UNIVERSITY OF ILLINOIS
DIGITAL COMPUTER LABORATORY
STATISTICAL LIBRARY

KSL 2.03 - 313

TITLE: Means, standard deviations, third and fourth moments about the means

TYPE: Entire program

CAPACITY: $v \leq 145$ where v is the number of variables; no practical limit on q, the sample size

DESCRIPTION: For each of a set of v variables, this routine will calculate the mean, the standard deviation, and the third and fourth moments about the mean. Much can be learned about the sample distribution from these four statistics. The mean is a measure of central tendency. The standard deviation is a measure of the dispersion of the distribution. The third moment indicates the amount and direction of skewness. The fourth moment is a measure of the kurtosis of the distribution.
The routine will read data either in the form of signed fractions or in the form of unsigned single digits (0, 1, ... 9). If means and standard deviations only are desired, computer time can be saved by suppressing the calculation of the third and fourth moments. In any event, this routine is preferred over K-17 (also K-8, KSL 2.01) for the $v(v + 1)/2$ cross-products are not calculated. For a large v, the saving in computer time may exceed 90%.

METHOD OF USE: 1. Read master program. Routine stops on 34084. A stop on FF003 indicates a sum check failure.
2. Read parameter tape. Routine stops on 2404N.
3. Read data tape. Routine ends on 24084.
Another problem can be begun by reading a new parameter tape.

PARAMETER TAPE PREPARATION:
There are four parameters (r, d, q, and p) for each problem. The parameters are punched on tape with sexadecimal terminating symbols as follows:
r N d J q F p L.

To read data consisting of single unsigned digits, set r = 0.
To read data consisting of signed fractions, set r = 1.
Set d equal to the number of decimal places desired in the means and standard deviations. The third and fourth moments are always punched to 10 decimal places.
q is the sample size or number of rows of data.
To suppress the third and fourth moments, set p = 0. To calculate and punch the moments, set p = 1.

DATA TAPE PREPARATION:

The data tape is punched by rows and consists of q rows of v variables each. Each row of the data tape is terminated by an N symbol. If an F follows a row instead of an N, the computer will stop and another section of the data tape can be inserted in the reader. By raising the black switch, the reading of the data tape is resumed.
When r = 0, an element of a row must be a single unsigned digit (0, 1, ... 9). If these are considered as integers, then the scaling on each will be 10^{-1}.
When r = 1, each row element must be punched as a signed fraction with any number from zero through 12 decimal digits.

SCALING IN THE RESULTS:

If the scaling on variable j is 10^{-p_j}, then in the results the means and standard deviations will also be scaled by 10^{-p_j}. The third moment will be scaled by 10^{-3p_j} and the fourth moment will be scaled by 10^{-4p_j}.

THE FORM OF THE RESULTS:

The means and standard deviations are printed out in parallel columns terminated by an N.
If moments are also calculated, these will follow in a second set of parallel columns terminated by an N.

A THREE-VARIABLE EXAMPLE:

Parameter tape

0N3J6F1L

Data Tape

232N 342N 151N 060N 470N 371N

Results

$\bar{X}_j$'s $\left\{\begin{matrix} +217 & +134 \\ +533 & +149 \\ +100 & +082 \end{matrix}\right\}_N$ s_j's

M_j^3's $\left\{\begin{matrix} -0007407407 & +0006025462 \\ -0009259259 & +0008074074 \\ +0000000000 & +0000666666 \end{matrix}\right\}_N$ M_j^4's

DURATION:

sec. (d = decimals, q = samples, v = variables)

Read master tape	30
Read data in	$.004(d + 1)vq$
Calculation	$.003vq$
Punch	$.035v(d + 1)$ means and standard deviations
	$.4v$ 3rd and 4th mom

FORMULAS: Let X_{ij} be the i^{th} observation ($i = 1, 2, \ldots q$) on the j^{th} variable ($j = 1, 2, \ldots v$). All summations below are over i from 1 through q.

