46L		
56	00 1F 00 1F		
57	L5 14F L4 8F		
58	46 59L 00 1F		
59	L5 ( )F 92 171F	by 58 from 54	
60	41 4F L4 7F		
61	46 64L L0 7F		

LOCATION	ORDER		NOTES	PAGE 5
62	L4 8F			
	42 65L			
63	42 66L			
	50 15F			
64	7J ( )F	by 61		
	66 16F			
65	41 F			
	L5 ( )F	by 62		
66	S0 F			
	40 ( )F	by 63		
67	L5 4F			
	L4 10F			
68	L0 9F			
	36 190F			
69	L4 9F			
	40 4F			
70	22 61F			
	41 4F	from 198F		
71	L4 7F	from 76		
	46 72L			
72	L5 ( )F	by 71		
	50 72L			
73	26 43F			
	L5 4F		Print factor loadings	
74	L4 10F			
	L0 9F			
75	36 77L			
	L4 9F			
76	40 4F			
	26 71L			
77	92 770F	from 75		
	92 999F		Print N	
78	L5 12F			
	L4 10F		Print spaces	
			Increase number of factors counter	

LOCATION	ORDER		NOTES	PAGE 5
79	40 12F L0 11F		Test for end of run	
80	36 87L 41 14F			
81	L4 13F 46 82L	from 86		
82	41 ( )F 22 59F	by 81		
83	L5 14F L4 10F	from 61F		
84	40 14F L0 9F		Clear locations where sums of cross-products are accumulated	
85	32 86L L4 9F			
86	26 81L 26 L	from 85		
87	41 4F L4 8F	from 80 from 93		
88	46 89L 41 F			
89	L5 ( )F 50 89L	by 88		
90	26 43F L5 4F			
91	L4 10F L0 9F		Print residual column sums	
92	36 94L L4 9F			
93	40 4F 22 87L			
94	92 770F OF F 00 70K	from 92	End	
	Code R 1 00 20K	from 125F	Square Root Routine	

LOCATION	ORDER		NOTES	PAGE 6
0	81 40F 40 14F		To make a forced pivot	
1	22 92F 00 F 00 17K			
0	92 770K L5 14F	from 90F		
1	40 F 82 40F		Print pivot number	
2	92 770F 22 91F 00 57K			
0	00 63F 00 63F	from 109F		
1	00 63F L5 4F			
2	26 101F 00 63F	from 162F		
3	00 63F 00 63F		Reduce r. a. r.	
4	26 163F 00 63F	from 150F		
5	00 63F 00 63F			
6	L5 4F 22 140F 00 190K			
0	41 5F L5 7F	from 148F from 9		
1	L4 5F 42 4L			
2	46 5L L5 8F			
3	L4 5F 42 5L			
4	46 6L 50 ( )F	by 1		

LOCATION	ORDER		NOTES	PAGE 7	
5	7J ( )F L4 ( )F	by 1 by 3	Adjust for absence of diagonal entry in column sums		
6	40 ( )F L5 5F	by 4			
7	L4 10F 40 5F				
8	L0 9F 32 150F				
9	22 L 00 F 00 16K				
10	00 F 00 100 000 000 000J 00 987K		Scaling constant		
0	L5 6F 46 99F				
1	40 F 46 94F				
2	40 7F 42 57F	waste			
3	46 175F L4 9F	waste			
4	40 13F L4 9F		Form dependent parameters		
5	40 8F 46 57F	waste			
6	46 58F 42 82F	waste			
7	46 177F 24 177F				
0	00 200K 41 4F 41 998F	from 120F			

LOCATION	ORDER	NOTES	PAGE 8
1	L4 7F 46 2L	from 7	
2	L5 ( )F L4 998F	by 1	
3	40 998F 00 60F		
4	00 60F L5 4F		
5	L4 10F L0 9F		
6	36 8L L4 9F		
7	40 4F 26 1L		
8	L5 6F L4 14F	from 6	
9	46 12L L5 8F		Check reduction to residuals
10	L4 14F 42 12L		
11	46 17L L5 998F		
12	L0 ( )F L0 ( )F	by 9 by 10	
13	40 1F 19 13F		
14	L2 1F 36 122F		
15	L5 998F 50 15L		
16	26 43F 92 189F		
17	L5 ( )F 50 17L	by 11	
18	26 43F 20 92F		

LOCATION	ORDER		NOTES	PAGE 9
0	00 218K 26 43F 41 179F			
1	14 13F	from 5		
2	46 2L 41 ( )F 15 179F	by 1		
3	14 10F 40 179F		Prepare for re-run of pivot in case of error	
4	10 9F 30 92F			
5	14 9F 26 1L 00 975K			
	Code X 6		Memory Sum Check	
	9500N 94NNK		Sum Check Number	
	24 999N		Control to D. O. I.	