UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE M 16 - 187

TITLE — Matrix Multiplication When the Common Dimension Is Large

TYPE — Complete program

DURATION — $n^2 (25d + .2p) + n (31.5p + 25d)$ milliseconds where

p = the common dimension of the matrices

n = the number of rows in the premultiplier plus the number of columns in the post-multiplier

d = the number of characters in each element of the product matrix

DESCRIPTION — Forms the matrix product ${}_jC_k = {}_jA_p \, {}_pB_k$ and then punches the matrix ${}_jC_k$ by columns, an N following the last element of each column.

CAPACITY — $j + k \leq 40$; no limit on p

INSTRUCTIONS TO ILLIAC OPERATOR

1. Program tape
2. Parameter tape Black up
3. Data tape Black up
4. Continue input of program tape Black up

Ends on OF order from 018.

PREPARATION OF TAPES — The following parameter tape is punched

p S n N d F j L

where p, n, d, and j are as defined above.

Before the elements of the matrices A and B are punched on the data tape they must be scaled by a constant which will make (a) all of the elements less than unity and (b) each of the row sums of squares of the premultiplier A and the column sums of squares of the postmultiplier B less than ten. (The program scales each of the elements of the product matrix by 10^{-1} while forming the product.) The scaled elements are arranged in the form of an n by p matrix composed

of the scaled premultiplier followed by the transpose of the scaled postmultiplier $\begin{bmatrix} A \\ \cdots \\ B^T \end{bmatrix}$. This composite matrix is punched by columns with an N after the last element in each of the columns.

EXAMPLE

Suppose we want to find the product matrix where

$$ {}_3A_2 = \begin{bmatrix} .5 & .6 \\ 1.0 & .7 \\ .3 & .2 \end{bmatrix}, \quad {}_2B_4 = \begin{bmatrix} 7 & 3 & 1 & 6 \\ 4 & 5 & 2 & 9 \end{bmatrix}. $$

In this case we have to scale the elements of A and B by 10^{-1}. The product matrix will be scaled by the square of this, 10^{-2}, and an additional 10^{-1} which is added by the code. Since there will be three scaling zeros, to get answers to three decimal places we will have to take d equal to 6.

The parameter tape is punched as follows:

spaces 2 S 7 N 6 F 3 L spaces.

The data tape is punched as follows:

+05 +10 +03 +7 +3 +1 +6 N
+06 +07 +02 +4 +5 +2 +9 N

The printed results will be:

+000590
+000980
+000290
N

+000450
+000650
+000190
N

+000170
+000240
+000070
N

+000840
+001230
+000360
N

DATE July 7, 1955
CODED BY R. J. Twarog
APPROVED BY J. P. Nash

LOCATION	ORDER		NOTES	PAGE 1
	Routine X 1 - 18		Decimal Order Input	
	00 115K			
0	L5 28F			
	42 1L	from 3		
1	41 8F			
	41 ( )F	by 0		
2	F5 1L		— Clear numerical storage locations	
	42 1L			
3	L0 111F			
	32 1L			
4	22 35F			
	00 F			
	00 10K			
0	L5 2F	from 13		
	L0 5F		— Form j	
1	40 120F			
	26 19L			
2	00 F			
	00 F			
3	00 F			
	00 F			
4	41 3F			
	41 F			
5	81 4F	from 9		
	L0 112F			
6	32 9L			
	L4 112F			
7	50 F			
	74 112F			
8	S5 F			
	40 F		— Input parameters	
9	26 5L			
	42 10L	from 6		
10	L5 F			
	40 ( )F	by 9		

LOCATION	ORDER		NOTES PAGE 2
11	F5 3F		
	40 3F		
12	L0 113F		
	32 4L		
13	92 191F		
	26 0L		
14	00 F		
	00 F		
15	00 ( )F	by 21	
	00 ( )F	by 21	Dependent parameters storage
16	00 ( )F	by 22	
	00 ( )F	by 22	
17	80 F		
	00 ( )F	by 23	
18	00 121F		Beginning of numerical storage
	00 121F		
19	50 2F	from 1	
	00 59F		
20	L4 2F		
	40 F		
21	L4 18L		
	40 15L		Form dependent parameters
22	L4 F		
	40 16L		
23	L5 2F		
	42 17L		
24	L5 1F		
	40 9F		
25	24 999F		Stop to read in second part of master tape
	41 3F		
26	40 121F		
	50 26L		
27	26 85F		
	L5 18L		

LOCATION	ORDER	NOTES	PAGE 3
28	42 30L		
	42 31L		
29	L5 15L		
	42 32L		
30	46 32L		
	50 ( )F		
31	7J 7F		
	40 ( )F		
32	L4 ( )F		
	40 ( )F		
33	F5 30L		
	42 30L		
34	42 31L		
	L5 32L		
35	L4 114F		
	40 32L		
36	F5 3F		
	40 3F		
37	L0 17L		
	32 30L		
38	22 38L		
	40 3F		
39	L5 18L		
	46 46L		
40	L5 16L	-Form sums of products	
	42 47L		
41	46 47L		
	22 43L		
42	L5 114F		
	L4 46L		
43	46 46L		
	L5 18L		
44	42 46L		
	F5 3F		

LOCATION	ORDER	NOTES	PAGE 4
45	40 3F		
	41 2F		
46	50 ( )F		
	7J ( )F		
47	L4 ( )F		
	40 ( )F		
48	F5 46L		
	42 46L		
49	L5 114F		
	L4 47L		
50	40 47L		
	F5 2F		
51	40 2F		
	L0 3F		
52	36 46L		
	L4 3F		
53	L0 17L		
	36 42L		
54	F4 8F		
	42 8F		
55	L0 9F		
	32 25L	Test for last column of partitioned matrix	
56	92 63F		
	26 111F	Go to print out C	
	00 67K		
	Routine P 2 - 52	Print (A) with or without Sign to n Places	
	00 85K		
	Routine N 2 - 88	Input a Sequence of Decimal Fractions	
	00 111K		
0	N1 8F		
	41 1023F		
1	00 F		
	00 10F		

LOCATION	ORDER	NOTES	PAGE 5
2	80 F	Constants	
	00 4F		
3	00 1F		
	00 1F		
	24 14N	Stop to read in parameters	
		Beginning of Part II of Program tape	
	00 111K		
0	41 31F		
	50 5F		
1	75 5F		
	26 10F		
	00 10K		
0	L5 5F		
	S4 F		
1	10 1F		
	36 2L		
2	L4 26F		
	42 3L		
3	41 30F		
	L5 ( )F		
4	50 S4		
	50 4L		
5	26 67F	Print the product matrix by columns	
	F5 30F		
6	40 31F		
	L0 5F		
7	36 9L		
	F4 3L		
8	42 3L		
	22 3L		
9	92 770F	Punch N	
	92 161F	Punch l.f.'s	
10	F5 31F		
	40 31F		

LOCATION	ORDER	NOTES
11	L0 120F	
	36 14L	
12	F5 31F	
	L4 3L	
13	42 3L	
	26 3L	
14	92 191F	Punch l.f.'s
	0F F	End
	00 7K	
	00 F	Scaling constant
	00 3162 2776 6500 J	
	24 115N	Go to clear numerical storage