Mean (central tendency)

$$\bar{X}_j = \frac{1}{q} \Sigma X_{ij}$$

Standard deviation (dispersion)

$$s_j = (M_j^2)^{1/2} = \left[\frac{1}{q} \Sigma (X_{ij} - \bar{X}_j)^2\right]^{1/2}$$

Third moment (skewness)

$$M_j^3 = \frac{1}{q} \Sigma (X_{ij} - \bar{X}_j)^3$$

Fourth moment (kurtosis)

$$M_j^4 = \frac{1}{q} \Sigma (X_{ij} - \bar{X}_j)^4$$

If the distribution is symmetric, $M_j^3 = 0$.
When M_j^3 is negative, the distribution is skewed to the left; when positive, the distribtuion is skewed to the right. For purposes of comparison, use
$A_j = M_j^3/s_j^3$.

For a flat distribution, M_j^4 will tend to be large; for a steep distribution, M_j^4 will tend to be small. For purposes of comparison, use $B_j = M_j^4/s_j^4$.

For a normal curve, A_j will be equal to zero and B_j will be equal to three.

NOTES:

1. The routine compares the number of variables in subsequent rows with the number in the first row. If these do not agree for any row, the computer will stop on FF000 at location 0S1.
2. If the variance, s_j^2, is negative due to rounding errors and outside the tolerance limit (10^{-10}), the machine will stop on FF0S6 at location 0N6. If a negative variance is within the tolerance limit, it is set to zero and the calculation is continued.

DATE October 12, 1960
PROGRAMMED BY Freda Fischer
APPROVED BY J N Snyder

nj

LOCATION			ORDER	NOTES
Abs.	Rel.	Sym.		
			00 11K	
11	0	(2)	00F 002F	
		(10)	00F 0010F	
		(145)	00145F 00145F	
		(299)	FF299F 00299F	
		(CL)	50F 411024F	
16		(J)	00F 000000000000100J	
		(D2)	001F 00F	
		(D6)	6F299F 00299F	
		(Z)	00F 00F	
20	9	(P16)	00K	
76		(N12)	00K	
115		(R1)	00K	
			00K	Read in digits
124	0	(N13)	K5F 427L	
			465L 50(Z)	
			814F L0(10)	
			327L L4(10)	
			66(10) S5F	
129			40F L55L	
			L4(D2) 465L	
131	7		221L 22F	
			00K	
132	0	(L)	193F 401F	Read parameters
			92575F 41F	
		(L1)	814F L0(10)	
			32(L2) L4(10)	
			50F 74(10)	
			S5F 40F	
138		(L2)	26(L1) 421(L2)	
			L5F 40F	
			L51F L41F	
141	9		401F 321(L)	

Abs.	Rel.	Sym.		
142	10		L44F 406F	
			50(Z) 1938F	
			664F S5F	
			408F 50(Z)	
			L55F 0039F	
			405F L52F	
			0039F 402F	
			L53F 0020F	
			46(P1) 46(P2)	
			92139F L52F	
152	20		36(L3) 26(L4)	
		(L3)	50299F 50(L3)	Read first row digits
			24(N13) L0(2)	
			L0(2) 343(L3)	
			L55(N13) 263(L4)	
		(L4)	50299F 50(L4)	Read fractions
			24(N12) L0(2)	
			302(L4) L521(N12)	
			409F 1020F	
			L0(299) 4210F	Number of variables
162	30		L5(299) L4(145)	
			42(L5) 92707F	
		(L5)	J0F 41F	
			F5(L5) 42(L5)	
			L0(CL) 32(L5)	
			417F 26(L8)	
		(L6)	50299F 50(L6)	
			26(N13) L0(2)	
			L0(2) 343(L6)	
			L55(N13) 263(L7)	
172	40	(L7)	50299F 50(L7)	
			26(N12) L0(2)	
			302(L7) L521(N12)	
175	43		L09F 40F	

LOCATION			ORDER	NOTES	PAGE 3	KSL 2.03
Abs.	Rel.	Sym.				
176	44		L3F 36(L8)			
		(T)	FFF 00F	incorrect number variables R.H.A. has tally		
		(L8)	L5(299) 40(T)			
			42(D6) L5(T)			
			46(L11) 463(L11)			
			464(L11) 469(L11)			
182	50		L4(145) 421(L11)			
			462(L11) L4(145)			
			424(L11) 465(L11)			
			L4(145) 427(L11)			
			468(L11) L4(145)			
			429(L11) 46(L9)			
		(L11)	50F 7J8F			
			40F L4F			
			40F 50F			
			7JF 40F			
192	60		50F L4F			
			40F L55F	Store $\Sigma \frac{x^2}{q} = \nu_2$		
			32(L9) 7JF			
			40F L4F			
			40F 50F	Store $\Sigma \frac{x^3}{q} = \nu_3$		
			7JF L4F			
		(L9)	40F F5(T)	Store $\Sigma \frac{x^4}{q} = \nu_4$		
			L4(D2) 40(T)			
			46(D6) L0(D6)			
			L010F 321(L8)			
202	70	(L10)	F57F 427F			
			L06F 322(L10)			
			26(S) L52F			
			36(L6) 26(L7)			
			00K			
206	0	(-)	L5(299) L4(145)	Compute M_j^3 and M_j^4		
			40(T) 42(D6)			
208	2		L5(T) 461(S4)			

LOCATION			ORDER	NOTES
Abs.	Rel.	Sym.		
209	3		463(S4) 423(S4)	
			4213(S4) 4216(S4)	
			L4(145) 424(S4)	
			4614(S4) 46(S1)	
			42(S2) 421(S3)	
			L4(145) 421(S4)	
			4218(S4) 4619(S4)	
216	10		L4(145) 4211(S4)	
			4613(S4) 26(S4)	
		(S4)	L55F 36(S1)	
			50F 75F	$M_j^4 = \nu_4 - 4\bar{x}\,\nu_3 + 6\bar{x}^2\,\nu_2 - 3\bar{x}^4$
			0020F 402F	
			50F 7JF	
			40F 50F	
			75F 001F	
			401F 001F	
			L41F 403F	
226	20		50F 75F	
			401F 001F	
			L41F 404F	
			L53F L4F	
			L02F L04F	
			40F 50F	Store M^4
			75F 401F	$M_j^3 = \nu_3 - 3\bar{x}\,\nu_2 + 2\bar{x}^3$
			001F	
			L41F	
			402F 50F	
			75F 001F	
236	30		L02F L4F	
			40F 26(S1)	Store M_j^3
		(S1)	L5F L0F	$(\nu_2 - \frac{j}{x^2})$
			401F 41F	
			L51F 32(S3)	
241	35		L4(J) 32(S2)	

LOCATION			ORDER	NOTES
Abs.	Rel.	Sym.		
242	36	(S2)	FF2F 41F	
		(S3)	262(S3) 50(S3)	
			26(R1) 40F	Store s_j
			F5(T) L4(D2)	
246	40		40(T) 46(D6)	
			L0(D6) L010F	
248	42		362(S) L5(299)	
			00K	
249	0	(P)	L4(145) 40(T)	Print routine
			42(D6) L5(T)	
			423(P) L4(145)	
			422(P1) L5F	
		(P1)	5012F 50(P1)	
			26(P16) 92963F	
			J0F L5F	
		(P2)	5012F 50(P2)	
			26(P16) 92131F	
			92515F F5(T)	
259	10		L4(D2) 40(T)	
			46(D6) L0(D6)	
			L010F 321(P)	
			92770F 92535F	
			L55F 32(P5)	
		(PR)	92147F L5(299)	
			L4(145) L4(145)	
			L4(145) 40(T)	
			42(D6) L5(T)	
			425(PR) L4(145)	
269	20		422(P3) L5F	
		(P3)	5010F 50(P3)	
			26(P16) 92963F	
			J0F L5F	
		(P4)	5010F 50(P4)	
274	25		26(P16) 92131F	

Abs.	Rel.	Sym.	ORDER	NOTES
275	26		92515F F5(T)	
			L4(D2) 40(T)	
			46(D6) L0(D6)	
			L010F 323(PR)	
279	30	(P5)	92770F 92575F	
280	31		92131F 24(L)	
			OOK	Sum check
281	0		L3F 34(L)	
			FF3F 26(L)	
283	2		F72889F 211040F	
			26L 261N	